Dissipative Engineering of Open Quantum Systems

Thesis Proposal, University of Pennsylvania

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21 May 2024

Outline

1. When are quantum systems open?
2. Example: dissipative quantum anharmonic oscillator
3. Response theory for open systems
   - Applications to spin chains
4. Other open systems and future directions

Open Systems: Examples

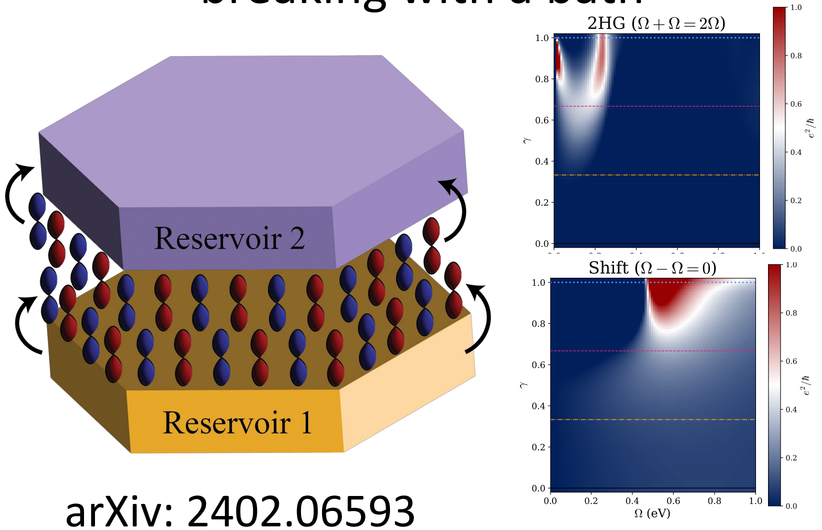
Bosons/Photons

	Closed	Open
Classical	LC oscillator	Parametric amplifier; laser
Quantum	Quantum harmonic oscillator	Leaky cavity ; qubit with dephasing

Fermions/Electrons

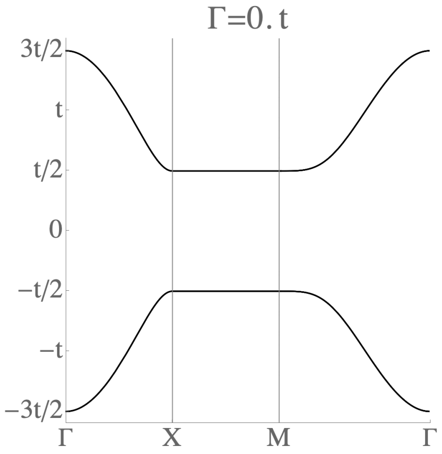
	Closed	Open
Classical	Cyclotron orbits	Ohm's law systems
Quantum	Meisner effect ; single-particle Z_2/Chern topology	System with leads; ex. Non-equilibrium GFs

Centrosymmetry breaking with a bath

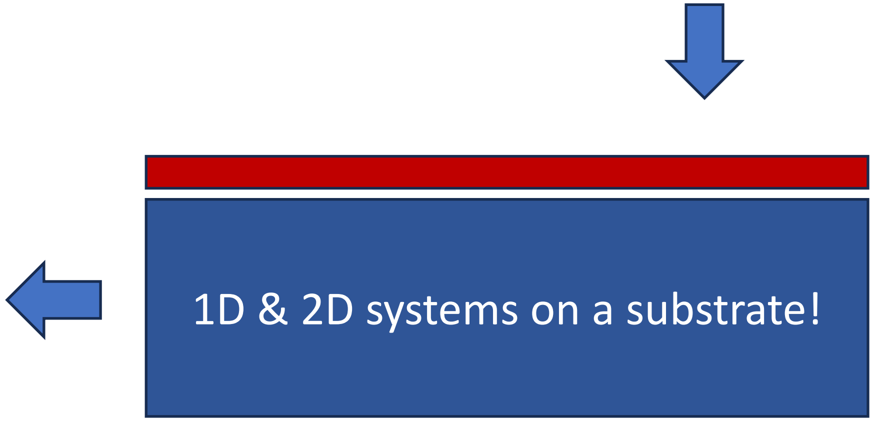


arXiv: 2402.06593

Band Flattening in a Chern insulator



PRB 106, 161109 (2022)



Open Systems: Formulation

Closed Systems

- Hermitian
- Real eigenvalues
- Orthogonal eigenstates
- Fermi-Dirac and Bose-Einstein distributions

Open Systems

- Non-Hermitian
- Complex eigenvalues
- Bi-orthogonal eigenstates
- Distribution function given by G^K
  - In Lindblad formalism this does not recover FD and BE distribution functions in the limit of weak dissipation

Classical Systems

- Waves: amplitudes and phases
  - May have unbounded gain/loss
- Maxwell-Boltzmann dist. is normalized
- Classical correlations
- Im(Eigenvalues) unrestricted

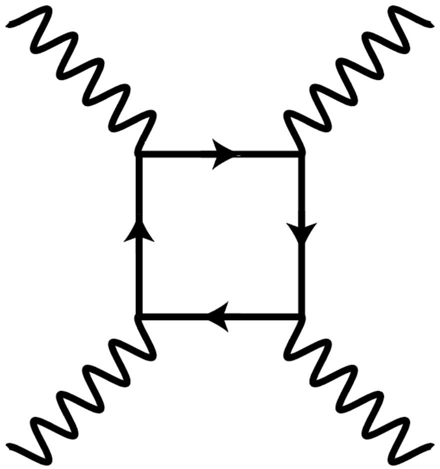
Quantum Systems

- Wavefunctions: normalized
- Mixed states: normalization of trace—probabilities *must* sum to one
- Entanglement
- Im(Eigenvalues) < 0

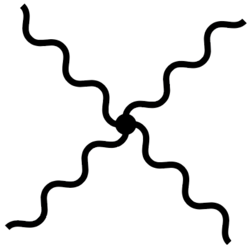
EXAMPLE: DISSIPATIVE QUANTUM ANHARMONIC OSCILLATOR

A minimal example system that provides introduction to the relevant formalism

Quantum Anharmonic Oscillator

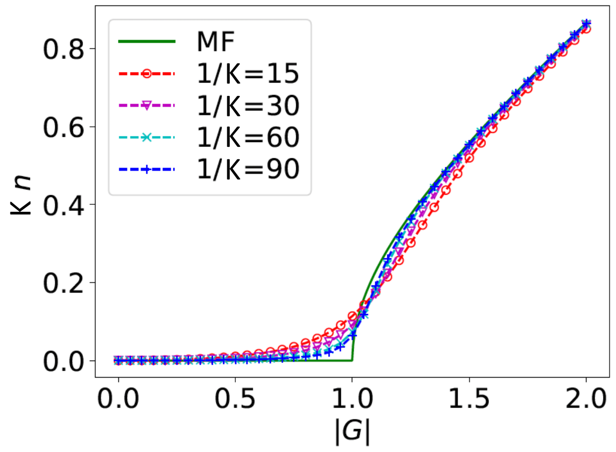


- In free space light hardly ever scatters off light
- In engineered systems such as capacitively coupled waveguides photons can strongly interact
- Describe using an anharmonic (Kerr) oscillator Hamiltonian



$$\hat{H} = \omega_0 \hat{a}^\dagger \hat{a} + \frac{1}{2} K \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a}$$

- Consider single photon losses $\hat{J} = \gamma \hat{a}$ too
- Relevant for device modeling, ex. transmon qubits
- Dissipative phase transition: with a drive G $a^{+}a$ cos(ωt) there is a phase transition from <$a^{+}a$> = 0 to <$a^{+}a$> > 0



PRA 103, 033711 (2021)

Quantum Master Equations

- Time evolution of a mixed quantum state $\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i|$ is described by a master equation $\partial_t \rho = \mathcal{L}[\rho]$, where for closed systems

$$\mathcal{L}[\rho] = -iH\rho + i\rho H$$

- The evolution is "quantum" if and only if
  - Positive eigenvalues: $p_i \geq 0$
  - Trace preservation: $\mathrm{Tr}[\rho(t)] = \Sigma_i\, p_i = 1$ for all times
  - Hermiticity: $\rho = \rho^\dagger$
- Now one can imagine other generators $\mathcal{L}$ that are "quantum"
  - The most general time-local master equation is the Lindblad master equation

$$\mathcal{L}[\rho] = -iH\rho + i\rho H - \sum_i \Gamma_i (J_i^\dagger J_i \rho + \rho J_i^\dagger J_i - 2 J_i \rho J_i^\dagger)$$

  - J_i are "jump" operators, and trace preservation is clear from cyclicity

Wherefore Jumps?

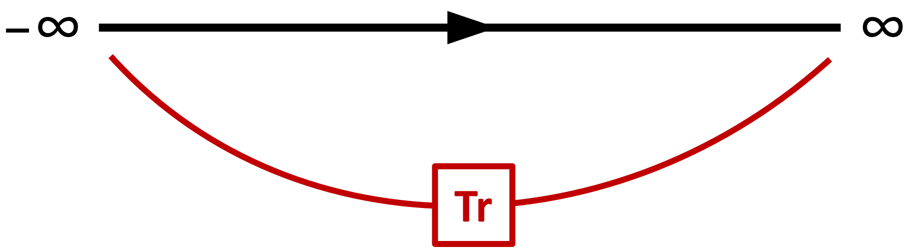
- We have the generator of time translation

$$\mathcal{L}[\rho] = \underbrace{-iH\rho + i\rho H}_{\text{Hermitian evolution}} - \sum_i \Gamma_i(\underbrace{J_i^\dagger J_i\rho + \rho J_i^\dagger J_i}_{\text{Anti-Hermitian evolution}} - \underbrace{2J_i\rho J_i^\dagger}_{\text{Jumps}})$$

- Effective non-Hermitian Hamiltonian doesn’t give the full picture!
- J_i comes from opening the system to an environment
  - Photon gain/loss, pair gain/loss in an optical cavity
  - Electronic gain/loss through coupling to a substrate
  - Dephasing through coupling to charge noise in a superconducting circuit
  - Engineered dissipation on a quantum simulator

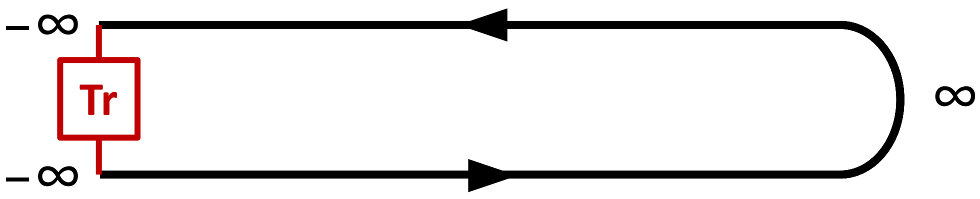
Path Integrals

- A very useful object is the partition function $Z = \mathrm{Tr}[\rho] = 1$
  - Functional derivatives generate correlation functions
- The Trace is a contraction of dangling indices on a contour
  - Equilibrium—final state is the initial state up to a phase



Note: time evolution
$|\psi(t)\rangle = U\,|\psi(-\infty)\rangle$

  - Non-equilibrium (“Keldysh” contour)—final state is unrelated to initial state



Note: time evolution
$\rho = |\psi(-\infty)\rangle\langle\psi(-\infty)|$
$\rho(t) = U\,\rho(-\infty)\,U^{+}$

Path Integrals (Continued)

- We can write Z = e^{iS} where the action is

$$iS[a^+, a^-] = \ln(\rho(-\infty)) \ + \ i\int_{-\infty}^{\infty} dt \ [\bar{a}^+ i\partial_t a^+ - \bar{a}^- i\partial_t a^- - i\hat{\mathcal{L}}[a^+, a^-]]$$

- Where time translation is generated by some $\mathcal{L}$ such as

$$\mathcal{L}[\rho] = -iH\rho + i\rho H - \sum_i \Gamma_i (J_i^\dagger J_i \rho + \rho J_i^\dagger J_i - 2J_i \rho J_i^\dagger)$$

where $\mathcal{L}$ is recast in terms of fields on each contour (±)

$$\hat{\mathcal{L}} = -iH_+ + iH_- - \sum_i \Gamma_i (J_{i,+}^\dagger J_{i,+} + J_{i,-}^\dagger J_{i,-} - 2J_{i,+} J_{i,-}^\dagger)$$

Example: Dissipative Nonlinear QHO

- Consider the normal-ordered bosonic Hamiltonian

$$\hat{H} = \omega_0 \hat{a}^\dagger \hat{a} + \tfrac{1}{2} K \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a}$$

- Subject to single photon losses $J = \gamma a$
- One can then "Keldysh" rotate to (anti)symmetric representation a^c, a^q

$$a^+ = \frac{a_c + a_q}{\sqrt{2}}, \quad a^- = \frac{a_c - a_q}{\sqrt{2}}, \quad \bar{a}^+ = \frac{\bar{a}_c + \bar{a}_q}{\sqrt{2}}, \quad \bar{a}^- = \frac{\bar{a}_c - \bar{a}_q}{\sqrt{2}}$$

- So that the action is

$$iS[a^c, a^q] = i\int_{-\infty}^{\infty} dt \begin{pmatrix} \bar{a}^c \\ \bar{a}^q \end{pmatrix} \begin{pmatrix} 0 & i\partial_t - \omega_0 - i\gamma \\ i\partial_t - \omega_0 + i\gamma & 2i\gamma \end{pmatrix} \begin{pmatrix} a^c \\ a^q \end{pmatrix} \quad \Big\} \, S_0$$

$$-i\int_{-\infty}^{\infty} dt \, \frac{K}{2} (\bar{a}^c a^q + \bar{a}^q a^c)(\bar{a}^c a^c + \bar{a}^q a^q) \quad \Big\} \, S_{int}$$

Perturbative Expansion & Wick's Theorem

- The argument of the exponential is a scalar and for small *K* we expand

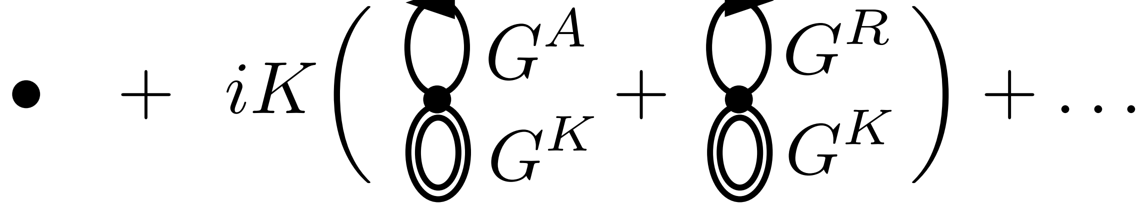
$$Z = e^{iS} = e^{iS_0 + iS_{\text{int}}} = e^{iS_0} e^{iS_{\text{int}}} = e^{iS_0}(1 + iS_{\text{int}} - S_{\text{int}}^2 + \dots)$$

- Now for bosonic modes weighted by $e^{i\sum_{ij}\bar{a}_i G_{ij}^{-1} a_j} = e^{-\sum_{ij}\bar{a}_i (iG_{ij})^{-1} a_j}$ the Wick theorem is $\langle a_i \bar{a}_j\rangle = iG_{ij}, \quad \langle a_i a_j \bar{a}_k \bar{a}_l\rangle = iG_{ik} iG_{jl} + iG_{il} iG_{jk}$
- For the problem at hand

$$Z \approx \int \mathcal{D}[\bar{a}, a]\Big(1 - i\frac{K}{2}(\bar{a}^c a^q + \bar{a}^q a^c)(\bar{a}^c a^c + \bar{a}^q a^q)\Big)e^{iS_0}$$

- Or, evaluating using Wick's Theorem we find the corrections to the partition function

$$1 + iK(G^A + G^R)G^K + \dots =$$

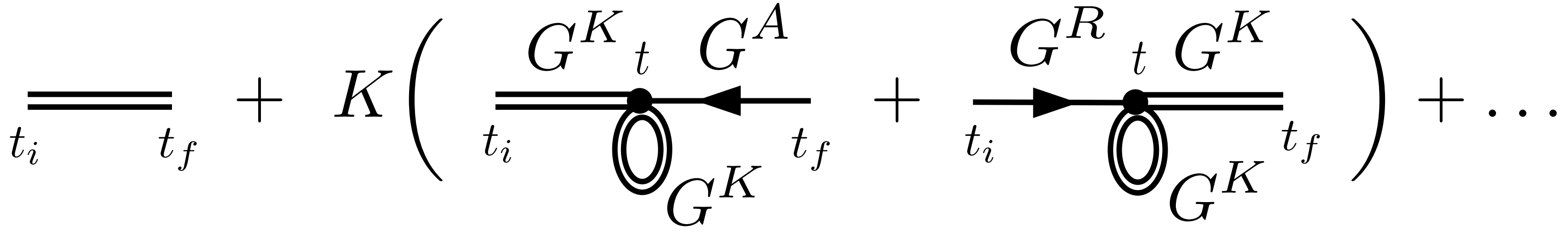


Ex. Density

- We may be interested in the density

$$n = \langle \bar{a}^- a^- \rangle + \langle \bar{a}^+ a^+ \rangle = \langle \bar{a}^c a^c \rangle + \langle \bar{a}^q a^q \rangle$$

- To lowest order we have $\langle \bar{a}^c a^c \rangle = \langle G^K \rangle$ and $\langle \bar{a}^q a^q \rangle = \langle 0 \rangle = 0$
- Now to next order we have the correction terms like $\frac{1}{2} iK \langle \bar{a}^c a^c \; \bar{a}^c a^q \bar{a}^c a^c \rangle$
- Evaluating Wick's theorem and simplifying only two diagrams survive

$n(t_i, t_f) =$ 

- The density is then a contour integral over all times (frequencies)

RESPONSE THEORY &
RESPONSE OF OPEN SPIN CHAINS

A systematic framework to calculate response properties for open quantum systems

+

Applications to free (XY) and many-body (XXZ) transverse field Heisenberg spin chains

Boundary-Dissipative Spin Chains

- Time evolution in the limit of continuous measurement by a memoryless bath $i\dot{\rho} = \mathcal{L}[\rho]$

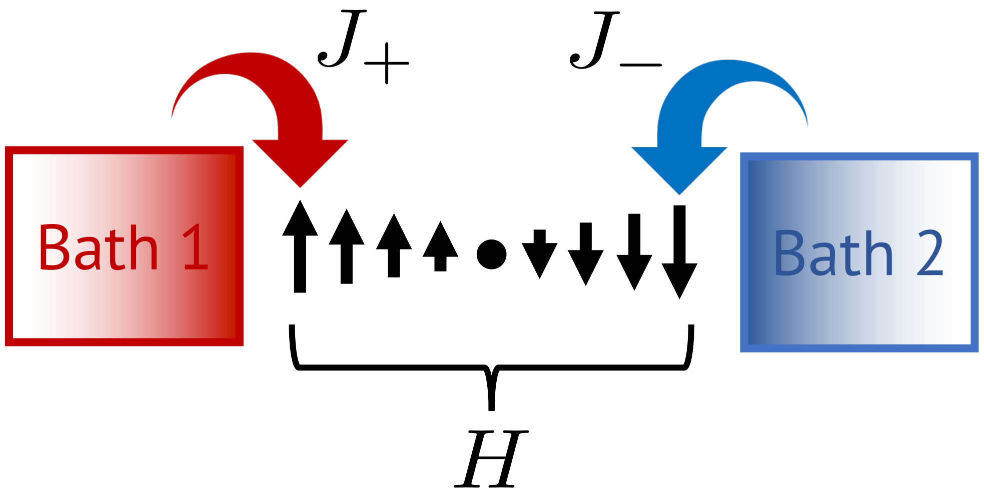
$$i\dot{\rho} = (\mathcal{L}^{\text{coh.}} + \mathcal{L}^{\text{n.h.}} + \mathcal{L}^{\text{jump}})[\rho]$$

where,

$$\mathcal{L}^{\text{coh.}}[\rho] = [H, \rho]$$

$$\mathcal{L}^{\text{n.h.}}[\rho] = -i\frac{\Gamma}{2}\sum_m \{J_m^\dagger J_m, \rho\}$$

$$\mathcal{L}^{\text{jump}}[\rho] = i\frac{\Gamma}{2}\sum_m 2J_m \rho J_m^\dagger$$

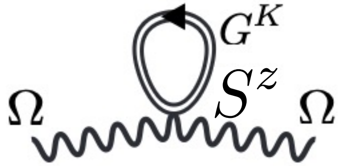


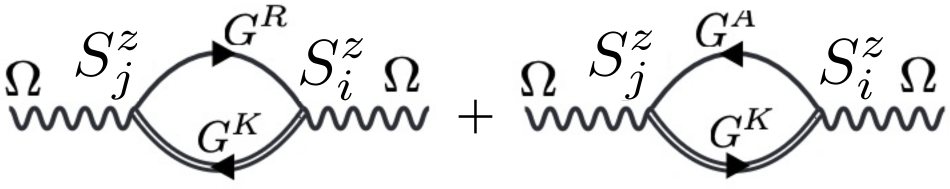
Spin chain with boundary dissipation. Jump operators linear in spins map to jump operators linear in fermions under Jordan-Wigner transformation.

Energy scales
J: spin-spin coupling
h: transverse field
Γ: dissipation strength

Green’s Functions

- We want to probe the steady state with static density response $\langle S^z \rangle_{\mathrm{ss}}$ and dynamic response $\mathcal{S}^z_{i,j}(\Omega) = \int dt\ e^{-i\Omega t} \langle [S^z_i(t), S^z_j(0)] \rangle_{\mathrm{ss}} \theta(t)$

$\langle S^z \rangle_{\mathrm{ss}} =$ 

$\mathcal{S}^z_{i,j}(\Omega) =$ 

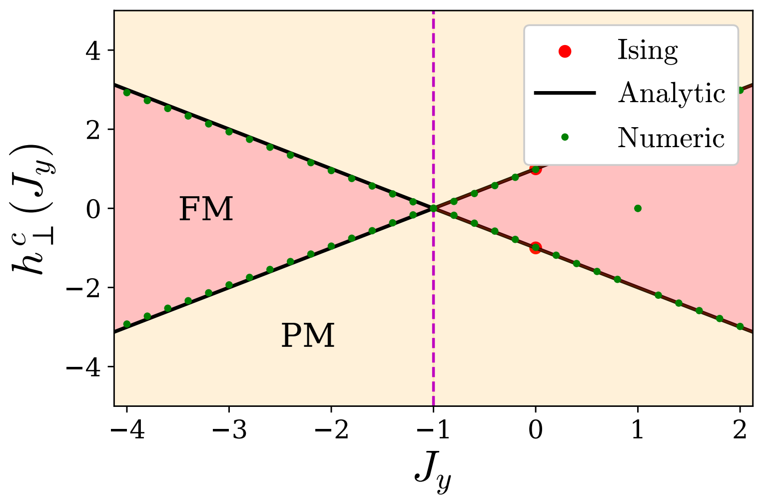
- These can be expressed in terms of Green’s functions
  - Closed system: G^R and G^A
  - Open system: G^R, G^A and a Keldysh Green’s function G^K
- Jump terms lead to complex self energy!
- Wick’s theorem can be used to reduce multi-point correlation functions to two-point correlation functions

Lindblad-Keldysh GFs:
- Thompson and Kamenev, Ann. Phys. 455, 169385 (2023)
- McDonald and Clerk, Phys. Rev. Res. 5, 033107 (2023)

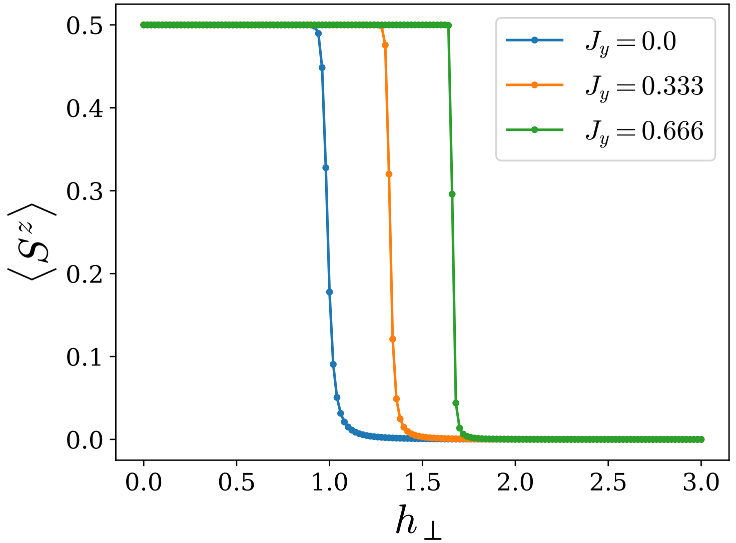
Ising and XY Models: Closed Systems I

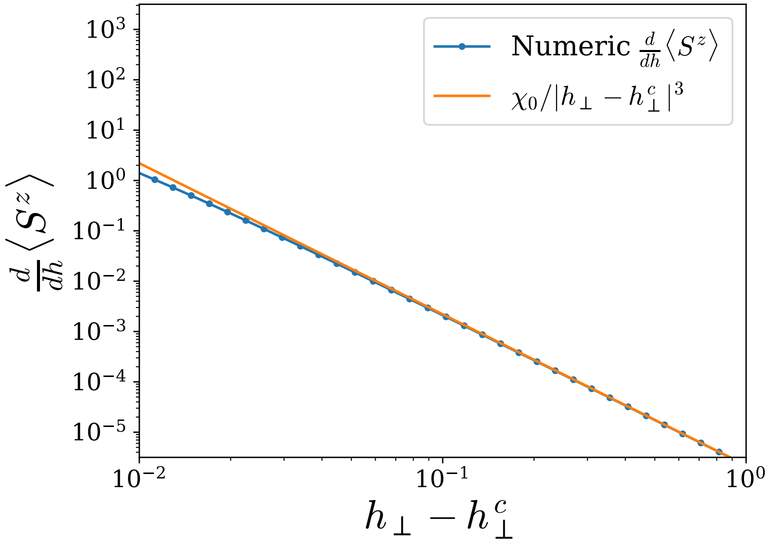
- Transverse field XY spin chain is paradigmatic example of gapped/gapless quantum matter

$$H_{XY} = \sum_{n=1}^{N-1} J_n^x S_n^x S_{n+1}^x + J_n^y S_n^y S_{n+1}^y + \sum_{n=1}^{N} h_n S_n^z$$



- Jordan-Wigner transformation to fermions
- Find magnetization
  - Paramagnetic to ferro-magnetic transition
- Susceptibility diverges with a power law



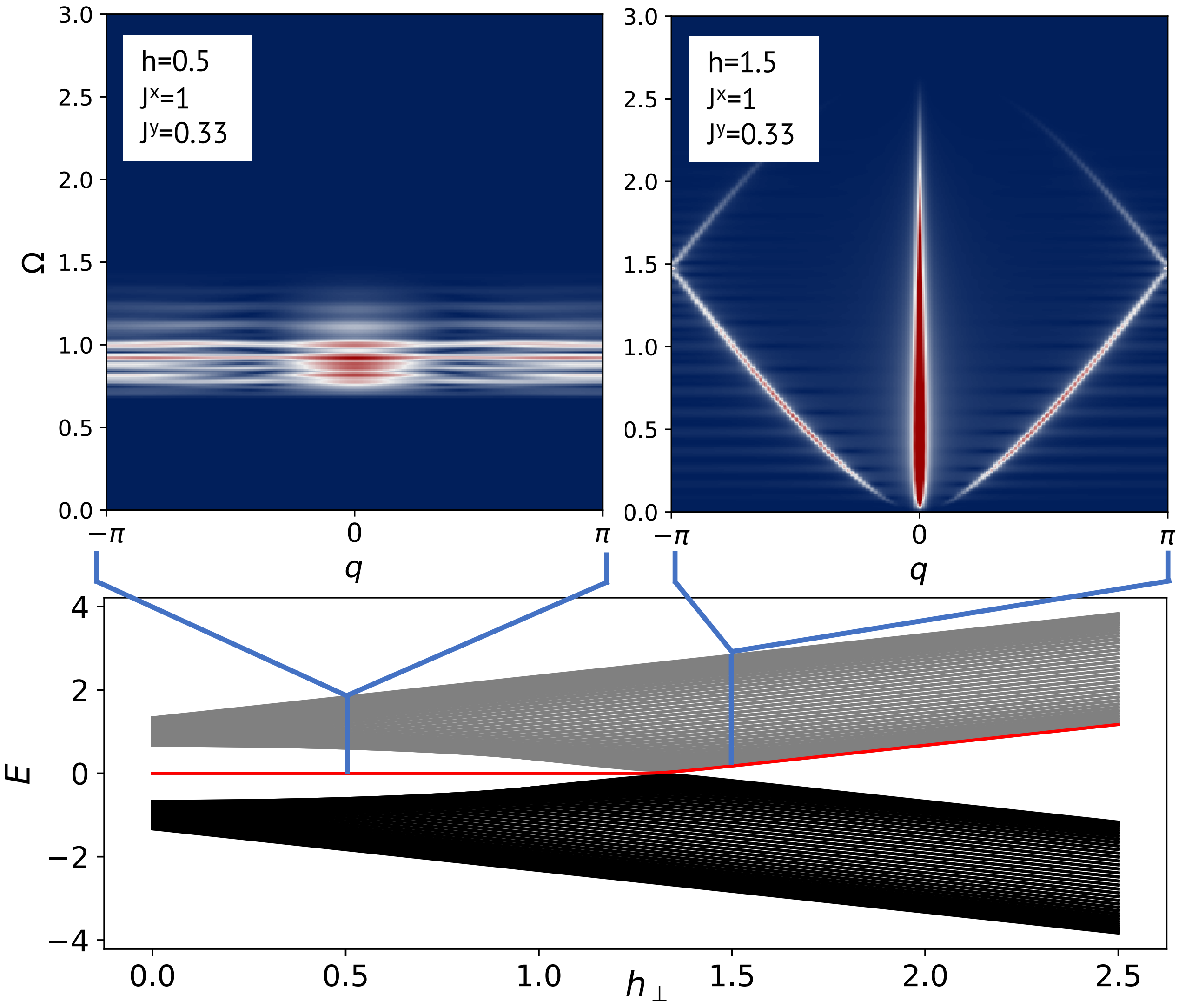


Ising and XY Models: Closed Systems II

- Dynamic correlation function

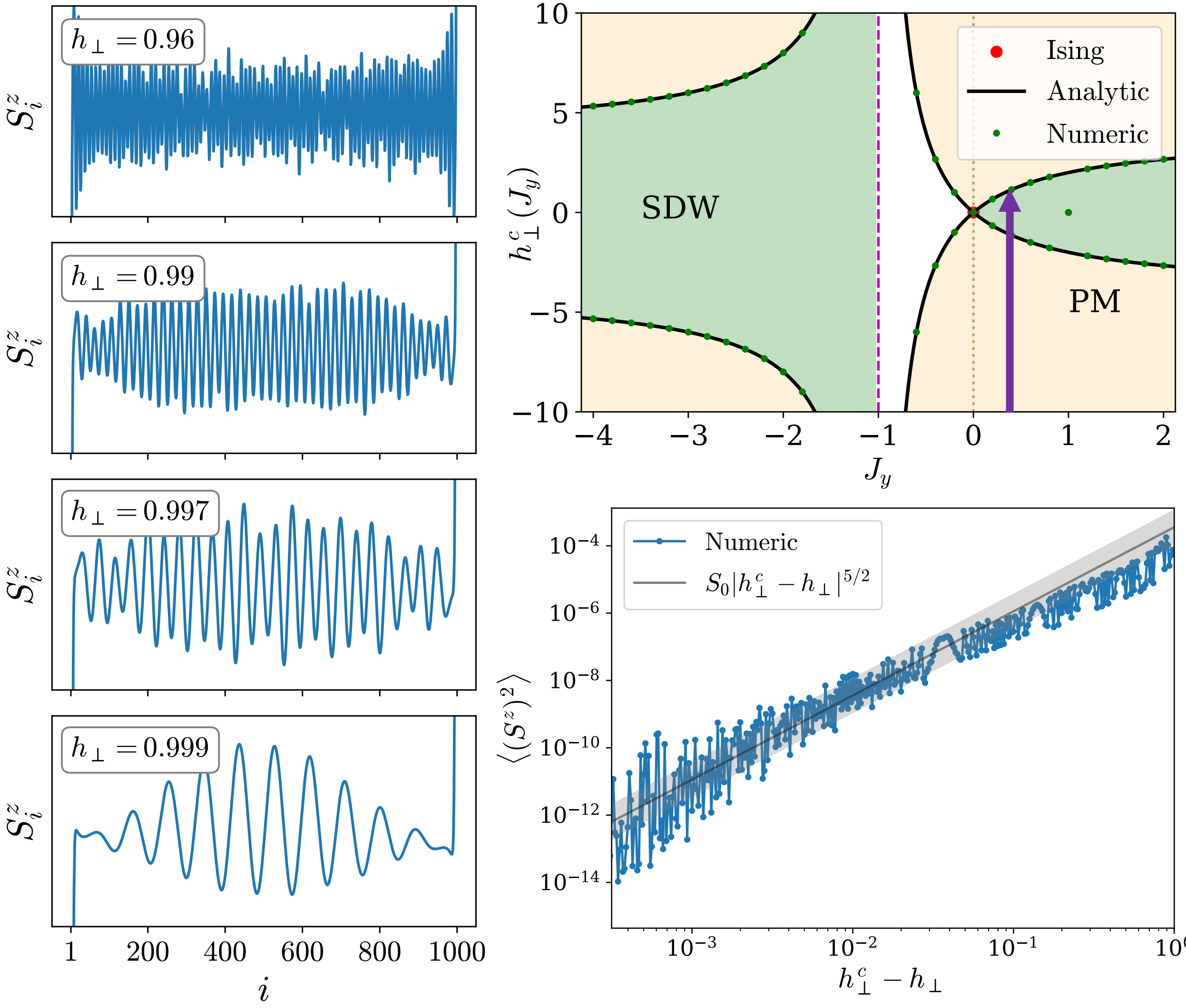
$$\chi_{ij}(\Omega) = \sum_{n>0} \frac{\langle u_0|S_i^z|u_n\rangle\langle u_n|S_j^z|u_0\rangle}{\Omega + i\eta - (\epsilon_n - \epsilon_0)}$$

- Probe excitations above $|u_0>$
- See the structure of gapped and gapless excitations
- All negative energy states are filled (think Pauli blocking)



Ising and XY Models: Open Systems I

- Spin chain with boundary dissipation S^+ on left, S^- on right
- New phase: spin-density wave
  - Magnetization $\langle S^z \rangle$ vanishes, but its higher moments do not
  - Phase boundary is very different from equilibrium phase boundary
- Wavelength diverges and $\langle (S^z)^2 \rangle$ exhibits power-law scaling on near critical point h^c
- Choose model with $J^x=1$, $J^y=1/3$ so that $h^c = 1$

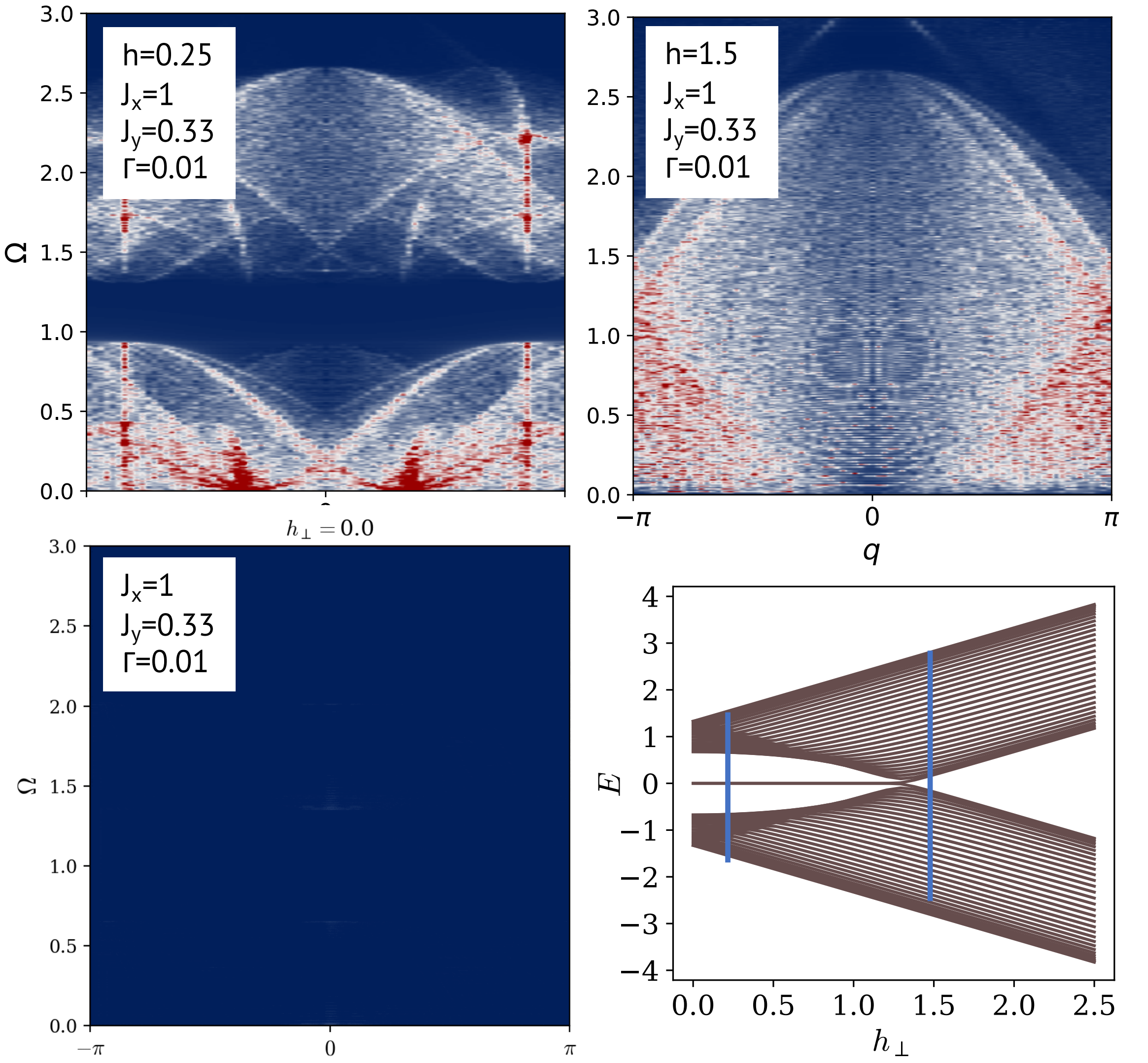


Ising and XY Models: Open Systems II

- Dynamic correlation function

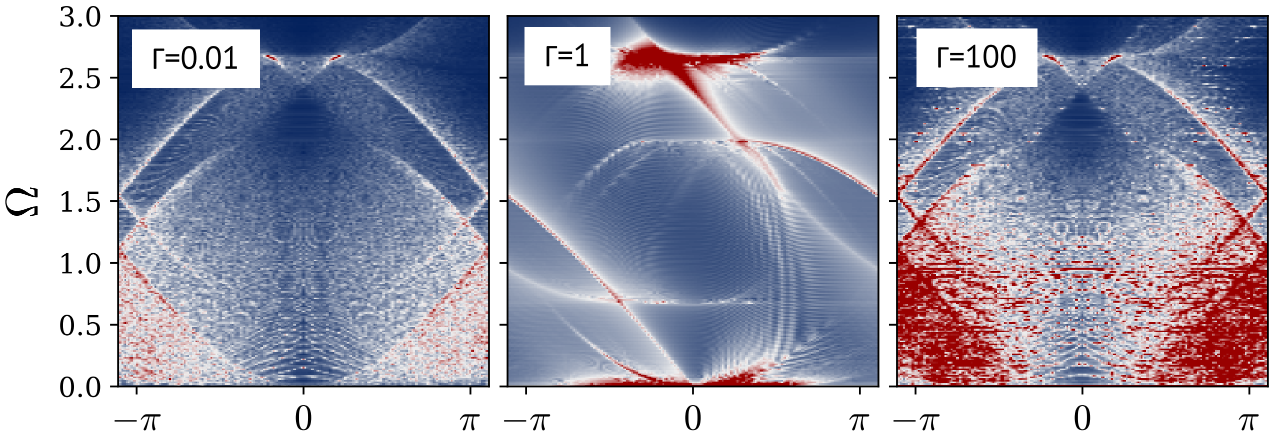
$$\mathcal{S}^z_{i,j}(\Omega) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left(\mathrm{Tr}[S^z_i G^R(\omega) S^z_j G^K(\omega+\Omega)] + \mathrm{Tr}[S^z_i G^K(\omega-\Omega) S^z_j G^A(\omega)] \right)$$

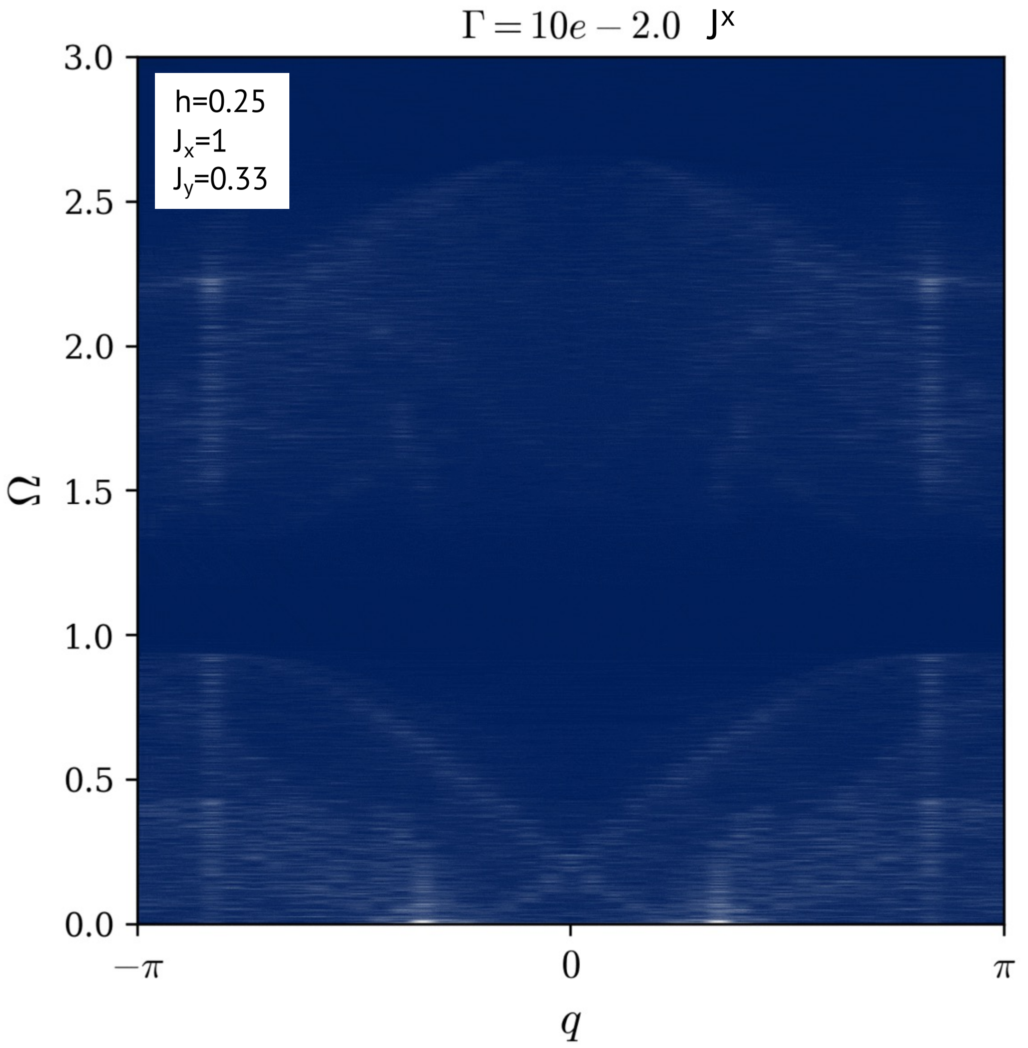
- Fractional occupation given by distribution function for ρ_{ss}
  - States at all energies contribute
  - Spectral gaplessness for h < h^c
- Vertical lines at q_{SDW} from SDW
- Dispersing modes from $2q_{SDW}$



Ising and XY Models: Open Systems III

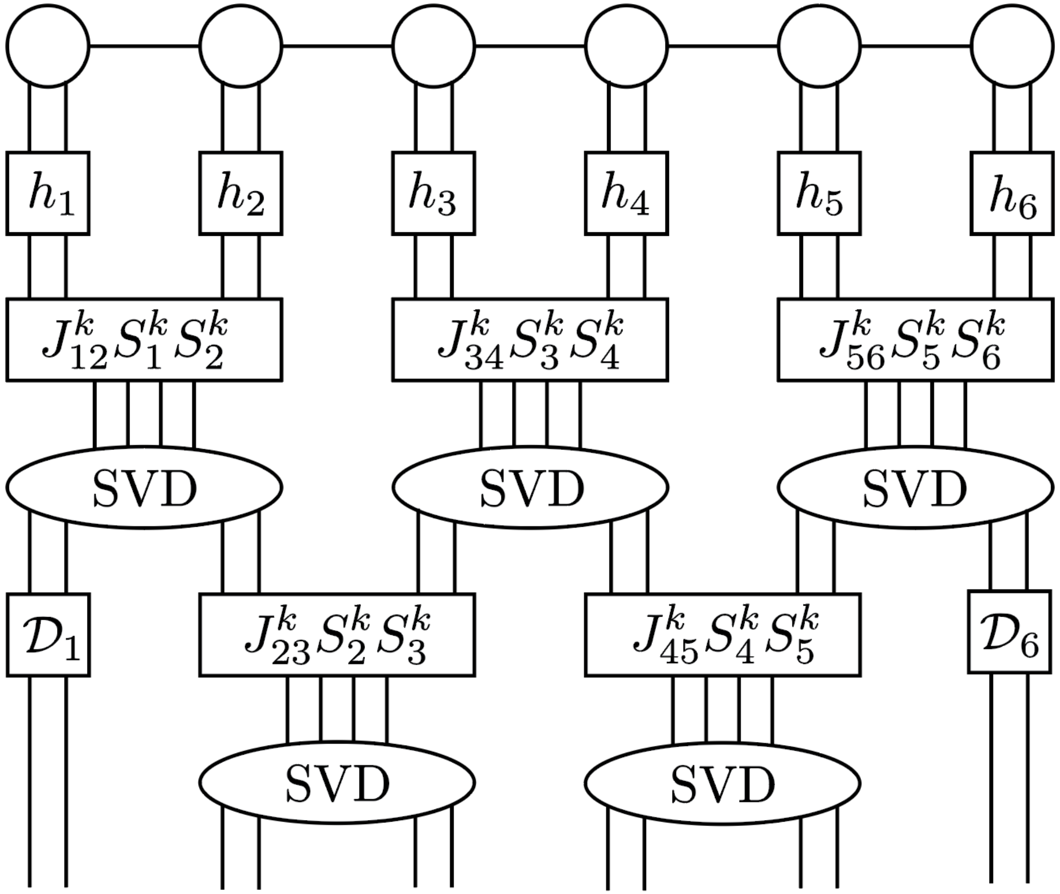
- Strong coupling (Γ~J)
  - Inversion-symmetry broken by dissipation
  - Large amplitude response
- Weak (Γ << J) and ultra-strong (Γ >> J)
  - Restoration of inversion symmetry
    - Bulk is relatively isolated from boundaries
  - Small amplitude response





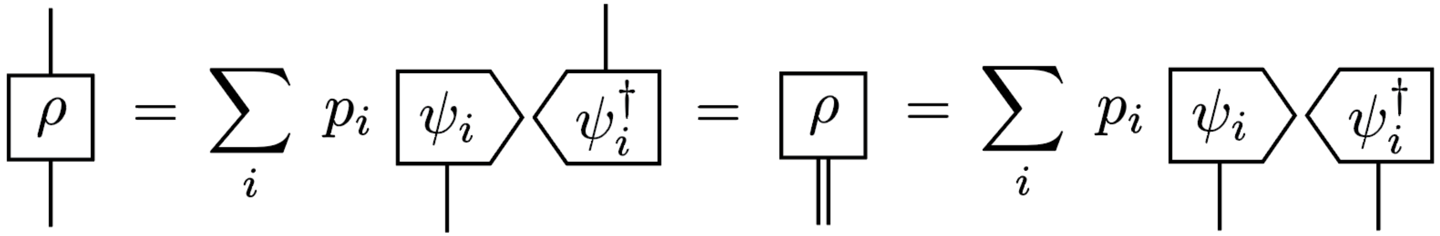
Many-Body Spin Chains: TEBD Approach to NESS

- Now, not all spin chains map to free fermions! Need to do a many-body problem: use field theory + numerics
- Steady state properties are the key
- Trotterized time evolution followed by decomposition into tensors with fixed "bond dimension" χ
  - Circles are an initial product state
  - Rectangles are exp(-i O dt) for boxed O
  - One cycle in evolution is visualized
- Iterate until steady state is reached



How to simulate open quantum systems

- Singular value decomposition still works for non-Hermitian systems
- One method is to use “vectorization” of the local Hilbert space



- Larger bond dimension, but still simulating the evolution of an MPS
- Evolution given by $|\rho(t)> = e^{L_{vec}\, t}|\rho(0)>$

$$L_{\mathrm{vec}} = -1 \otimes iH + iH^\top \otimes 1 - \sum_i \gamma_i((J_i^\dagger J_i)^\top \otimes 1 + 1 \otimes J_i^\dagger J_i - 2(J_i^\dagger)^\top \otimes J_i)$$

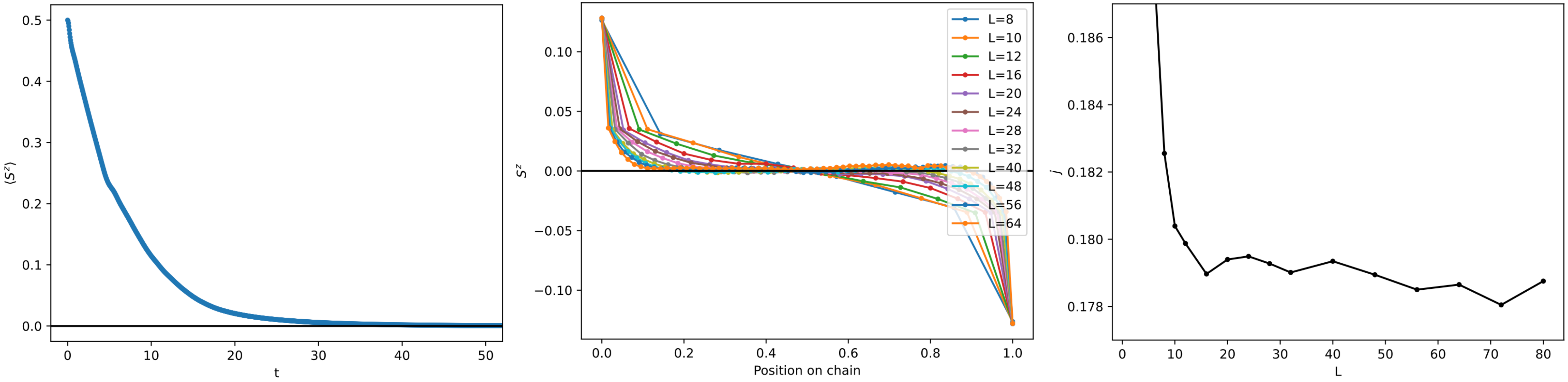
See PRL 93, 207204 (2004), and PRL 93, 207205 (2004)

XXZ model transport

- We consider the model $H = \sum_{i=1}^{L-1} J^x S_i^x S_{i+1}^x + J^y S_i^y S_{i+1}^y + J^z S_i^z S_{i+1}^z + \sum_{i=1}^{L} h^x S_i^x + h^y S_i^y + h^z S_i^z$
- With jumps given by $J_1^+ = \left(\frac{1-\tanh(\mu_L)}{1+\tanh(\mu_L)}\right)^{1/4} S_1^+, \quad J_1^- = \left(\frac{1+\tanh(\mu_L)}{1-\tanh(\mu_L)}\right)^{1/4} S_1^-, \quad J_L^+ = \left(\frac{1-\tanh(\mu_R)}{1+\tanh(\mu_R)}\right)^{1/4} S_L^+, \quad J_L^- = \left(\frac{1+\tanh(\mu_R)}{1-\tanh(\mu_R)}\right)^{1/4} S_L^-$
- Can calculate arbitrary correlation functions
- Here $\langle S^z_n \rangle$ and $j = \langle S^x_n S^y_{n+1} - S^y_n S^x_{n+1} \rangle$

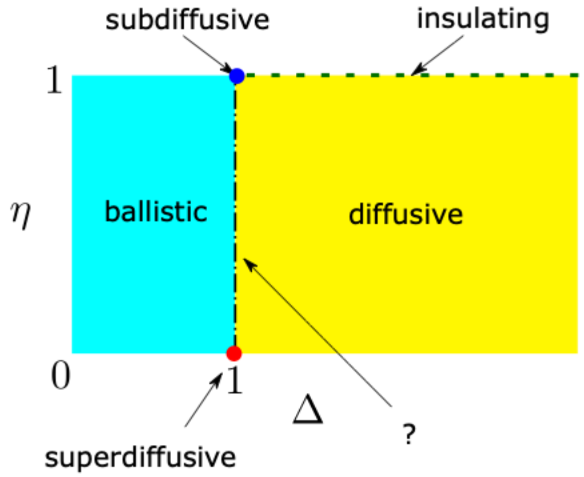
Prosen, J. Stat. Mech. (2009) P02035

$\mu_L = 0.22$, $\mu_R = -0.22$
$J_x = J_y = 1$, $J_z = 0.5$
$h_x = h_y = h_z = 0$

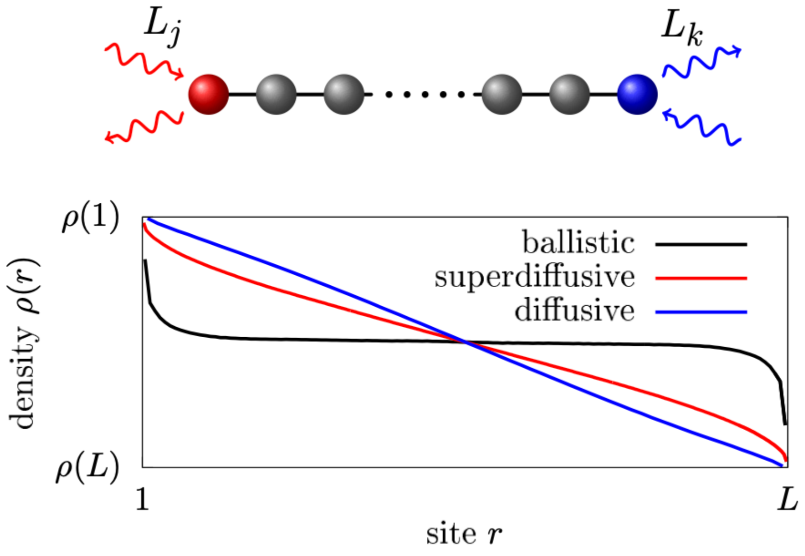


XXZ transport (continued)

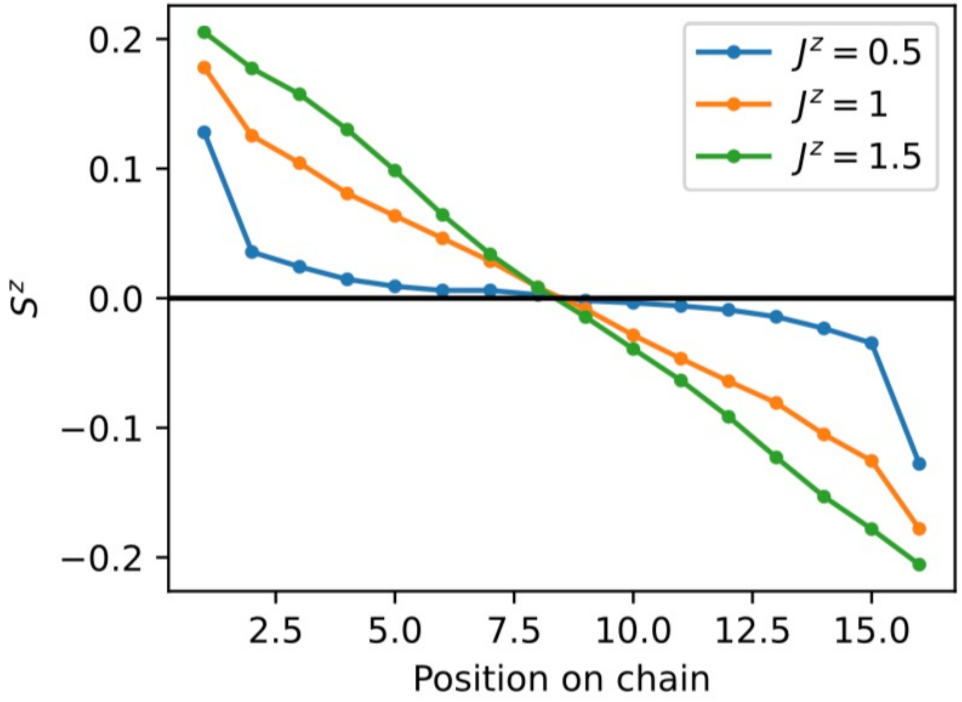
- Can see unusual scaling of the spin transport
- KPZ point of "superdiffusive" dynamics
- Understood as vanishing of Drude weight in an integrable system



RMP 94, 045006 (2022)



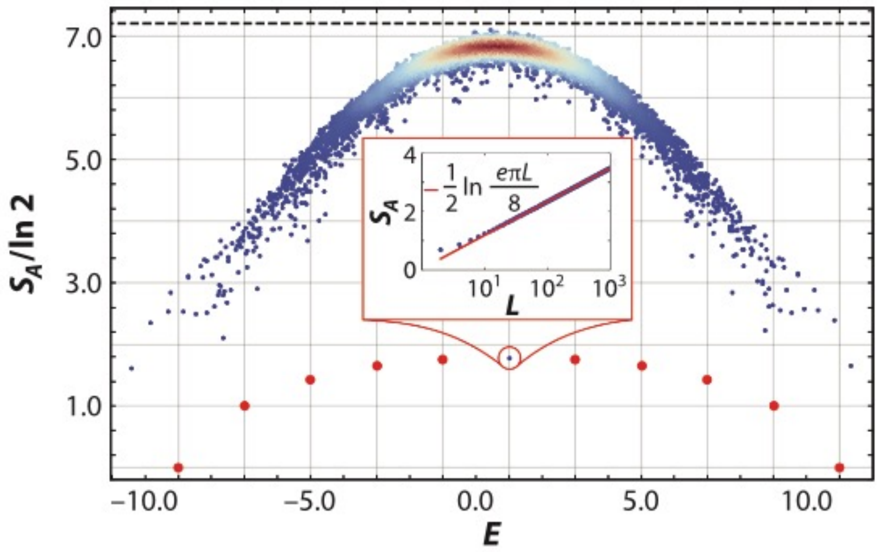
Bertini, RMP 93, 25003 (2021)



Possible Directions on Spin Chains

- Competing time scales in spin ladders
  - Ex. Diffusive chain coupled to ballistic chain
- Different regimes of coupling
  - The weak/ultra-strong dissipation duality
- Coordinated gain and loss
  - Sublattice structure/dimerization, a la SSH
- Targeting low-entropy states
  - Scar states in the middle of the spectrum
  - Impossible in equilibrium: pure state vs thermal mixture
  - Dissipation can reduce entropy!
- New types of chains, ex. surface/toric code cylinders
  - Local dissipation acts differently on vertex (charge) and plaquette (flux) operators
  - Relevant for error corrected quantum computing efforts

Low entropy scar states in spin-1 XY chain



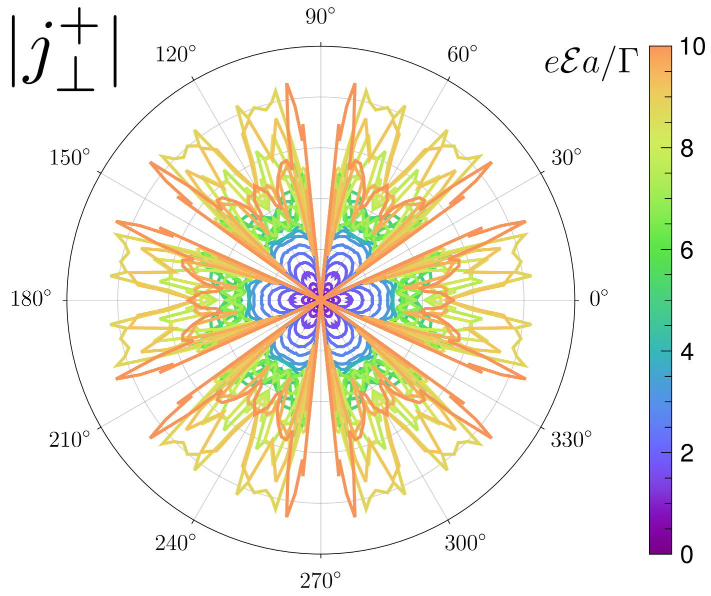
PRL 123, 147201 (2019)

OTHER OPEN SYSTEMS & FUTURE DIRECTIONS

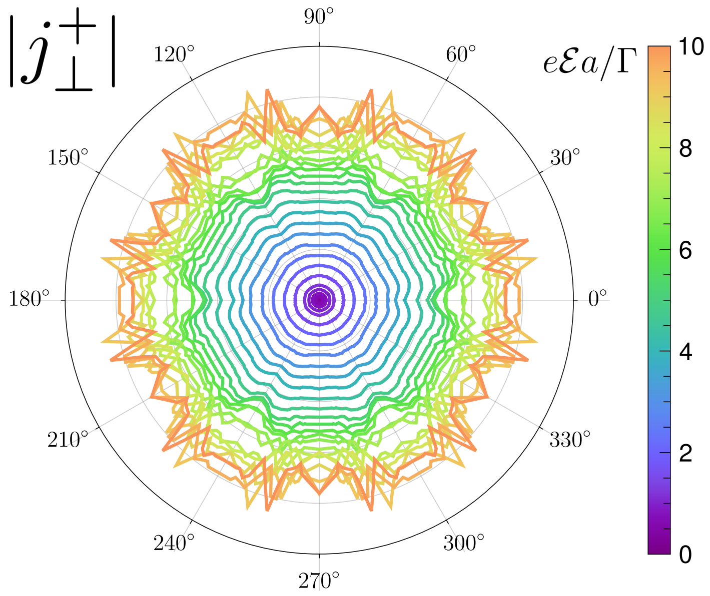
Non-Equilibrium Green's Functions (Leads)

- Work with Christophe, Steven, and Gene
- DC electric field in a 2D material with some quantum geometry
- Probe the geometry using the response
- Use dissipation to avoid Stark localization
    - Calculate currents using non-equilibrium GFs

Trivial



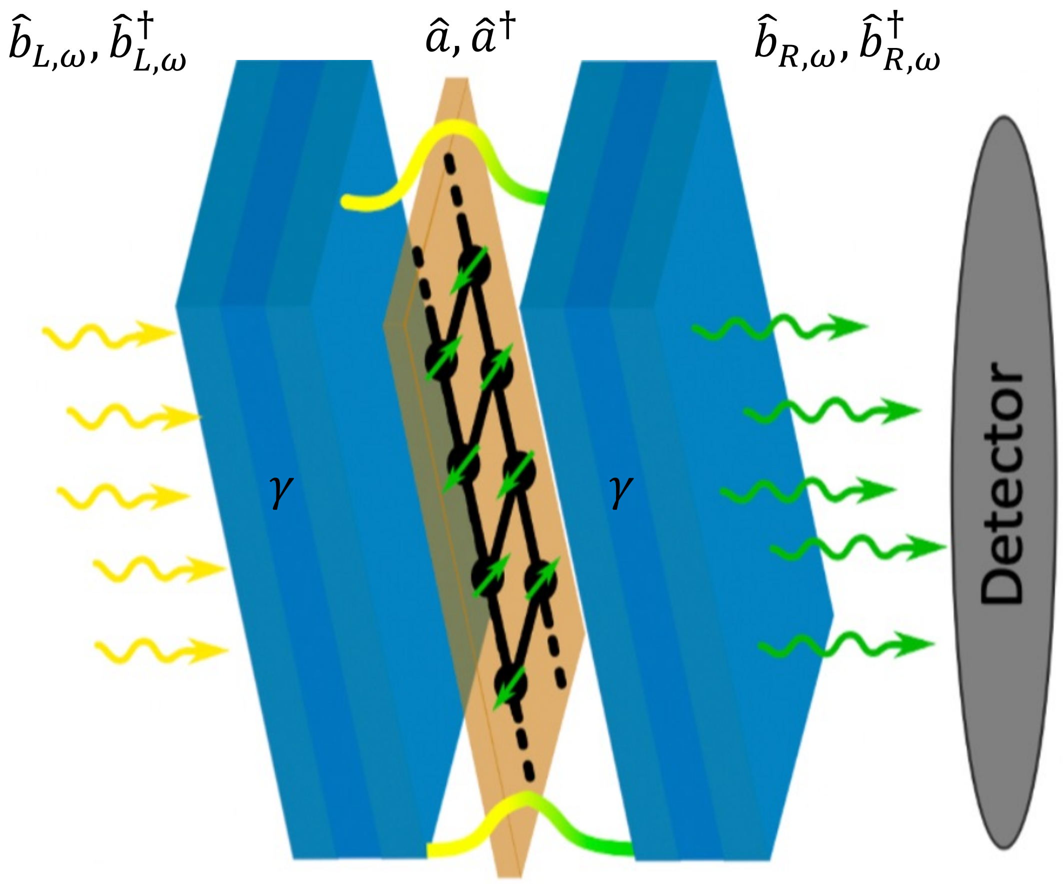
Chern

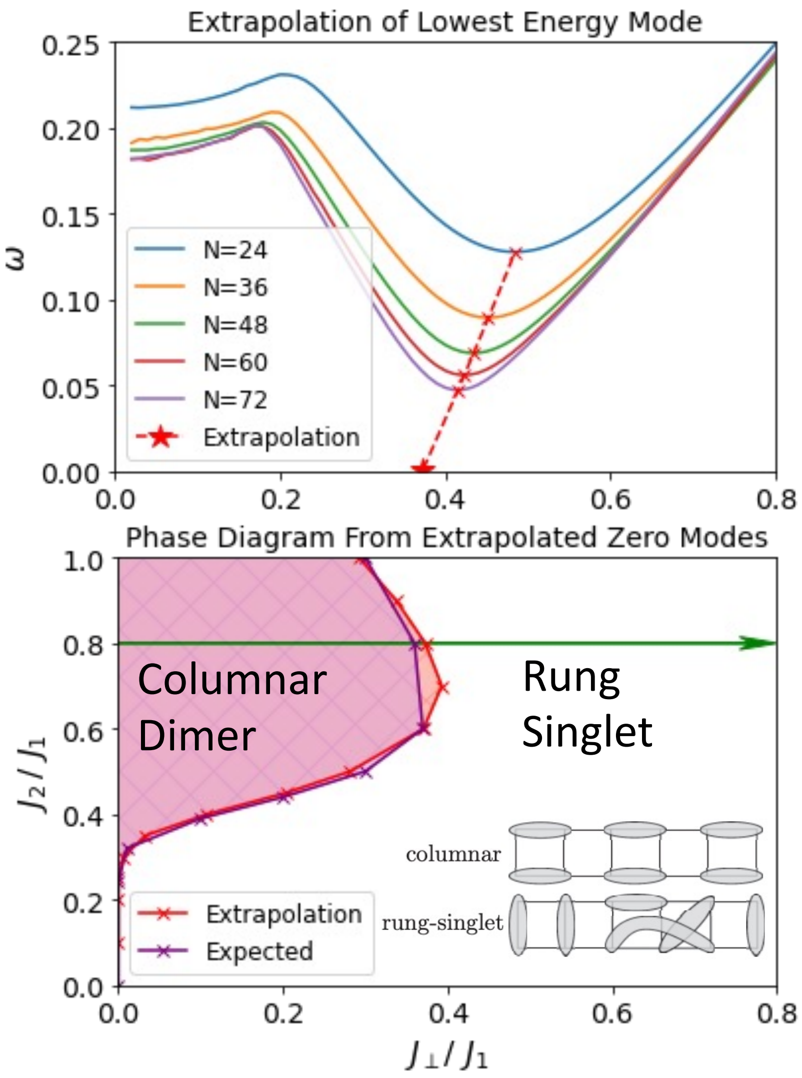


arXiv: 2401.17368

Cavities and Optics

- With Ben Kass and Martin
- Use output light to extrapolate cavity phases



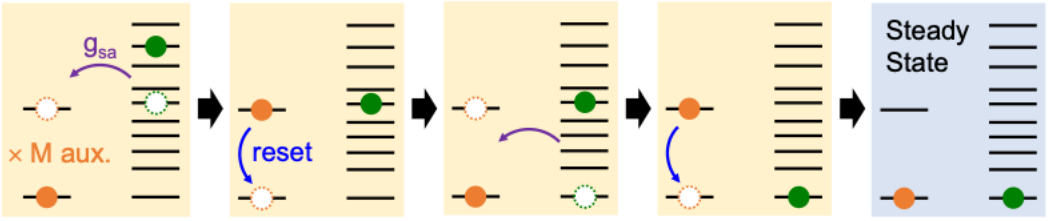


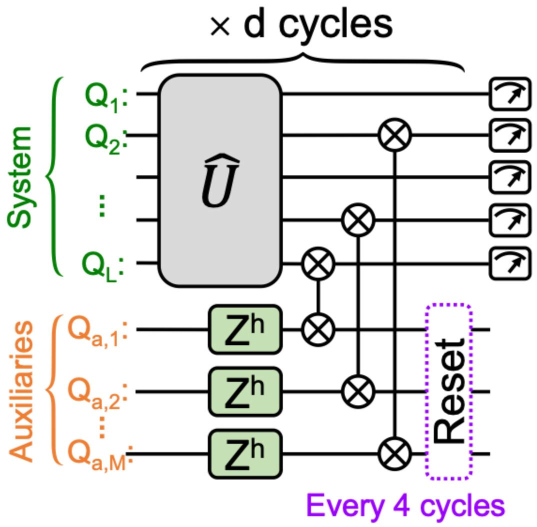
Example: Dissipative Cooling

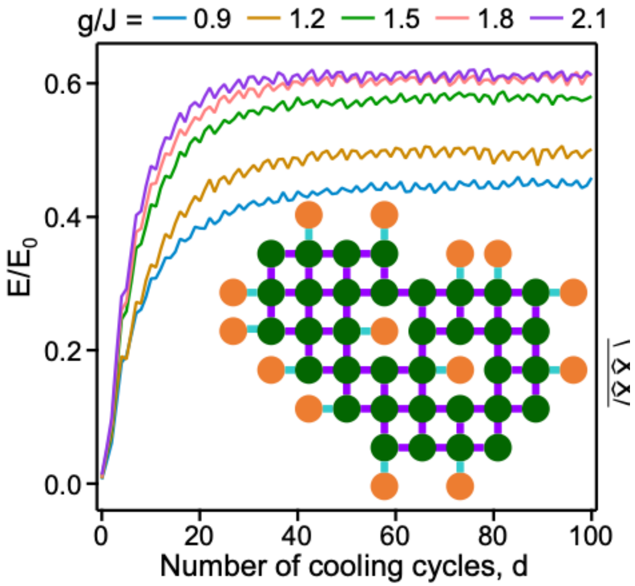
- Repeatedly reset ancillas to "cold" ferromagnetic alignment
- Stroboscopic time evolution
    - Trotterized Hamiltonian
    - Partial SWAP gates between system and ancilla

$$i\mathrm{SWAP}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & i\sin(\theta) & 0 \\ 0 & i\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Get to 60-80% of the (negative) ground state energy







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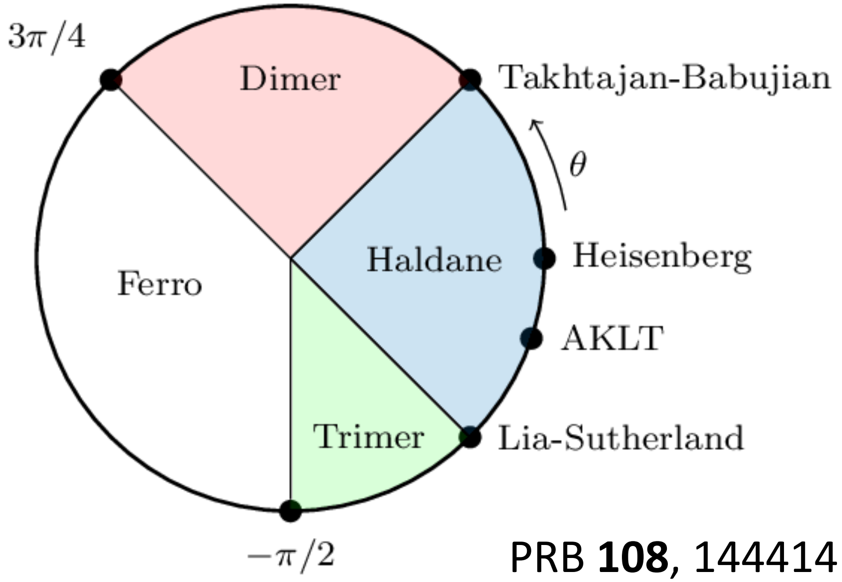
Quantum Simulators

$$H_{BLBQ} = \sum_i J^{(1)} \vec{S}_i \cdot \vec{S}_{i+1} + J^{(2)} (\vec{S}_i \cdot \vec{S}_{i+1})^2$$

$$H_{AKLT} = \sum_i 1\, \vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i \cdot \vec{S}_{i+1})^2$$

- Dissipation as a method to:
  - Rapidly prepare desired states
  - Stabilize states
  - Realize non-equilibrium states
- Of particular interest: 3-state systems
  - LSM type theorems: gapped and interesting
  - Spin-1, or Z_3 parafermionic models
  - Dissipative preparation of ground states



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- Also: this summer I will be an intern at HRL Labs in Los Angeles
  - Decoherence of electronic spin qubits by a bath of nuclear spins

Conclusion

- Dissipative engineering is applicable to a wide range of physical systems
  - Optical cavities and lattices
  - Spin chains
  - 2D electron systems
  - Quantum simulators
- **Key idea: use selectively engineered dissipation to realize (steady) states with desirable properties that are not possible in equilibrium**

- 4 peer reviewed publications and 2 preprints so far
- Targeting PhD completion near the end of 2026

Thank you!