Long Lived Flat Bands in Fermionic Lindbladian Systems

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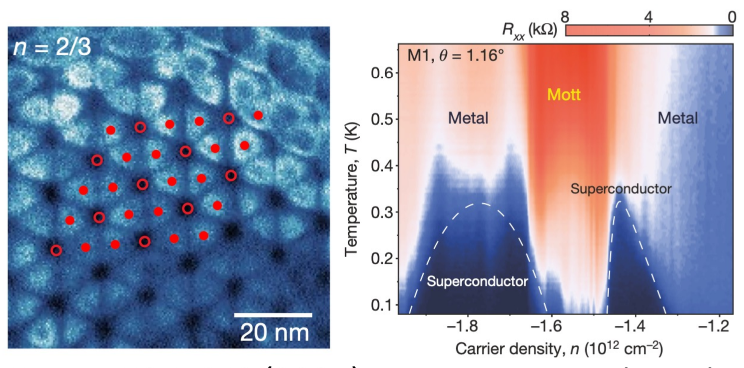
Flat Bands

Correlated Physics in Magic Angle TBG

The Coulomb interactions in magic angle TBG dominate kinetic behaviors leading to correlated behaviors like

- Superconductivity
- Charge orders
- Magnetic orders
- Fractionalized states
- And more!



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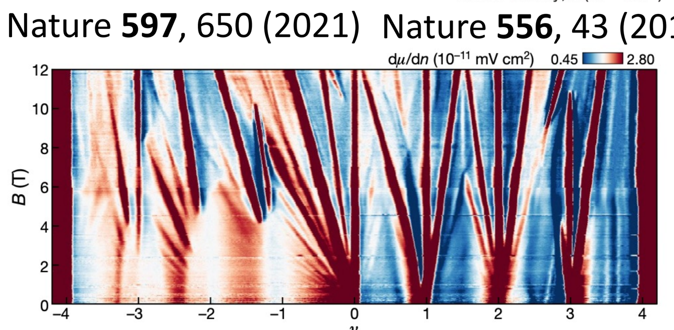


Nature **600**, 439 (2021)

Where Do Flat Bands Come From?

Truly flat bands can originate in isolated atomic insulators, or from more subtle kinetic (eg. Lieb and Kagomé lattices) or quantum interference (eg. the twisted bilayer graphene (TBG), or the 1D diamond chain lattice). Each of these origins necessarily involves the Rank-Nullity theorem.

Symmetry Protected Flat Bands

Formation of a Dark Space

In quantum optics subject to dissipation, a “dark state” is a state that is long lived. Borrowing this concept, we use the term ”dark space” to describe the long-lived subspace of a dissipative (fermionic) system. Now suppose that

$$\mathcal{D}^{-1} L \mathcal{D} = L \qquad \mathcal{D} = \eta_3 \otimes \tau_1$$

holds, where we refer to $\mathcal{D}$ as the “dark space” symmetry operator. If this holds, then $A = B$ and $C = C^\top$ and also L_{dis} is Hermitian! With some algebra [3] this means L_{dis} has N particle-like and N hole-like zero modes. If there are fewer jump operators M than orbitals N then there will be an additional $N-M$ particle like, and hole-like zero modes.

Projection Into Dark Space

In the large Γ limit, the effective Lindbladian (the only non-trivial evolution) is generated by

$$\widetilde{L}_{ij} = \langle \phi_i | L_{\mathrm{coh}} | \phi_j \rangle$$

where ϕ_i , ϕ_j are the states that span the dark space. Generically this projection will lead to N dispersive bands, but charge conjugation symmetry requires paired modes where $\epsilon(\boldsymbol{k}) \leftrightarrow -\epsilon(\boldsymbol{k})$ and hence $\mathrm{Tr}(\widetilde{L}) = 0$, so if N is odd then there must be at least one long lived zero mode $\epsilon = 0$.

When Does $\mathcal{D}^{-1} L \mathcal{D} = L$?

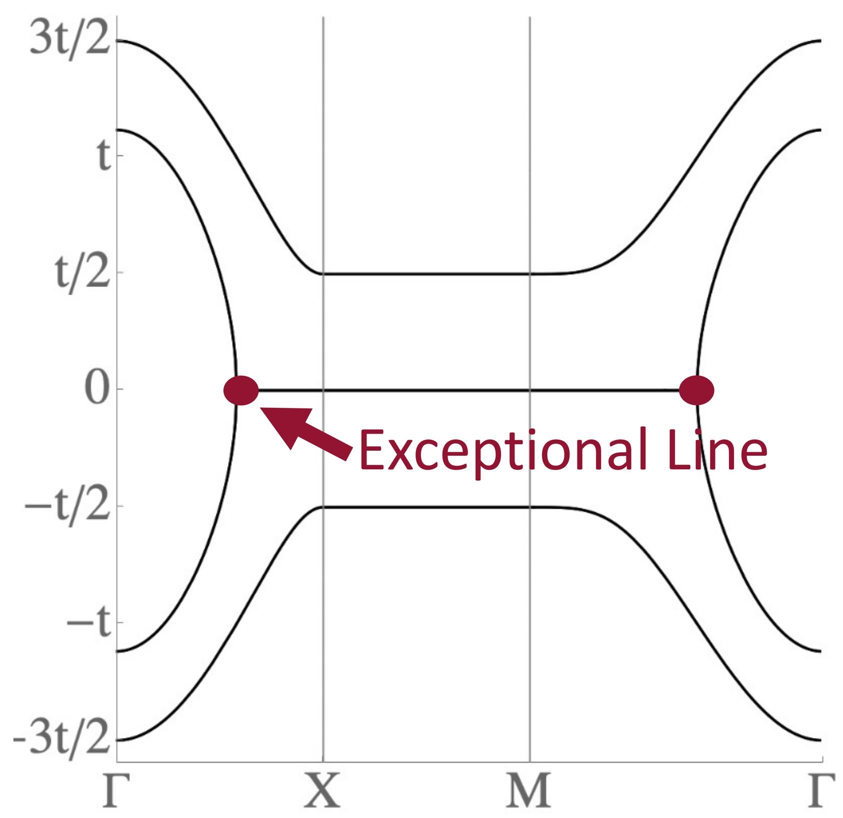
Since $\mathcal{D}$ imposes $A = B$ and $C = C^\top$ an ansatz which fulfills this requirement is $(b_{m,1}, \ldots, b_{m,N}) = e^{iS}(a_{m,1}, \ldots, a_{m,N})$ Where is a real valued symmetric matrix which occurs in a superconducting substrate for any electronic system.

Topology of Bloch Oscillations

Exceptional Surfaces

“Exceptional points” where spectral degeneracies and coalescence of eigenvectors coincide are a hallmark of non-Hermitian systems. In higher dimensions these exceptional points can connect to form exceptional lines and surfaces. Figure at right: inside the exceptional line, states have finite lifetimes while outside they are long lived.

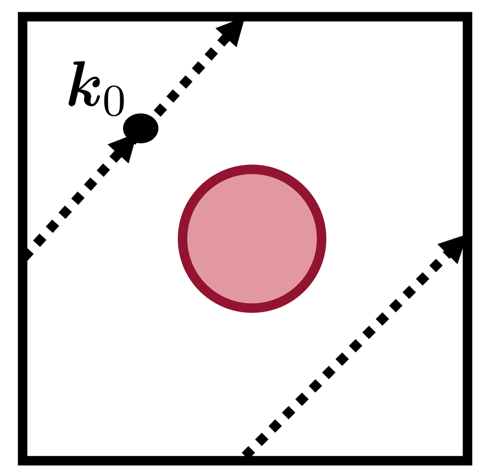
Dissipative QWZ ($\Gamma < \Gamma_c$)



Bloch Oscillations

Bloch oscillations occur when an electron is subject to a periodic potential and an electric field gradient. In the Lindbladian case the motion of the electron can wind around (or through) an exceptional surface.

$$\boldsymbol{k}(t) = \boldsymbol{k}_0 + (V_x, V_y)t$$



Floquet Lindbladians

While one of the assumptions for an operator to be of Lindbladian superoperator form is that the time-evolution operator be time-independent, one can side-step this by constructing a time-evolution operator that is periodic and time and whose Floquet time-evolution operator is of Lindblad form. This can be used to study Bloch oscillations.

$$\mathcal{L}(t) = \sum_f e^{if\Omega t} \mathcal{L}_f \implies \mathcal{L}_F = \begin{pmatrix} \ldots & \mathcal{L}_{-1} & \mathcal{L}_{-2} & \mathcal{L}_{-3} \\ \mathcal{L}_1 & \mathcal{L}_0 & \mathcal{L}_{-1} & \mathcal{L}_{-2} \\ \mathcal{L}_2 & \mathcal{L}_1 & \mathcal{L}_0 - \Omega & \mathcal{L}_{-1} \\ \mathcal{L}_3 & \mathcal{L}_2 & \mathcal{L}_1 & \ldots \end{pmatrix}$$

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