Long Lived Flat Bands in Fermionic Lindbladian Systems

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Abstract

Exceptional points in parameter space where eigenvalues and eigenvectors coalesce are a key feature of non-Hermitian systems. Non-Hermiticity can originate with dissipation processes that lead to the gain or loss of energy and/or particles. In quantum systems, these processes can be represented via Lindbladian superoperators that determine the time evolution of the density matrix of a system subjected to continuous measurements by a memoryless bath. For quadratic Hamiltonians the Lindbladian can be reformulated in terms of right and left superfermions/superbosons with a low-dimension matrix representation. We showed (in arXiv:2203.07453) conditions under which symmetries of the dissipation (jump operators) ensure the formation of flat bands that are protected from dissipation. Here we review this work and outline our more recent work on Bloch oscillations and their possible signatures of winding topology around exceptional points and surfaces in the Brillouin zone.

Lindbladian Superoperators

Mathematical Definition

The Lindbladian superoperators are the class of time-independent time evolution superoperators $\mathcal{L}[\rho]=i\dot{\rho}$ of the density matrix ρ that are completely positive trace preserving (CPTP) and Markovian (memoryless) [1].

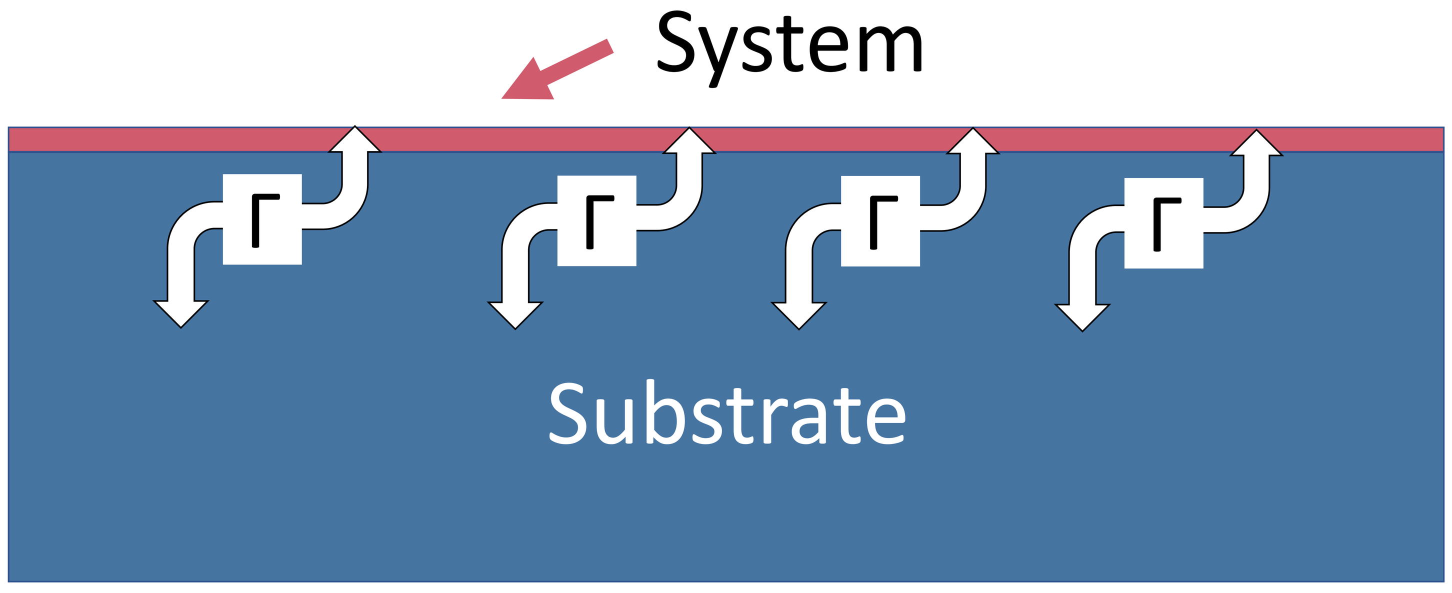
These superoperators extend the coherent evolution of the von Neumann equation $[H,\rho]=i\dot{\rho}$ by considering the evolution of a subsystem subject to jump operators J that change particle number and energy of (introduce non-Hermiticity to) the subsystem. Explicitly, they read

$$\mathcal{L}[\rho]=[H,\rho]-i\frac{\Gamma}{2}\sum_m\left(\{J_m^\dagger J_m,\rho\}-2J_m\rho J_m^\dagger\right)$$

where J_m are normalized and Γ sets the scale of jumps.

Physical Systems and Assumptions

We can interpret the jump operators as measurement operators in the limit of continuous measurement by a memoryless observer. Physically this could correspond to (1) qubits in a quantum circuit subject to continuous measurements, (2) photons in a leaky cavity subject to jumps to and from free space modes, or (3) a low dimensional electronic system subject to jumps to and from a substrate. For the electronic case, the substrate must have a much larger DOS than the system and this DOS must be flat over the energy range of interest for the evolution to be Markovian. The CPTP constraint limits the eigenspectrum of the Lindbladian to eigenvalues with negative (or zero) imaginary part which correspond to finite (or long lived) states with lifetimes $\tau(\epsilon)=\hbar/\mathrm{Im}(\epsilon)$. We postulate that we can study a set of time dependent systems by considering the case where the Floquet time evolution superoperator is Lindbladian.



Single Particle Lindbladian

Complex Fermion Representation

For a quadratic Lindbladian we can extend Prosen's "third-quantization" algebra [2] to place different parity sectors on equal footing. Doing so enables a concise matrix representation of the Lindbladian as expressed in terms of "left" and "right" super-fermions whose eigenfunctions are the normal modes of the full Lindbladian [3]. It reads

$$\hat{\mathcal{L}}=\mathbf{\Phi}^\dagger[L_{\mathrm{coh}}(\boldsymbol{k})-iL_{\mathrm{dis}}(\boldsymbol{k})]\mathbf{\Phi}$$

where the basis is $\mathbf{\Phi}_{\boldsymbol{k}}=(\boldsymbol{\ell}_{\boldsymbol{k}},\boldsymbol{r}_{\boldsymbol{k}},\boldsymbol{\ell}^\dagger_{-\boldsymbol{k}},\boldsymbol{r}^\dagger_{-\boldsymbol{k}})$ where the "left" and "right" super-fermions are given by

$$\ell_{\boldsymbol{k},\alpha}\rho=c_{\boldsymbol{k},\alpha}\rho\mathcal{P}\qquad r_{\boldsymbol{k},\alpha}\rho=\rho c^\dagger_{\boldsymbol{k},\alpha}\mathcal{P}$$

Matrix Form

The single particle matrix forms are

$$L_{\mathrm{dis}}=\frac{\Gamma}{2}\begin{pmatrix} A-B & -2B & C-C^\top & 2C^\top \\ -2A & B-A & -2C & C-C^\top \\ (C^\top-C)^* & -2C^* & (B-A)^\top & 2A^\top \\ 2C^\dagger & (C^\top-C)^* & 2B^\top & (A-B)^\top \end{pmatrix}$$

and $L_{\mathrm{coh}}=\mathrm{diag}(H,H,-H^\top,-H^\top)$

Where the blocks are (implicit sum over m)

$$A_{\alpha,\beta}=a^*_{m,\alpha}a_{m,\beta},\quad B_{\alpha,\beta}=b_{m,\alpha}b^*_{m,\beta},\quad C_{\alpha,\beta}=a^*_{m,\alpha}b_{m,\beta}$$

In terms of the coefficients of the jump operators

$$J_m(\boldsymbol{k})=\sum_\alpha a_{m,\alpha}(\boldsymbol{k})c_{\boldsymbol{k},\alpha}+b_{m,\alpha}(\boldsymbol{k})c^\dagger_{-\boldsymbol{k},\alpha}$$

Symmetries

We can write in terms of pseudospins for particles/holes η and left/right contours τ to make the symmetries manifest. L has Bougilibov-de Gennes form, so we expect

- Charge conjugation symmetry
  $\mathcal{C}^{-1}L^\top\mathcal{C}=-L\qquad \mathcal{C}=\eta_1\otimes\tau_0$
- Time reversal symmetry ("contour-reversal symmetry")
  $\mathcal{T}^{-1}(iL)^*\mathcal{T}=iL\qquad \mathcal{T}=\eta_2\otimes\tau_2$
- Chiral symmetry
  $\mathcal{S}^{-1}(iL)^\dagger\mathcal{S}=-iL\qquad \mathcal{S}=i\eta_3\otimes\tau_2$

(note the differences from Hermitian systems; see [4])