OTOCs

Let the OTOC be Garcia-Mata et al, Scholarpedia 18, 55237 (2023)

$$C(t) = \langle [A(t), B(0)]^\dagger [A(t), B(0)] \rangle$$

which is

$$C(t) = \langle B^\dagger(0)A^\dagger(t)A(t)B(0)\rangle - \langle B^\dagger(0)A^\dagger(t)B(0)A(t)\rangle - \langle A^\dagger(t)B^\dagger(0)A(t)B(0)\rangle + \langle A^\dagger(t)B^\dagger(0)B(0)A(t)\rangle$$

Now the first and last terms are "more trivial" correlators than the middle two since they aren't truly out of time order. Let us focus on the middle two. Now there are many equivalent ways to write the expectation value as the generalization of a Keldysh contour, see FIG 1 for two examples.

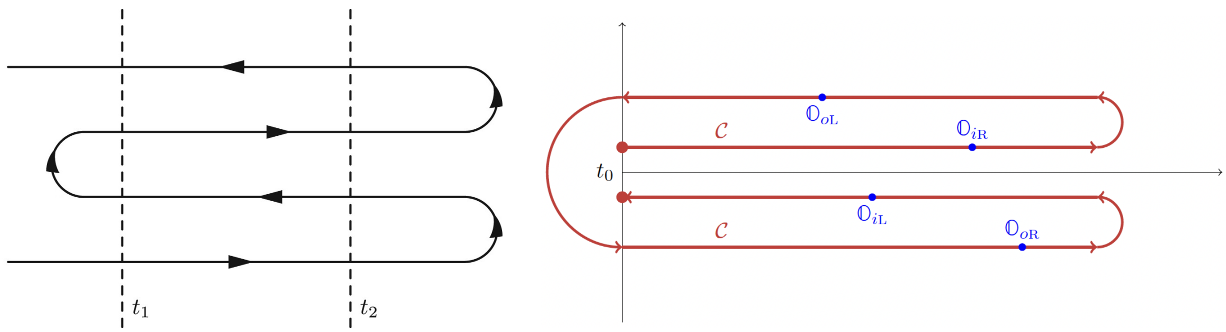
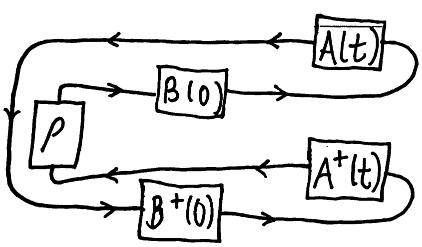


FIG 1. Aleiner et al, Ann. Phys. 375, 378 (2016), (right) Haehl et al, JHEP 6, 79 (2017)

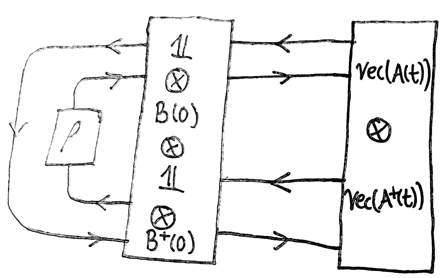
Tensor Notation for Contour Evaluation of OTOCs

Now let us write the OTOC in tensor notation where horizontal position is time (for notational clarify left of time 0 and right of time t there are no dynamics with evolution only occurring in between these times)



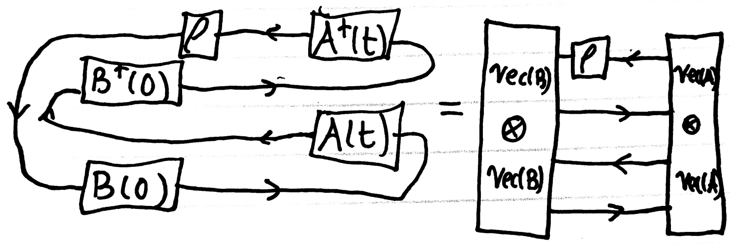
$= \langle A^\dagger(t) B^\dagger(0) A(t) B(0) \rangle$

This immediately suggests to us an alternate representation



$= \langle A^\dagger(t) B^\dagger(0) A(t) B(0) \rangle$

where we note that this is *not* just a tensor product of the vectorization of the A and B operators but has a different structure. If we were to insist on having it be a tensor product of the vectorization of *both* A and B operators, we would have to put ρ somewhere else (which seems messy)



Now Martin had suggested that the dynamics would be just a vectorization of the Louvillian superoperator, but this doesn't have quite the right structure– instead we need a direct product of two super-operators! This enables us to write the full expectation value in terms of tensor contractions in a visually straight-forward fashion