Simulability of Lindbladians

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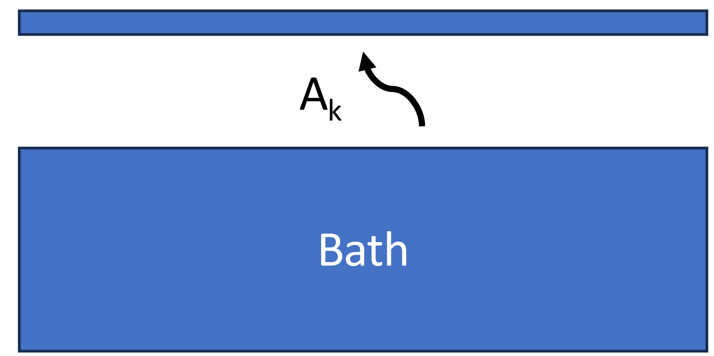
The System and Simulability

- Free fermions in the system
- Dissipation: Quadratic jumps to a bath (two or one fermions)
  - Particle number is not conserved!

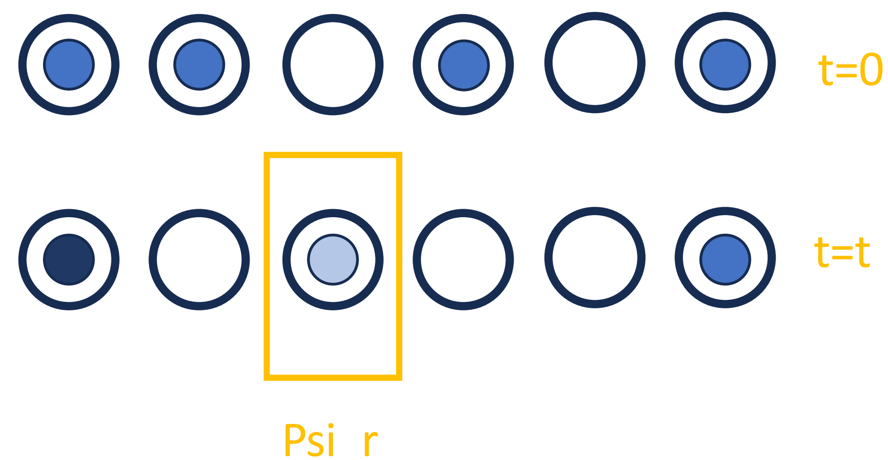


- Example: simulate P(r|r’;t) classically

$$P(\mathbf{r}|\mathbf{r}') = \langle \psi_{\mathbf{r}} | \rho(t) | \psi_{\mathbf{r}} \rangle, \quad \rho(0) = |\psi_{\mathbf{r}'}\rangle\langle\psi_{\mathbf{r}'}|,$$

Start in a random product state.

Can you get rho(t) within a tolerance?



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Time Evolution and Lindblad Master Equation

Formal solution $U(t) = \mathcal{T} e^{i \int_0^t d\bar{t}\ \mathcal{L}(\bar{t})}$ where $\mathcal{L}$ generates

$$\frac{d\rho}{dt} = -i[H(t), \rho] + \sum_{k=1}^{k_A} A_k(t)\rho A_k^\dagger(t) - \frac{1}{2}\{A_k^\dagger(t)A_k(t), \rho\},$$

Jumps (measurement) Non-Hermitian gain/loss

can build most general quadratic + linear Hamiltonian H and jumps A

$$H(t) = \frac{i}{2}\sum_{i,j=0}^{2L-1} \alpha_{ij}(t)\gamma_i\gamma_j + \sum_{i=0}^{2L-1} \beta_i(t)\gamma_i, \qquad A_k(t) = \frac{i}{2}\sum_{i,j=0}^{2L-1} a_k^{ij}(t)\gamma_i\gamma_j + \sum_{i=0}^{2L-1} b_k^i(t)\gamma_i + d_k(t),$$

not particle conserving—use Majorana fermion operators

$$\gamma_{2n} = c_n + c_n^\dagger, \qquad \gamma_{2n+1} = -i(c_n - c_n^\dagger)$$

How to express everything on the same footing? Use an *ancilla*!

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Complexity

- We have seen c.f. 662 that ground state properties are often classically simulable
  - Tensor networks/DMRG/MPOs, etc
- Area/Volume law for entanglement entropy
  - Area law has short range entanglement: can be approximated by products
  - Volume law is more difficult
- Lindbladians
  - For linear A_k there is a third quantized solution
  - For quadratic jumps simulability is often hard

TABLE I. Comparison between different types of noninteracting fermion dynamics with additional dissipation. For simplicity, we provide examples for two modes out of L, denoted by numbers 1 and 2. The symbol & means that both operators are present in the set $\mathcal{A}(t)$ with factors equal in absolute value. Abbreviations EC1, EC2, EC3 stand for easy class 1, 2, and 3 described in the text.

Type	Examples of A_k	Complexity
Dephasing	$c_1^\dagger c_1$	Easy (EC1)
Particle shuffle	$c_1^\dagger c_2$ & $c_2^\dagger c_1$	
Classical fluctuations	$c_1^\dagger$ & c_1	Easy
Classical pair fluctuations	$c_1^\dagger c_2^\dagger$ & $c_1 c_2$	
Mixing unitaries	$2c_1^\dagger c_1 - 1 + i(c_2^\dagger + c_2)$	Easy (EC2)
Single-particle loss or gain	c_1 OR $c_1^\dagger$	Easy (EC3)
Incoherent hopping	$c_1^\dagger c_2$	Hard
Pair loss or gain	$c_1 c_1$ OR $c_1^\dagger c_1^\dagger$	

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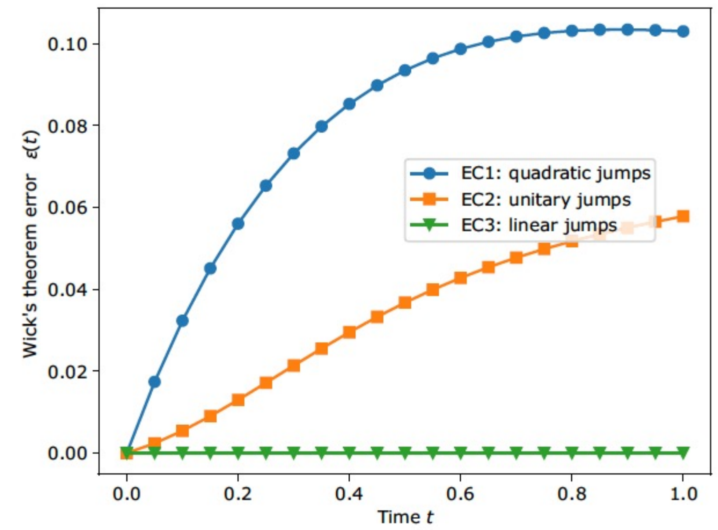
Non-Interacting Fermions and Wick's Theorem



- Often, we will want correlation functions like

$$\langle \psi_1 \psi_2 \dots \psi_{2n-1} \psi_{2n} \rangle = \frac{\delta^{2n}}{\delta \psi_1 \dots \delta \psi_{2n}} Z[\psi_1, \dots, \psi_N]$$
$$= \frac{\delta^{2n}}{\delta \psi_1 \dots \delta \bar{\psi}_{2n}} \int \mathcal{D}[\psi_1, \dots, \psi_N] e^{iS}$$

- In the case that S is quadratic, this is a Gaussian integral which we can do! The result is a sum of all two-point correlation functions (with -1s)

$$\langle \psi_1 \psi_2 \psi_3 \psi_4 \rangle = \langle \psi_1 \psi_2 \rangle - \langle \psi_1 \psi_3 \rangle + \langle \psi_2 \psi_4 \rangle$$

- For non-quadratic integrals, there will be a deviation from Wick's Thm

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Quantum Computing using Dissipation + Hopping

- Consider a charge qubit composed of two sites |0> and |1>
- Hopping between the two site can be a general one-qubit operation
  - Just accumulate a phase
- Two qubit operations can be realized using hopping between qubits at the same time as dissipation
  - Ex. CZ is generated by $H = J(c_2^\dagger c_3 + c_3^\dagger c_2)$ along with $A = \Gamma c_3 c_4$, $\Gamma \gg J$
- Then realize universal logic gates—build quantum computing logic!
  - Pauli matrices, plus phase shift, plus CZ is universal

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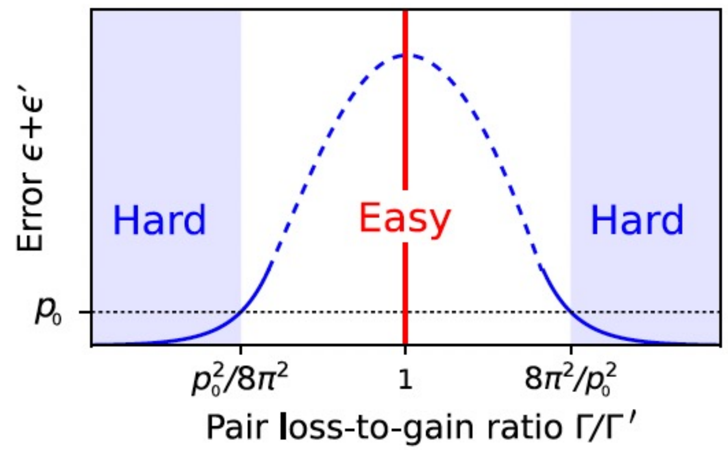
Is dissipation always bad?



- No!
- At least, no if it is controlled
- Quantum information result: if a process is as hard as general quantum computing, it can be used to realize quantum computing
  - Interacting fermionic systems are hard: 2^N versus a polynomial
- Dissipation is just a system with hopping/interactions where we choose to only focus on the behavior of a subsystem's
- Can use dissipation to realize quantum computing

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