Motivation and Introduction

The self energy is a key concept in quantum field theory. In electronic systems it enters in the expression for dressed propagators which correspond to quasiparticle excitations in the system. Quasiparticles are inherently unstable and the imaginary part of the self energy gives the lifetime of the quasiparticle.

Self Energy

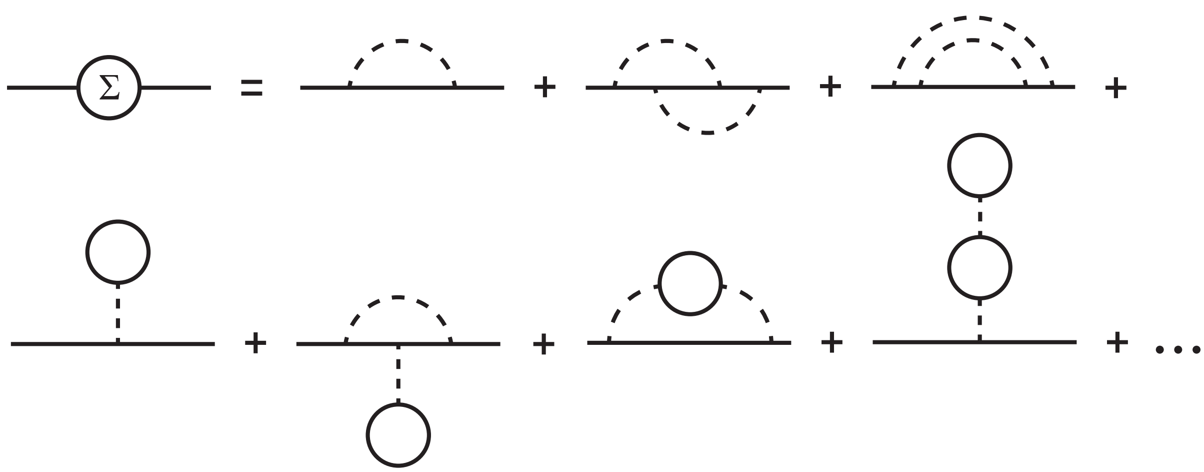
For the dressed propagator we are interested not only the propagator of the electron, but the propagator taking into account the interactions with the surrounding material. We have

$$\widetilde{G} = G + G\Sigma_R G$$

where Σ_R is the reducible self energy corresponding to all internal loops. Now, this can be rewritten recursively in terms of the irreducible self energy Σ

$$\begin{aligned}\widetilde{G} &= G + G\Sigma\widetilde{G} \\ &= G + G\Sigma G + G\Sigma G\Sigma\widetilde{G} \\ &= G + G\Sigma G + G\Sigma G\Sigma G + G\Sigma G\Sigma G\Sigma\widetilde{G}\end{aligned}$$

This is accounted for by internal loops corresponding to virtual (off-shell) processes. Pictorially, the phonon-mediated terms contributing to the self-energy Σ of the electron in a material to two loops are



where there are symmetry factors corresponding to ways to rewrite the diagrams with electrons and holes, and where the balloon diagrams involve soft-phonons with momentum zero. There can also be photon mediated Coulomb interactions. Now, these allow one to evaluate the self-energy by evaluating the loop integrals (using some renormalization procedure).

With the self energy we have the formal solution

$$\widetilde{G} = \frac{G}{1 - G\Sigma} = \frac{1}{G^{-1} - \Sigma}$$

which can be obtained by summing the geometric series, or by inverting the Green's function as a linear operator. Recall that the single particle inverse Matsubara Greens function is $G^{-1}(\boldsymbol{k}, \omega_n) = i\omega_n - H(\boldsymbol{k})$. (For the retarded Greens function take $i\omega_n \to \omega + i\eta$)

<u>*Dressed Propagator Example*</u>

We have the dressed propagator

and the bare electronic propagator

$$G(\boldsymbol{k}, \omega) = \frac{1}{\omega - H(\boldsymbol{k}) + i\eta}$$

and phononic propagator

$$g(\boldsymbol{k}, \omega)$$

vertex rule (here assumed to be a real scalar)

$$V$$

To one loop order we have (where we are sloppy with factors of i and symmetry factors)

$$\begin{aligned} \widetilde{G}(\boldsymbol{k}, \omega) = G(\boldsymbol{k}, \omega) &+ G(\boldsymbol{k}, \omega) V g(0,0) V \bigg(\int \frac{d^D k'}{(2\pi)^D} \int \frac{d\omega'}{2\pi} G(\boldsymbol{k}', \omega') \bigg) G(\boldsymbol{k}, \omega) \\ &+ G(\boldsymbol{k}, \omega) V \bigg(\int \frac{d^D k'}{(2\pi)^D} \int \frac{d\omega'}{2\pi} G(\boldsymbol{k}', \omega') g(\boldsymbol{k} - \boldsymbol{k}', \omega - \omega') \bigg) V G(\boldsymbol{k}, \omega) \end{aligned}$$

which can be evaluated and yield the self energy

$$\Sigma(\boldsymbol{k}, \omega) = G^{-1} - \widetilde{G}^{-1}$$

<u>*Self Energy and Scattering*</u>

Now this dressed propagator is the propagator for the single particle electronic quasiparticle which differs from the electron in both its propagator, and in the fact that it is not really a stable particle but instead has a lifetime. This lifetime is given by the inverse of the self energy

$$\tau = \frac{\hbar}{2\,\mathrm{Im}(\Sigma)}$$

This lifetime is the period over which the dressed propagator quasiparticle may be treated as a particle with a defined momentum and energy.