RC Circuits Charging and Discharging

We note that the current is the change in charge on the capacitor as a function of time:

$$I = \frac{dQ}{dt}$$

Using Kirchoff's Loop Rule:

$$V = IR + \frac{Q}{C}$$

Substituting for I:

$$R\frac{dQ}{dt} + \frac{1}{C}Q - V = 0$$

By the method of characteristic equations:

$$\lambda = -\frac{1}{RC}$$

This gives us a homogeneous solution of the form:

$$Q_h(t) = c_1 e^{-t/RC}$$

For the particular solution, we see the following (time-independent) form will suffice:

$$Q_p(t) = CV$$

So, the solution is:

$$Q(t) = Q_h(t) + Q_p(t) = CV(1 + c_1 e^{-t/RC})$$

With the boundary value $Q(0) = Q_0$, we find:

$$c_1 = \frac{Q_0 - CV}{CV}$$

Thus:

$$Q(t) = Q_h(t) + Q_p(t) = CV \;+\; (Q_0 - CV)e^{-t/RC}$$