Anisotropic optical response in the quasi-one-dimensional topological insulators Bi₄Br₄ and Bi₄I₄

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Layered van der Waals Materials

- Direction-dependent bond strength leads to natural cleavage planes
- Good cleavage planes makes for easy access to surface and edge states
- Many materials are stacked 2D layers



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D. Brenner, et al. JPCM 14 783 (2002)
 Z. Liu, et al. PRB 85, 205418 (2012)

Quasi-One Dimensional Bismuth Halides

- Bi₄Br₄ and Bi₄I₄ are 1D chains that align to form 3D materials^{1,2,3}
- Two natural cleavage planes
 - (001) surface: trivial insulator¹
 - (100) surface: weak/strong Tl¹

 C.C. Liu, et al. PRL **116**, 066801 (2016)
 C. Yoon, et al. arXiv 2005.14710 (2020)
 H. G. von Schnering, et al Z. Inorg. Chem. 438, **37** (1978). H. von Benda, et al. Z. Inorg. Chem. 438, **53** (1978).



Phases of the Bismuth Halides



1. C.C. Liu, et al. PRL **116**, 066801 (2016)

2. C. Yoon, et al. arXiv 2005.14710 (2020)

3. R. Noguchi, et al. Nature 566, 518 (2019)

Bulk Conductivity

- Bulk optical conductivities are anisotropic
 - $\sigma_{bb} \sim \sigma_{aa} \sim 10-50 \sigma_{cc}$
- Kubo formula calculation
 - Single particle
 - Low Temperature



Comparison to Experiment

- Experiment
 - Crystals grown on CaF₂ substrate by flux method
 - Linearly polarized light shined on exposed (001) surface
 - Measured absorbance
- Model
 - Fermi energy 137 meV below center of bulk band gap
 - Beer-Lambert Law
 - $A = (1-R)(1-e^{-l\mathcal{A}})$



Experimental data from P. Mao, et al. arXiv:2007.00223 (2020).

(100) Surface σ

- Strand direction is much more conductive than inter-strand conduction
 - We can model as an elliptic Dirac cone
- Conductivity is large
 - Graphene in Dirac-cone limit is $e^2/4\hbar$



Elliptic Dirac Cone

- Let there be distinct Fermi velocities in *x* and *y*
- Optical absorbance • $A_{dir} = \pi \alpha \frac{v_{dir}^2}{v_x v_y}$
- If we multiply/divide by $v_{\rm dir}^2/v_b v_c$, we should see the same behavior





Kubo Formula Integration

• Kubo formula (non-interacting, non-relativistic, low temperature):

$$\sigma_{\mu\nu} = i \frac{e^2}{\hbar} \sum_{s,s'} \int_{BZ} \frac{d\mathbf{k}}{(2\pi)^{\dim}} \frac{1}{\epsilon_{s'} - \epsilon_s} \frac{\langle s, \mathbf{k} | \hbar \hat{v}_{\mu} | s', \mathbf{k} \rangle \langle s', \mathbf{k} | \hbar \hat{v}_{\nu} | s, \mathbf{k} \rangle}{\hbar \omega - (\epsilon_{s'} - \epsilon_s) + i\eta}$$

• Velocity operators (in gradient approximation):

$$\widehat{v}_{\mu} = rac{\partial \mathcal{H}}{\partial k_{\mu}}$$

• Hamiltonians were obtained in our group's previous work¹

1. C. Yoon, et al. arXiv 2005.14710 (2020)

Conclusions

- Bi₄X₄ are quasi-1D materials with easy access to surface and edge states
- Our predictions for the bulk agree with experiment for $\hbar\omega>{
 m gap}$
- We predict that the (100) side surface exhibits anisotropic optical conductivity by a factor of ~15 due to anisotropic Fermi velocities
- The peak optical conductivity for (100) surface states is predicted to exceed that of graphene in the infrared region by factors of 10-20

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Questions?

Supplemental Slides

Bulk Tight-Binding Energy Bands



(100) Surface Tight-Binding Energy Bands



Monoclinic Lattice and Reciprocal Lattice



 $\overline{\mathbf{Y}}$

Bulk bb absorbance



Reprocessing Bulk o

Complex dielectric function

$$\epsilon = 1 + i \frac{4\pi}{\omega} \sigma$$

Other properties follow from textbook formulas

Where the diagonal elements of the refractive index tensor are given by:

$$n_{\mu\mu}(\omega) = \sqrt{\frac{|\epsilon_{\mu\mu}(\omega)| + \operatorname{Re}[\epsilon_{\mu\mu}(\omega)]}{2}} \tag{8}$$

and the diagonal elements of the extinction coefficient tensor κ are given by:

$$\kappa_{\mu\mu}(\omega) = \sqrt{\frac{|\epsilon_{\mu\mu}(\omega)| - \operatorname{Re}[\epsilon_{\mu\mu}(\omega)]}{2}} \tag{9}$$

The absorption coefficient is:

$$A_{\mu\mu}(\omega) = \frac{2\omega}{c} \kappa_{\mu\mu} \tag{10}$$

Following the Beer-Lambert law, the absorbance per length ℓ is:

$$\mathcal{A}_{\mu\mu}(\omega) = (1 - \mathcal{R}_{\mu\mu})(1 - \exp(-\ell A_{\mu\mu})) \qquad (11)$$

The reflectance is:

$$\mathcal{R}_{\mu\mu}(\omega) = \left|\frac{1 - \widetilde{n}_{\mu\mu}}{1 + \widetilde{n}_{\mu\mu}}\right|^2 \tag{12}$$

$$=\frac{(n_{\mu\mu}-1)^2 + \kappa_{\mu\mu}^2}{(n_{\mu\mu}+1)^2 + \kappa_{\mu\mu}^2}$$
(13)

Reprocessing Surface σ

$$\mathcal{A}_{\mu\mu} = 2\pi\alpha \frac{\operatorname{Re}(\sigma_{\mu\mu})}{e^2/\hbar} \left| \frac{2}{1 + \tilde{n}_{\mu\mu}^{\operatorname{sub}} + 2\pi\alpha \, \sigma_{\mu\mu}/(e^2/\hbar)} \right|^2 \quad (14)$$
$$\mathcal{R}_{\mu\mu} = \left| \frac{1 - \tilde{n}_{\mu\mu}^{\operatorname{sub}} - 2\pi\alpha \, \sigma_{\mu\mu}/(e^2/\hbar)}{1 + \tilde{n}_{\mu\mu}^{\operatorname{sub}} + 2\pi\alpha \, \sigma_{\mu\mu}/(e^2/\hbar)} \right|^2 \quad (15)$$

$$\mathcal{T}_{\mu\mu} = \operatorname{Re}(\widetilde{n}_{\mu\mu}^{\mathrm{sub}}) \left| \frac{2}{1 + \widetilde{n}_{\mu\mu}^{\mathrm{sub}} + 2\pi\alpha \, \sigma_{\mu\mu}/(e^2/\hbar)} \right|^2 \qquad (16)$$

Bulk Operators I

For all bulk systems $v^a = v_L^a$ and $v^b = v_L^b$. Layer

$$H_L = M\sigma_z + D + t_a\sigma_y(\sin(q_1) + \sin(q_2)) + t_b\sigma_x s_z \sin(q_2 - q_1)$$
(1)

$$M = m_0 + m_a(\cos(q_1) + \cos(q_2)) + m_b \cos(q_2 - q_1)$$
⁽²⁾

$$D = d_0 + d_a(\cos(q_1) + \cos(q_2)) + d_b\cos(q_2 - q_1)$$
(3)

$$v_L^a = \left[-(m_a \sigma_z + d_a)(\sin(q_1) + \sin(q_2)) + t_a \sigma_y(\cos(q_1) + \cos(q_2)) \right] a/2\hbar$$
(4)

$$v_L^b = \left[-(m_b \sigma_z + d_b) \sin(q_2 - q_1) + t_b \sigma_x s_z \cos(q_2 - q_1) \right] b/2\hbar$$
(5)

$$v_L^c = 0 \tag{6}$$

Bulk Operators II

Beta

$$H_{\beta} = H_L + 2(d_c + m_c \sigma_z) \cos(q_3) + 2t_c \sigma_x s_y \sin(q_3) \tag{7}$$

$$v_{\beta}^{c} = \left[-2(d_{c} + m_{c}\sigma_{z})\sin(q_{3}) + 2t_{c}\sigma_{x}s_{y}\cos(q_{3})\right]c/\hbar$$

$$\tag{8}$$

Alpha Bismuth Bromide

$$\widetilde{H}_{\beta} = H_L + (d_c + m_c \sigma_z) [\tau_x + (\tau_x \cos(q_3) + \tau_y \sin(q_3))] + t_c \sigma_x s_y [\tau_y - (\tau_y \cos(q_3) - \tau_x \sin(q_3))]$$
(9)

$$H_{\alpha-\mathrm{Br}} = H_{\beta} + (d'_0 + m'_0\sigma_z)\tau_z + t'_c\sigma_y s_y [-\tau_x + (\tau_x\cos(q_3) + \tau_y\sin(q_3))] + (d'_c + m'_c\sigma_z)s_y [\tau_y + (\tau_y\cos(q_3) - \tau_x\sin(q_3))$$
(10)

$$v_{\alpha-Br}^{c} = \left[(d_{c} + m_{c}\sigma_{z} + t_{c}^{\prime}\sigma_{y}s_{y})(-\tau_{x}\sin(q_{3}) + \tau_{y}\cos(q_{3})) + (t_{c}\sigma_{x} - d_{c}^{\prime} - m_{c}^{\prime}\sigma_{z})s_{y}(\tau_{y}\sin(q_{3}) + \tau_{x}\cos(q_{3})) \right]c/\hbar$$
(11)

Alpha Bismuth Iodide

$$H_{\alpha-I} = H_L + t\sigma_x \tau_z + t'\sigma_y s_y \tau_z + \left[(d_c + m_c \sigma_z) \tau_x + (d'_c + m'_c \sigma_z) (\tau_x \cos(q_3) + \tau_y \sin(q_3)) \right]$$
(12)

$$+ \sigma_x s_y [t_c \tau_y - t'_c (\tau_y \cos(q_3) - \tau_x \sin(q_3))]$$
(13)

$$v_{\alpha-I}^{c} = \left[(d_{c}' + m_{c}' \sigma_{z}) (-\tau_{x} \sin(q_{3}) + \tau_{y} \cos(q_{3})) + \sigma_{x} s_{y} t_{c}' (\tau_{y} \sin(q_{3}) + \tau_{x} \cos(q_{3})) \right] c/\hbar$$
(14)

(100) Surface Operators I

Beta Bismuth Bromide

$$h_{\beta-\mathrm{Br}} = (d_0 - 2d_a + d_b)1 + \eta t_b s_z q_b + 2d_c \cos(q_c) + 2\eta t_c s_y \sin(q_c)$$
$$v_{\beta-\mathrm{Br}}^b = \eta t_b s_z b$$
$$v_{\beta-\mathrm{Br}}^c = [-2d_c \sin(q_c) + 2\eta t_c s_y \cos(q_c)]c$$

Beta Bismuth Iodide

$$\begin{split} h_{\beta\text{-I}} &= (d_0 - 2d_a + d_b + 2d_c)1 + \eta t_b s_z q_b + 2\eta t_c s_y q_c \\ v^b_{\beta\text{-I}} &= \eta t_b s_z b \\ v^c_{\beta\text{-I}} &= 2\eta t_c s_y c \end{split}$$

(100) Surface Operators I

Alpha Bismuth Bromide

$$\begin{aligned} h_{\alpha-\mathrm{Br}} &= (d_0 - 2d_a + d_b)1 + \eta t_b s_z q_b + d_c (\tau_x + (\tau_x \cos(q_c) + \tau_y \sin(q_c))) \\ &+ \eta t_c s_y (\tau_y - (\tau_y \cos(q_c) - \tau_x \sin(q_c))) + d'_c s_y (\tau_y + (\tau_y \cos(q_c) - \tau_x \sin(q_c))) + d'_0 \tau_z \\ v^b_{\alpha-\mathrm{Br}} &= \eta t_b s_z b \\ v^c_{\alpha-\mathrm{Br}} &= [d_c (-\tau_x \sin(q_c) + \tau_y \cos(q_c)) + \eta t_c s_y (\tau_y \sin(q_c) + \tau_x \cos(q_c)) - d'_c s_y (\tau_y \sin(q_c) + \tau_x \cos(q_c))]c \end{aligned}$$

Alpha Bismuth Iodide

$$\begin{aligned} h_{\alpha-\mathrm{I}} &= (d_0 - 2d_a + d_b)1 + \eta t_b s_z q_b + (d_c \tau_x + d'_c (\tau_x \cos(q_c) + \tau_y \sin(q_c))) + \eta s_y (t_c \tau_y - t'_c (\tau_y \cos(q_c) - \tau_x \sin(q_c)) + \eta t \tau_z \\ v_{\alpha-\mathrm{I}}^b &= \eta t_b s_z b \\ v_{\alpha-\mathrm{I}}^c &= [d'_c (-\tau_x \sin(q_c) + \tau_y \cos(q_c)) - \eta t'_c s_y (\tau_y \sin(q_c) + \tau_x \cos(q_c))]c \end{aligned}$$