# Anisotropic optical response in the quasi-one-dimensional topological insulators $\mathrm{Bi}_{4} \mathrm{Br}_{4}$ and $\mathrm{Bi}_{4} \mathrm{I}_{4}$ 

APS March Meeting - 15 March 2021
Spenser Talkington (UCLA)
Fan Zhang (UT Dallas)


## Layered van der Waals Materials

- Direction-dependent bond strength leads to natural cleavage planes
- Good cleavage planes makes for easy access to surface and edge states
- Many materials are stacked 2D layers


Image from J. Clark, Chemistry Libretexts, 14.4 (2021). CC BY-NC-SA 3.0.

1. D. Brenner, et al. JPCM 14783 (2002)
2. Z. Liu, et al. PRB 85, 205418 (2012)

## Quasi-One Dimensional Bismuth Halides

- $\mathrm{Bi}_{4} \mathrm{Br}_{4}$ and $\mathrm{Bi}_{4} \mathrm{I}_{4}$ are 1 D chains that align to form 3D materials ${ }^{1,2,3}$
- Two natural cleavage planes
- (001) surface: trivial insulator ${ }^{1}$
- (100) surface: weak/strong TI ${ }^{1}$

1. C.C. Liu, et al. PRL 116, 066801 (2016)
2. C. Yoon, et al. arXiv 2005.14710 (2020)
3. H. G. von Schnering, et al Z. Inorg. Chem. 438, 37 (1978).
H. von Benda, et al. Z. Inorg. Chem. 438, 53 (1978).


## Phases of the Bismuth Halides




AA Stacking
$\alpha-\mathrm{Bi}_{4} \mathrm{I}_{4}$


AB Stacking

1. C.C. Liu, et al. PRL 116, 066801 (2016)
2. C. Yoon, et al. arXiv 2005.14710 (2020)
3. R. Noguchi, et al. Nature 566, 518 (2019)

## Bulk Conductivity

- Bulk optical conductivities are anisotropic
- $\sigma_{\mathrm{bb}} \sim \sigma_{\mathrm{aa}} \sim 10-50 \sigma_{\mathrm{cc}}$
- Kubo formula calculation
- Single particle
- Low Temperature



## Comparison to Experiment

- Experiment
- Crystals grown on $\mathrm{CaF}_{2}$ substrate by flux method
- Linearly polarized light shined on exposed (001) surface
- Measured absorbance
- Model
- Fermi energy 137 meV below center of bulk band gap
- Beer-Lambert Law

$$
\text { - } A=(1-R)\left(1-e^{-l \mathcal{A}}\right)
$$



Experimental data from P. Mao, et al. arXiv:2007.00223 (2020).

## (100) Surface $\sigma$

- Strand direction is much more conductive than inter-strand conduction
- We can model as an elliptic Dirac cone
- Conductivity is large
- Graphene in Dirac-cone limit is $e^{2} / 4 \hbar$





## Elliptic Dirac Cone

- Let there be distinct Fermi velocities in $x$ and $y$
- Optical absorbance
- $A_{\mathrm{dir}}=\pi \alpha \frac{v_{\mathrm{dir}}^{2}}{v_{x} v_{y}}$
- If we multiply/divide by $v_{\text {dir }}^{2} / v_{b} v_{c}$, we should see the same behavior



## Methods

| Our group's previous work in: C. Yoon, et al. arXiv 2005.14710 (2020) | Structure |
| :---: | :---: |
|  | $\downarrow$ First-principles DFT |
|  | MLWF tight-binding model <br> $\downarrow$ Best-fit nearest-neighbor tight-binding model |
|  | Hamiltonians and velocity operators <br> $\downarrow$ Integrate Kubo formula |
| The current work | Optical conductivity <br> $\downarrow$ Textbook formulas |
|  | Optical response propertie |

## Kubo Formula Integration

- Kubo formula (non-interacting, non-relativistic, low temperature):

$$
\sigma_{\mu \nu}=i \frac{e^{2}}{\hbar} \sum_{s, s^{\prime}} \int_{\mathrm{BZ}} \frac{d \boldsymbol{k}}{(2 \pi)^{\operatorname{dim}}} \frac{1}{\epsilon_{s^{\prime}}-\epsilon_{s}} \frac{\langle s, \boldsymbol{k}| \hbar \hat{v}_{\mu}\left|s^{\prime}, \boldsymbol{k}\right\rangle\left\langle s^{\prime}, \boldsymbol{k}\right| \hbar \hat{v}_{v}|s, \boldsymbol{k}\rangle}{\hbar \omega-\left(\epsilon_{s^{\prime}}-\epsilon_{s}\right)+i \eta}
$$

- Velocity operators (in gradient approximation):

$$
\hat{v}_{\mu}=\frac{\partial \mathcal{H}}{\partial k_{\mu}}
$$

- Hamiltonians were obtained in our group's previous work ${ }^{1}$


## Conclusions

- $\mathrm{Bi}_{4} \mathrm{X}_{4}$ are quasi-1D materials with easy access to surface and edge states
- Our predictions for the bulk agree with experiment for $\hbar \omega>$ gap
- We predict that the (100) side surface exhibits anisotropic optical conductivity by a factor of $\sim 15$ due to anisotropic Fermi velocities
- The peak optical conductivity for (100) surface states is predicted to exceed that of graphene in the infrared region by factors of 10-20


## Acknowledgements

- This work was supported by the National Science Foundation under Grant No. DMR-1921581 through the DMREF program, Grand No. DMR-1945351 through the CAREER program, and Grant No. PHYS1757503 through the REU program, by the Army Research Office under Grant No. W911NF-18-1-0416, and by a Barry Goldwater Scholarship.


Questions?

## Supplemental Slides

## Bulk Tight-Binding Energy Bands



## (100) Surface Tight-Binding Energy Bands

(100) Surface Tight-Binding Energy Bands for $\mathrm{Bi}_{4} \mathrm{X}_{4}$ Systems


## Monoclinic Lattice and Reciprocal Lattice


$\beta-\mathrm{Bi}_{4} \mathrm{Br}_{4}$ Brillouin Zones


## Bulk bb absorbance



## Reprocessing Bulk $\sigma$

- Complex dielectric function

$$
\epsilon=1+i \frac{4 \pi}{\omega} \sigma
$$

- Other properties follow from textbook formulas

Where the diagonal elements of the refractive index tensor are given by:

$$
\begin{equation*}
n_{\mu \mu}(\omega)=\sqrt{\frac{\left|\epsilon_{\mu \mu}(\omega)\right|+\operatorname{Re}\left[\epsilon_{\mu \mu}(\omega)\right]}{2}} \tag{8}
\end{equation*}
$$

and the diagonal elements of the extinction coefficient tensor $\kappa$ are given by:

$$
\begin{equation*}
\kappa_{\mu \mu}(\omega)=\sqrt{\frac{\left|\epsilon_{\mu \mu}(\omega)\right|-\operatorname{Re}\left[\epsilon_{\mu \mu}(\omega)\right]}{2}} \tag{9}
\end{equation*}
$$

The absorption coefficient is:

$$
\begin{equation*}
A_{\mu \mu}(\omega)=\frac{2 \omega}{c} \kappa_{\mu \mu} \tag{10}
\end{equation*}
$$

Following the Beer-Lambert law, the absorbance per length $\ell$ is:

$$
\begin{equation*}
\mathcal{A}_{\mu \mu}(\omega)=\left(1-\mathcal{R}_{\mu \mu}\right)\left(1-\exp \left(-\ell A_{\mu \mu}\right)\right) \tag{11}
\end{equation*}
$$

The reflectance is:

$$
\begin{align*}
\mathcal{R}_{\mu \mu}(\omega) & =\left|\frac{1-\widetilde{n}_{\mu \mu}}{1+\widetilde{n}_{\mu \mu}}\right|^{2}  \tag{12}\\
& =\frac{\left(n_{\mu \mu}-1\right)^{2}+\kappa_{\mu \mu}^{2}}{\left(n_{\mu \mu}+1\right)^{2}+\kappa_{\mu \mu}^{2}} \tag{13}
\end{align*}
$$

## Reprocessing Surface $\sigma$

$$
\begin{align*}
\mathcal{A}_{\mu \mu} & =2 \pi \alpha \frac{\operatorname{Re}\left(\sigma_{\mu \mu}\right)}{e^{2} / \hbar}\left|\frac{2}{1+\widetilde{n}_{\mu \mu}^{\mathrm{sub}}+2 \pi \alpha \sigma_{\mu \mu} /\left(e^{2} / \hbar\right)}\right|^{2}  \tag{14}\\
\mathcal{R}_{\mu \mu} & =\left|\frac{1-\widetilde{n}_{\mu \mu}^{\mathrm{sub}}-2 \pi \alpha \sigma_{\mu \mu} /\left(e^{2} / \hbar\right)}{1+\widetilde{n}_{\mu \mu}^{\mathrm{sub}}+2 \pi \alpha \sigma_{\mu \mu} /\left(e^{2} / \hbar\right)}\right|^{2}  \tag{15}\\
\mathcal{T}_{\mu \mu} & =\operatorname{Re}\left(\widetilde{n}_{\mu \mu}^{\mathrm{sub}}\right)\left|\frac{2}{1+\widetilde{n}_{\mu \mu}^{\mathrm{sub}}+2 \pi \alpha \sigma_{\mu \mu} /\left(e^{2} / \hbar\right)}\right|^{2} \tag{16}
\end{align*}
$$

## Bulk Operators I

For all bulk systems $v^{a}=v_{L}^{a}$ and $v^{b}=v_{L}^{b}$.
Layer

$$
\begin{align*}
& H_{L}=M \sigma_{z}+D+t_{a} \sigma_{y}\left(\sin \left(q_{1}\right)+\sin \left(q_{2}\right)\right)+t_{b} \sigma_{x} s_{z} \sin \left(q_{2}-q_{1}\right)  \tag{1}\\
& M=m_{0}+m_{a}\left(\cos \left(q_{1}\right)+\cos \left(q_{2}\right)\right)+m_{b} \cos \left(q_{2}-q_{1}\right)  \tag{2}\\
& D=d_{0}+d_{a}\left(\cos \left(q_{1}\right)+\cos \left(q_{2}\right)\right)+d_{b} \cos \left(q_{2}-q_{1}\right)  \tag{3}\\
v_{L}^{a}= & {\left[-\left(m_{a} \sigma_{z}+d_{a}\right)\left(\sin \left(q_{1}\right)+\sin \left(q_{2}\right)\right)+t_{a} \sigma_{y}\left(\cos \left(q_{1}\right)+\cos \left(q_{2}\right)\right)\right] a / 2 \hbar }  \tag{4}\\
v_{L}^{b}= & {\left[-\left(m_{b} \sigma_{z}+d_{b}\right) \sin \left(q_{2}-q_{1}\right)+t_{b} \sigma_{x} s_{z} \cos \left(q_{2}-q_{1}\right)\right] b / 2 \hbar }  \tag{5}\\
v_{L}^{c}= & 0 \tag{6}
\end{align*}
$$

## Bulk Operators II

Beta

$$
\begin{align*}
& H_{\beta}=H_{L}+2\left(d_{c}+m_{c} \sigma_{z}\right) \cos \left(q_{3}\right)+2 t_{c} \sigma_{x} s_{y} \sin \left(q_{3}\right)  \tag{7}\\
& v_{\beta}^{c}=\left[-2\left(d_{c}+m_{c} \sigma_{z}\right) \sin \left(q_{3}\right)+2 t_{c} \sigma_{x} s_{y} \cos \left(q_{3}\right)\right] c / \hbar \tag{8}
\end{align*}
$$

Alpha Bismuth Bromide

$$
\begin{equation*}
\widetilde{H}_{\beta}=H_{L}+\left(d_{c}+m_{c} \sigma_{z}\right)\left[\tau_{x}+\left(\tau_{x} \cos \left(q_{3}\right)+\tau_{y} \sin \left(q_{3}\right)\right)\right]+t_{c} \sigma_{x} s_{y}\left[\tau_{y}-\left(\tau_{y} \cos \left(q_{3}\right)-\tau_{x} \sin \left(q_{3}\right)\right)\right] \tag{9}
\end{equation*}
$$

$H_{\alpha-\mathrm{Br}}=\widetilde{H}_{\beta}+\left(d_{0}^{\prime}+m_{0}^{\prime} \sigma_{z}\right) \tau_{z}+t_{c}^{\prime} \sigma_{y} s_{y}\left[-\tau_{x}+\left(\tau_{x} \cos \left(q_{3}\right)+\tau_{y} \sin \left(q_{3}\right)\right)\right]+\left(d_{c}^{\prime}+m_{c}^{\prime} \sigma_{z}\right) s_{y}\left[\tau_{y}+\left(\tau_{y} \cos \left(q_{3}\right)-\tau_{x} \sin \left(q_{3}\right)\right)\right]$
(10)
$v_{\alpha-\mathrm{Br}}^{c}=\left[\left(d_{c}+m_{c} \sigma_{z}+t_{c}^{\prime} \sigma_{y} s_{y}\right)\left(-\tau_{x} \sin \left(q_{3}\right)+\tau_{y} \cos \left(q_{3}\right)\right)+\left(t_{c} \sigma_{x}-d_{c}^{\prime}-m_{c}^{\prime} \sigma_{z}\right) s_{y}\left(\tau_{y} \sin \left(q_{3}\right)+\tau_{x} \cos \left(q_{3}\right)\right)\right] c / \hbar$
Alpha Bismuth Iodide

$$
\begin{align*}
H_{\alpha-\mathrm{I}} & =H_{L}+t \sigma_{x} \tau_{z}+t^{\prime} \sigma_{y} s_{y} \tau_{z}+\left[\left(d_{c}+m_{c} \sigma_{z}\right) \tau_{x}+\left(d_{c}^{\prime}+m_{c}^{\prime} \sigma_{z}\right)\left(\tau_{x} \cos \left(q_{3}\right)+\tau_{y} \sin \left(q_{3}\right)\right)\right]  \tag{12}\\
& +\sigma_{x} s_{y}\left[t_{c} \tau_{y}-t_{c}^{\prime}\left(\tau_{y} \cos \left(q_{3}\right)-\tau_{x} \sin \left(q_{3}\right)\right)\right]  \tag{13}\\
& v_{\alpha-\mathrm{I}}^{c}=\left[\left(d_{c}^{\prime}+m_{c}^{\prime} \sigma_{z}\right)\left(-\tau_{x} \sin \left(q_{3}\right)+\tau_{y} \cos \left(q_{3}\right)\right)+\sigma_{x} s_{y} t_{c}^{\prime}\left(\tau_{y} \sin \left(q_{3}\right)+\tau_{x} \cos \left(q_{3}\right)\right)\right] c / \hbar \tag{14}
\end{align*}
$$

## (100) Surface Operators I

Beta Bismuth Bromide

$$
\begin{aligned}
h_{\beta-\mathrm{Br}} & =\left(d_{0}-2 d_{a}+d_{b}\right) 1+\eta t_{b} s_{z} q_{b}+2 d_{c} \cos \left(q_{c}\right)+2 \eta t_{c} s_{y} \sin \left(q_{c}\right) \\
v_{\beta-\mathrm{Br}}^{b} & =\eta t_{b} s_{z} b \\
v_{\beta-\mathrm{Br}}^{c} & =\left[-2 d_{c} \sin \left(q_{c}\right)+2 \eta t_{c} s_{y} \cos \left(q_{c}\right)\right] c
\end{aligned}
$$

Beta Bismuth Iodide

$$
\begin{aligned}
h_{\beta-\mathrm{I}} & =\left(d_{0}-2 d_{a}+d_{b}+2 d_{c}\right) 1+\eta t_{b} s_{z} q_{b}+2 \eta t_{c} s_{y} q_{c} \\
v_{\beta-\mathrm{I}}^{b} & =\eta t_{b} s_{z} b \\
v_{\beta-\mathrm{I}}^{c} & =2 \eta t_{c} s_{y} c
\end{aligned}
$$

## (100) Surface Operators I

## Alpha Bismuth Bromide

```
\(h_{\alpha-\mathrm{Br}}=\left(d_{0}-2 d_{a}+d_{b}\right) 1+\eta t_{b} s_{z} q_{b}+d_{c}\left(\tau_{x}+\left(\tau_{x} \cos \left(q_{c}\right)+\tau_{y} \sin \left(q_{c}\right)\right)\right)\)
    \(+\eta t_{c} s_{y}\left(\tau_{y}-\left(\tau_{y} \cos \left(q_{c}\right)-\tau_{x} \sin \left(q_{c}\right)\right)\right)+d_{c}^{\prime} s_{y}\left(\tau_{y}+\left(\tau_{y} \cos \left(q_{c}\right)-\tau_{x} \sin \left(q_{c}\right)\right)\right)+d_{0}^{\prime} \tau_{z}\)
\(v_{\alpha-\mathrm{Br}}^{b}=\eta t_{b} s_{z} b\)
\(v_{\alpha-\mathrm{Br}}^{c}=\left[d_{c}\left(-\tau_{x} \sin \left(q_{c}\right)+\tau_{y} \cos \left(q_{c}\right)\right)+\eta t_{c} s_{y}\left(\tau_{y} \sin \left(q_{c}\right)+\tau_{x} \cos \left(q_{c}\right)\right)-d_{c}^{\prime} s_{y}\left(\tau_{y} \sin \left(q_{c}\right)+\tau_{x} \cos \left(q_{c}\right)\right)\right] c\)
```

Alpha Bismuth Iodide

$$
\begin{aligned}
h_{\alpha-\mathrm{I}} & =\left(d_{0}-2 d_{a}+d_{b}\right) 1+\eta t_{b} s_{z} q_{b}+\left(d_{c} \tau_{x}+d_{c}^{\prime}\left(\tau_{x} \cos \left(q_{c}\right)+\tau_{y} \sin \left(q_{c}\right)\right)\right)+\eta s_{y}\left(t_{c} \tau_{y}-t_{c}^{\prime}\left(\tau_{y} \cos \left(q_{c}\right)-\tau_{x} \sin \left(q_{c}\right)\right)+\eta t \tau_{z}\right. \\
v_{\alpha-\mathrm{I}}^{b} & =\eta t_{b} s_{z} b \\
v_{\alpha-\mathrm{I}}^{c} & =\left[d_{c}^{\prime}\left(-\tau_{x} \sin \left(q_{c}\right)+\tau_{y} \cos \left(q_{c}\right)\right)-\eta t_{c}^{\prime} s_{y}\left(\tau_{y} \sin \left(q_{c}\right)+\tau_{x} \cos \left(q_{c}\right)\right)\right] c
\end{aligned}
$$

