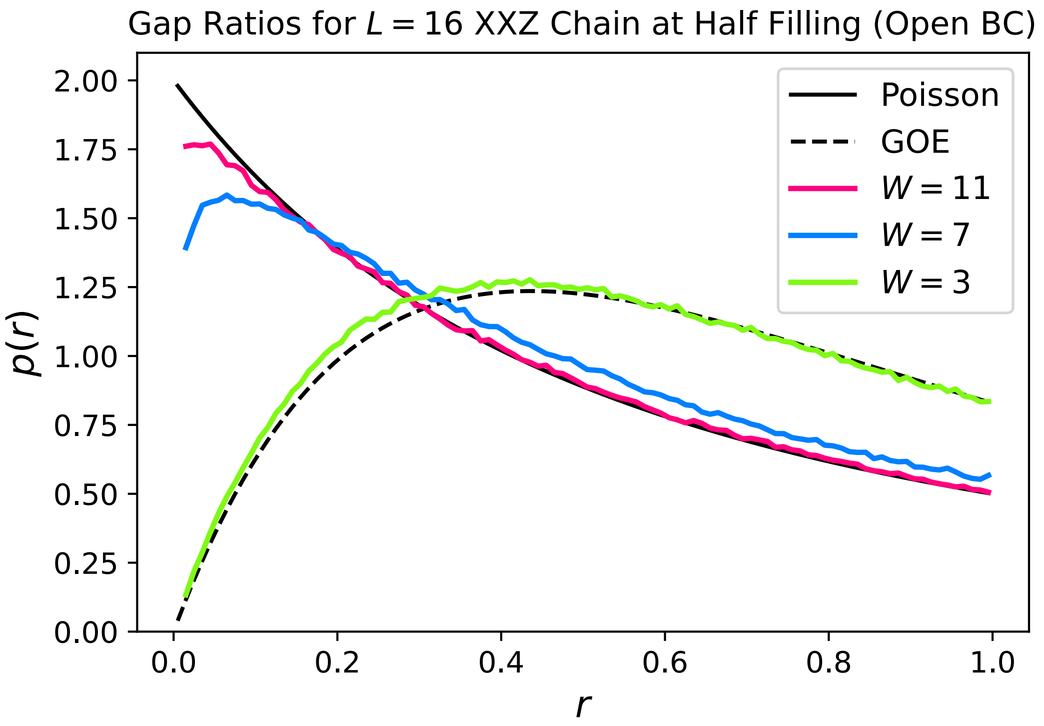
Level Statistics of Thermalization and Many-Body Localization

Spenser Talkington • 25 April 2023

Physics 662: Numeric Many-Body Methods

University of Pennsylvania



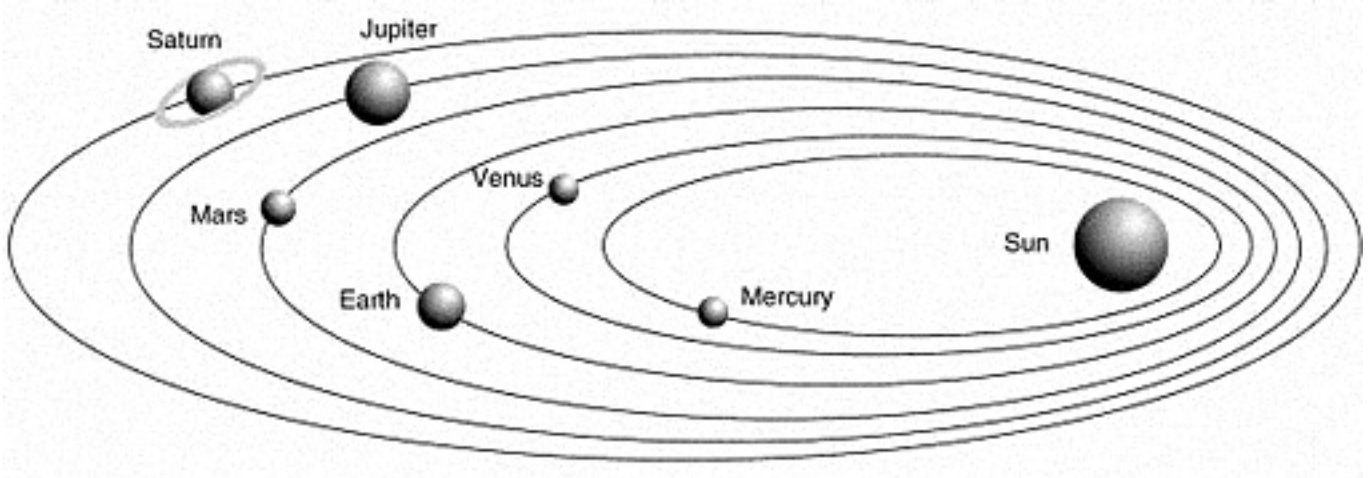
Localization

- How can we keep something somewhere?
  - Quantum tunneling would seem to prevent this
  - This is key to the storage of information
- With fermions Pauli blocking
  - Get a band insulator (in the charge sector)
- Quantum interference
  - Anderson localization!
  - Like in flat bands!
  - $\text{Im}(\Sigma) \neq 0$
    - (No Anderson localization in open quantum systems)
- Can many-body systems localize?

	Structure	Quantum interference
k-space	Trivial flat band	Topological flat band
x-space	Particle in box	Anderson localization

Integrability

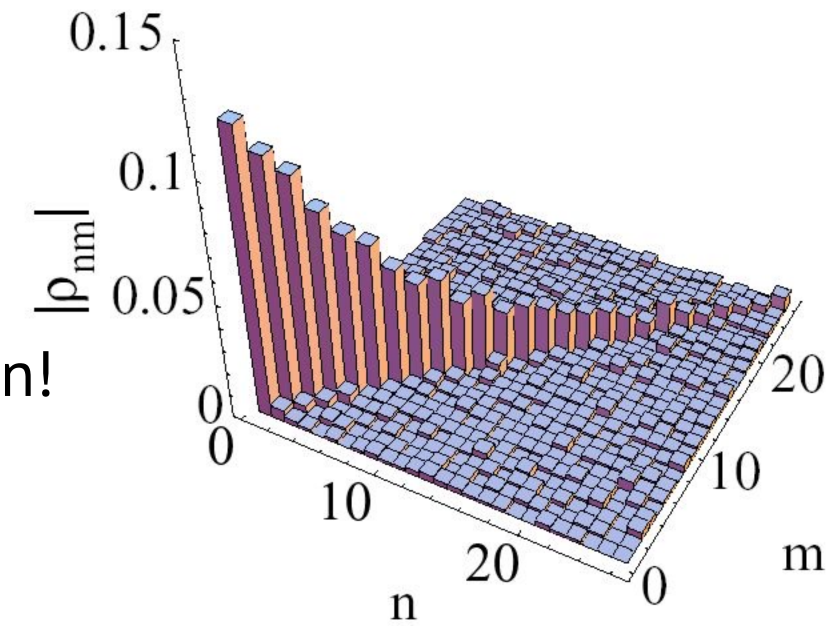
- When is a model solvable?
- Classical mechanics
  - Simple harmonic oscillator
    - Conserved E
  - Two-body gravitation
    - Runge-Lenz vector
    - Angular momentum



- Conserved quantities! Closed orbits and localization of information
- These quantities are *extensive*
- In quantum systems we look for extensive and *local* quantities
  - These may be a local (U(1)) charge, or something more exotic

Diffusivity and Thermalization

- Arbitrarily small perturbations generically lead to open trajectories
- These open trajectories are often diffusive
  - Trajectories fill parameter space
  - *Ergodicity*
- At long times information thermalizes
  - Stored in non-local degrees of freedom
  - Even pure states generated by unitary evolution!
- Can one avoid thermalization?
  - If it stays integrable
  - If there is many body Anderson localization
  - (Also driven and dissipative systems, local measurements)



Level Statistics

- Localized
  - States don't know about each other
  - Levels can be degenerate
  - Poisson statistics
- Delocalized
  - Random matrix theory statistics
  - Levels will repel
  - "Wigner-Dyson"/GOE/GUE/GSE
- Ratios are more robust than levels

$$r_n = \frac{\min(E_{n+1} - E_n, E_{n+2} - E_{n+1})}{\max(E_{n+1} - E_n, E_{n+2} - E_{n+1})}$$

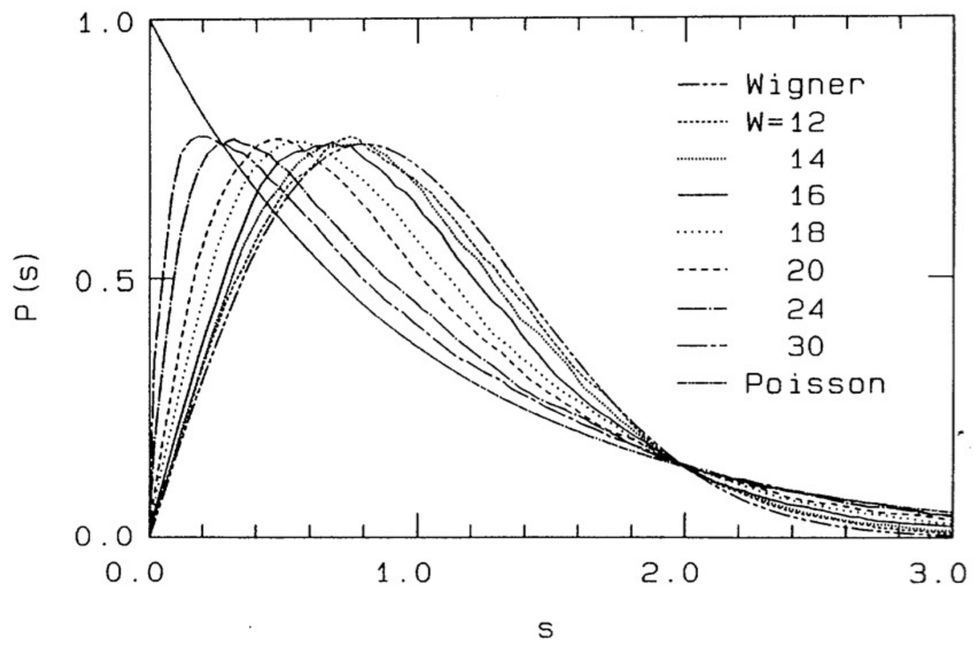


FIG. 1. Nearest-neighbor spacing function $P(s)$ for $L=12$ and different W. Wigner and Poisson functions $P_W(s)$ and $P_P(s)$ are also shown for comparison.

Shklovskii, *et al*, PRB 47, 11487 (1993)

XXZ Spin Model

- Let us consider $H = H_{XXZ} + H_{\|} + H_{\perp}$
- The XXZ model is integrable via the Bethe ansatz

$$H_{XXZ} = J \sum_{i=1}^{L-1} (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + J_z \sum_{i=1}^{L-1} S_i^z S_{i+1}^z$$

- We can break integrability with a parallel field (non-local JW string)

$$H_{\|} = h_x \sum_{i=1}^{L} S_i^x$$

- And can attempt to localize with a transverse field (onsite potential)

$$H_{\perp} = \sum_{i=1}^{L} h_i S_i^z$$

Jordan-Wigner Transformation

- Use our old friend to go from spins to fermions

$$S_i^x = \frac{1}{2}(S_i^+ + S_i^-)$$

$$S_i^y = \frac{1}{2i}(S_i^+ - S_i^-)$$

$$S_i^+ = e^{-i\pi \sum_{j<i} n_j} c_i^\dagger$$

$$S_i^- = e^{i\pi \sum_{j<i} n_j} c_i$$

$$S_i^z = c_i^\dagger c_i - \tfrac{1}{2}$$

$$H_{XXZ} = \frac{J}{2} \sum_{i=1}^{L-1} c_{i+1}^\dagger c_i + c_i^\dagger c_{i+1} + J_z \sum_{i=1}^{L-1} (n_i - \tfrac{1}{2})(n_{i+1} - \tfrac{1}{2})$$

$$H_\parallel = \frac{h_x}{2} \sum_{i=1}^{L} e^{-i\pi \sum_{j<i} n_j} c_i^\dagger + e^{i\pi \sum_{j<i} n_j} c_i$$

$$H_\perp = \sum_{i=1}^{L} h_i (n_i - \tfrac{1}{2})$$

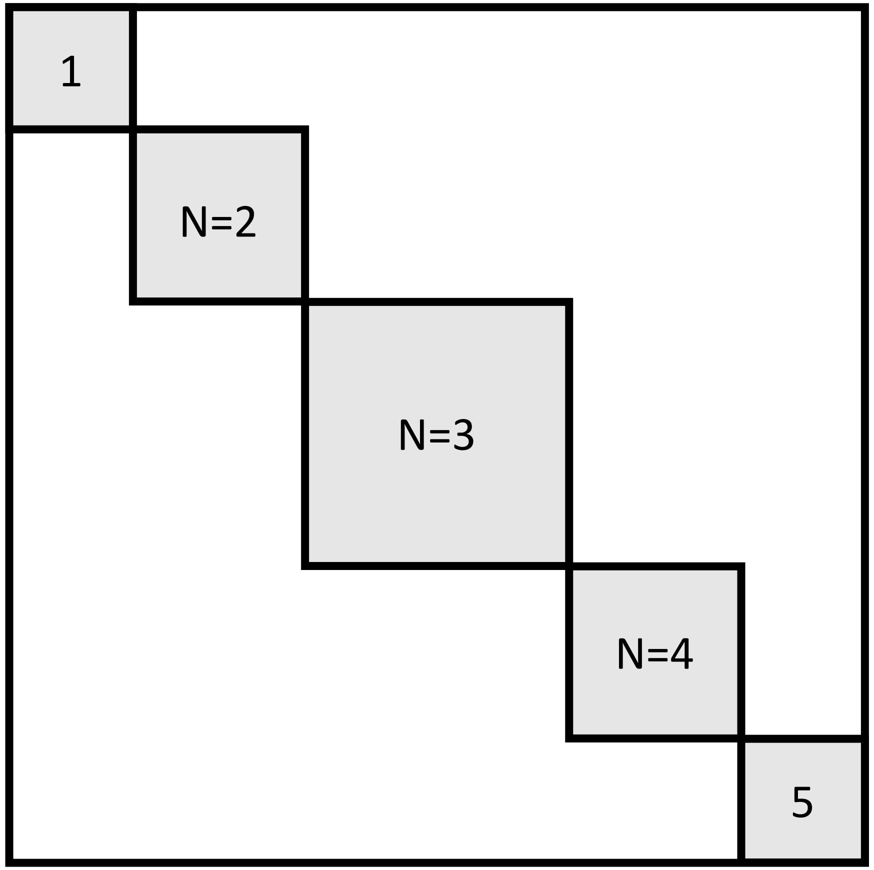
- $H_\parallel$ is highly nonlocal—no conserved particle number!

Make it Better for Numeric Study

- Make it conserve particle number
- But still need to break integrability
- Next nearest neighbor hopping!

$$H_{\parallel} \quad \mapsto \quad \sum_{i=1}^{L-2} c_i^{\dagger} c_{i+2} + c_{i+2}^{\dagger} c_i$$

- “Frustrates” locally conserved moments
- But still conserves total particle number
- This can be studied much faster!



Exact Diagonalization

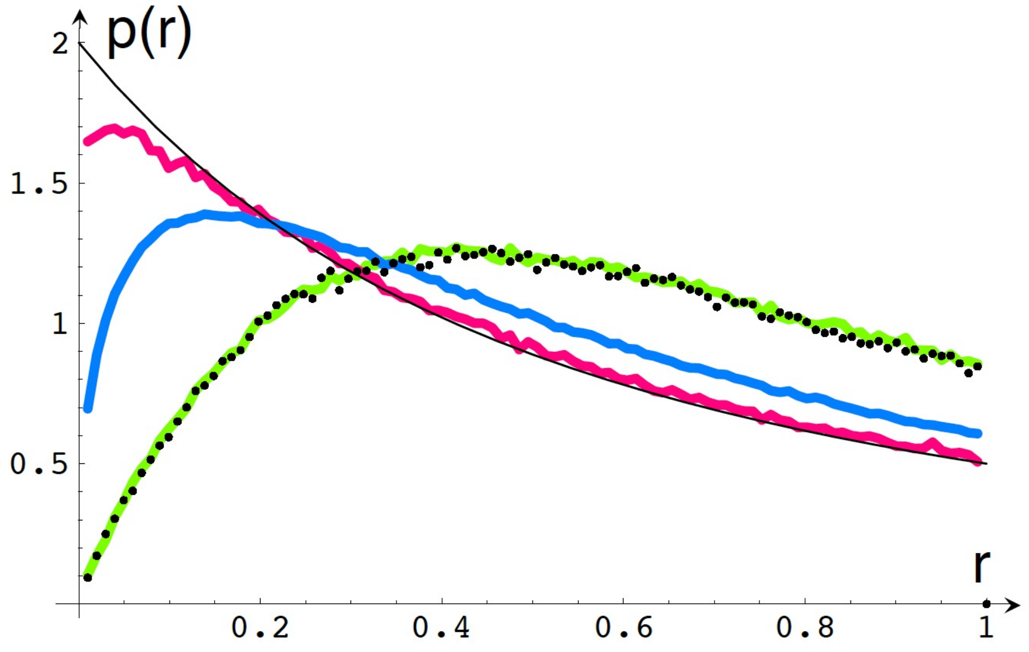
- Fixed particle number sector: half filling

$$\binom{10}{5} = 252 \quad < \quad 1024 = 2^{10} \qquad\qquad \binom{16}{8} = 12870 \quad < \quad 65536 = 2^{16}$$

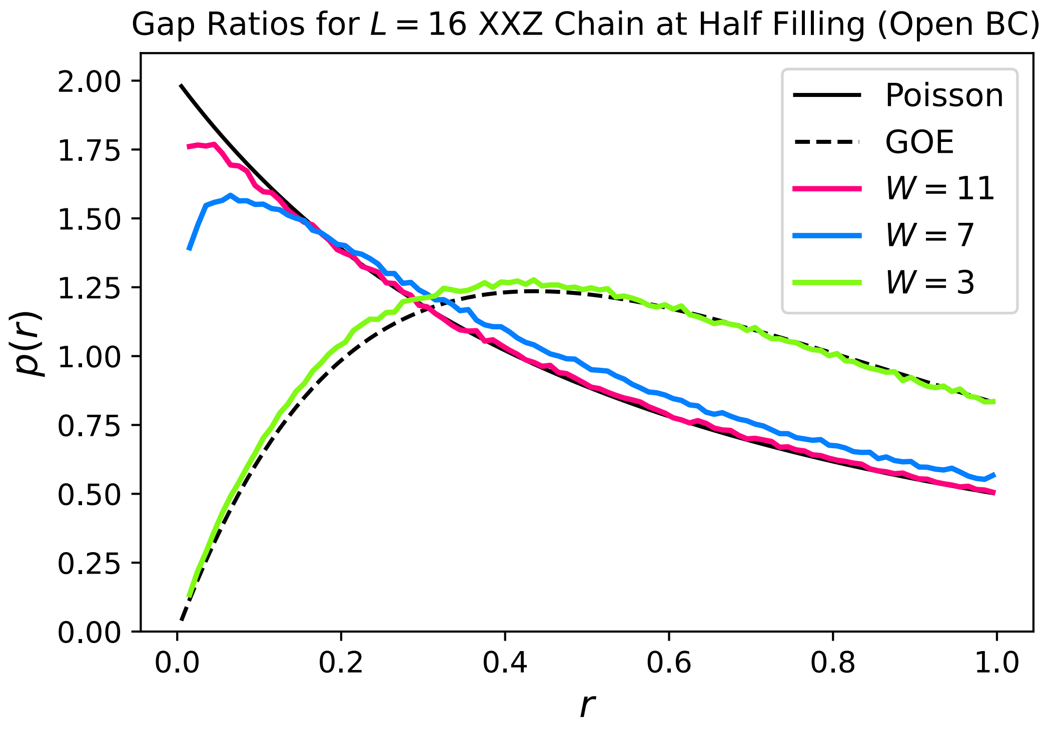
- Cache number operators, Hamiltonian, disorder term
  - Just update a scalar multiple
- Speedup
  - 125x from fixed particle number sector (diagonalization goes as N^3)
  - ~100x from caching: no matrix multiplications or constructions, just add
  - Overall: 10^4 speedup vs naïve implementation
  - Consequence: can study 16 spins vs 12 spins
  - This is still not the thermodynamic limit!

Results

- Reproduced level ratio statistics from a paradigmatic paper



Oganesyan and Huse, PRB 75, 155111 (2007)



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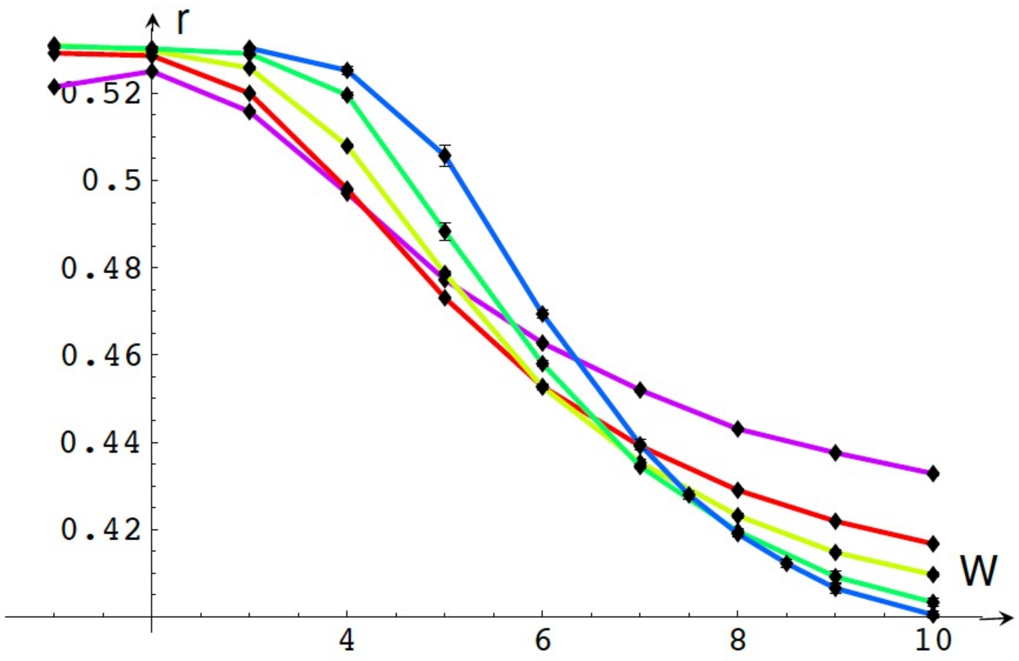

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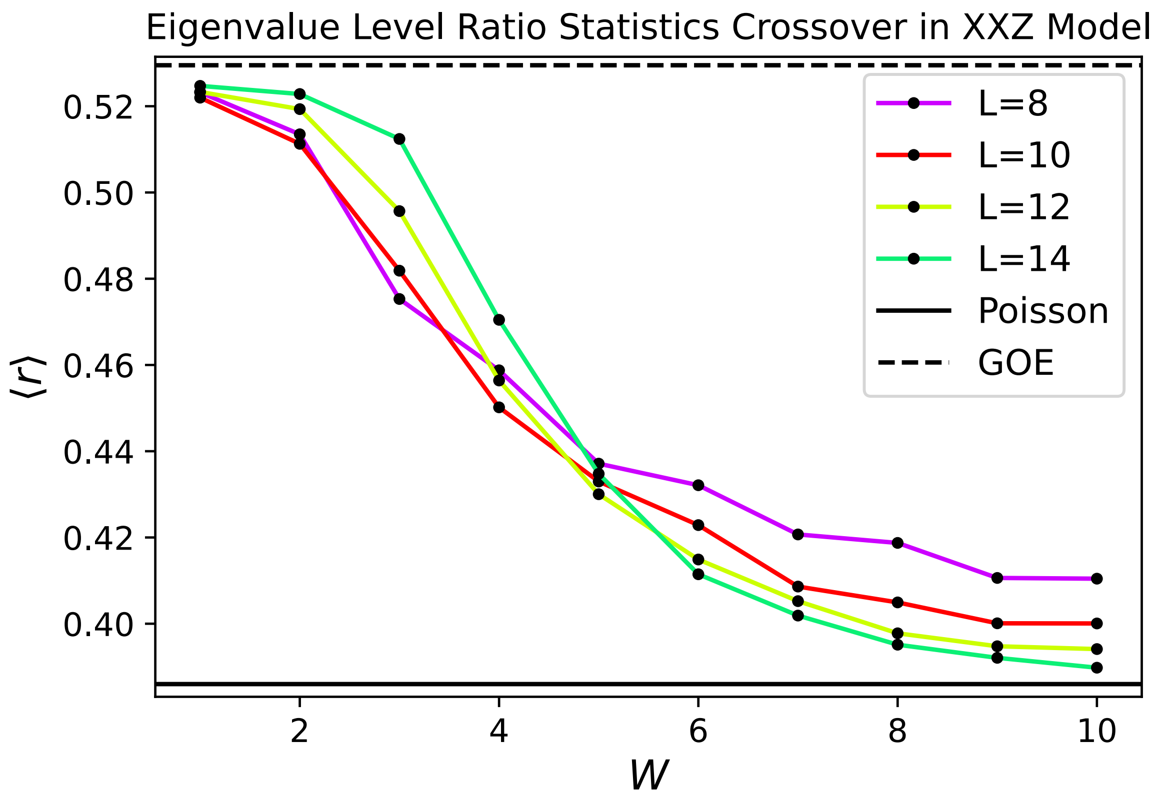

Controversy

- PRE 102, 062144 (2020)
  - Finite size scaling L<=18 indicates the absence of MBL: quantum chaos
- Ann. Phys. 427, 168415 (2021)
  - Maybe MBL is still present? L<=20
- PRE 104, 054105 (2021) and PRB 104, 201117 (2021)
  - It doesn't look like MBL is present at small disorder strengths L<=24
- Can you localize information forever?
  - Perhaps not
  - Does it matter for quantum computation?
    - Probably not

Outlook

- Localization of information is crucial for quantum computing devices
- In single particle quantum mechanics interference can lead to localization, but the picture in many body QM is less clear
- Level statistics are one tool to diagnose localization
  - Lyapunov exponents/entanglement/self energy are other tools
- At intermediate system sizes these seem to indicate localization
- At larger system sizes these no longer indicate localization
  - What’s up?