Optical Absorption of Graphene in the Dirac Cone Limit

Let the incident energy flux be:

$$W_{\text{incident}} = \frac{\omega^2}{4\pi c}|A|^2$$

And the absorbed power be:

$$W_{\text{absorbed}} = \langle w \rangle \hbar\omega$$

Where Fermi's Golden rule gives the transition rate at low temperature ($E_f = \hbar\omega/2$ is the final energy):

$$w = \frac{2\pi}{\hbar}|\langle f|H'|i\rangle|^2 D(E_f)$$

Now for graphene in the Dirac-Cone limit, with $k = \sqrt{k_x^2 + k_y^2}$ and $\theta = \arg(k_x + ik_y)$:

$$\begin{aligned} H_0 &= v\hbar(k_x\sigma_x + k_y\sigma_y) \\ &= v\hbar k \begin{pmatrix} 0 & e^{-i\theta} \\ e^{i\theta} & 0 \end{pmatrix} \end{aligned}$$

As above, we consider the perturbation:

$$\begin{aligned} H' &= -\frac{e|A|}{\hbar c}\frac{\partial H_0}{\partial k_x} \\ &= -\frac{ev|A|}{c}\sigma_x \end{aligned}$$

So Fermi's Golden rule gives:

$$w = \frac{2\pi}{\hbar}\left(\frac{ev|A|}{c}\right)^2 \sin^2(\theta) D(E_f)$$

Where the average may be evaluated over all states:

$$\langle w \rangle = \frac{1}{2\pi}\int_0^{2\pi} d\theta\ w = \frac{\pi}{\hbar}\left(\frac{ev|A|}{c}\right)^2 D(E_f)$$

The number of states per volume is (with $E = \hbar v k$):

$$\frac{N}{V} = \left(\frac{1}{2\pi}\right)^2 \int 2\pi k dk = \int \frac{2\pi E dE}{(2\pi\hbar v)^2}$$

So the density of states is (with $E_f = \hbar\omega/2$):

$$D(E_f) = \frac{\hbar\omega}{4\pi(\hbar v)^2}$$

So, we see that the absorption is:

$$\begin{aligned} P(\omega) &= \frac{\langle w \rangle \hbar\omega}{W_{\text{incident}}} \\ &= \hbar\omega\frac{\pi}{\hbar}\left(\frac{ev|A|}{c}\right)^2 \frac{\hbar\omega}{4\pi(\hbar v)^2}\frac{4\pi c}{\omega^2|A|^2} \\ &= \frac{\pi e^2}{\hbar c} \\ &= \pi\alpha \end{aligned}$$

Which is independent of frequency.