Floquet Lindbladians?

Penn CM Theory Journal Club

Spenser Talkington

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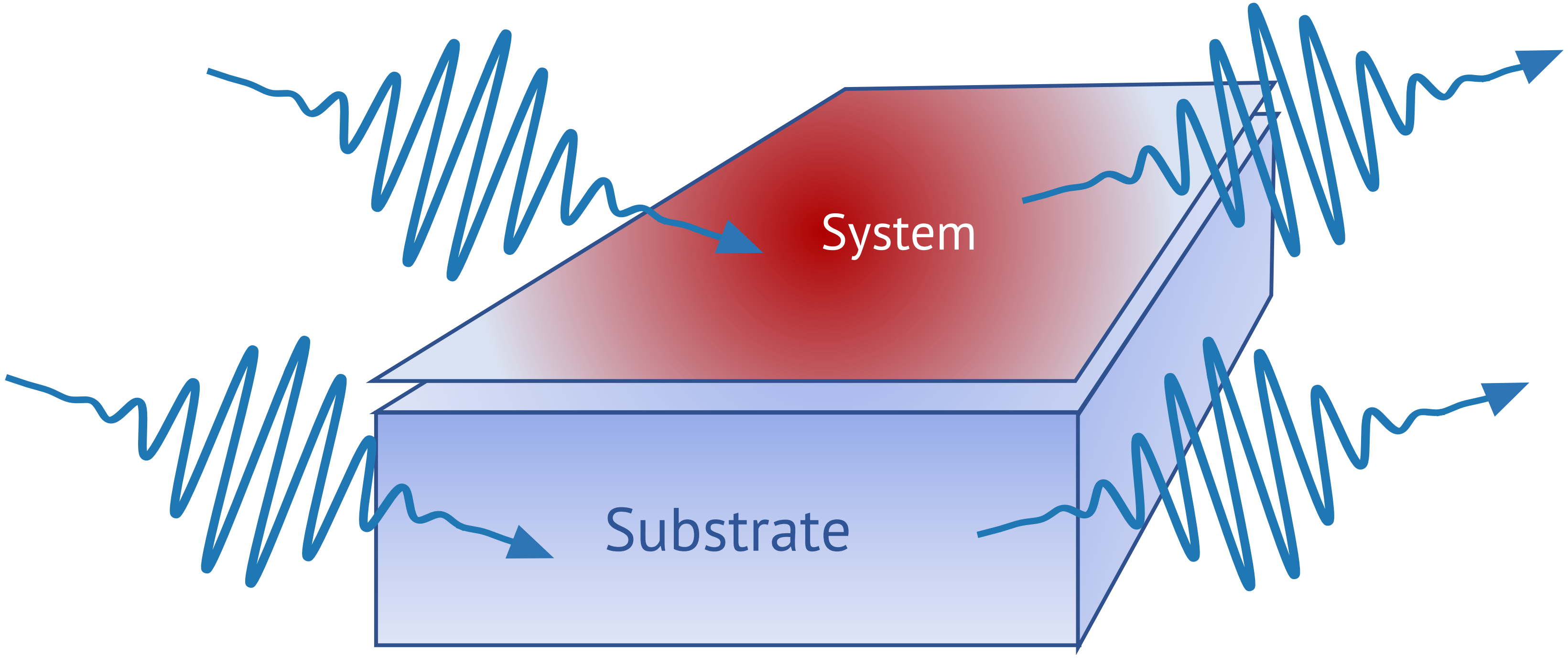


The Question

- Can we study the strong-driving limit of dissipative systems?
  - How to do so with a minimum of pain?
  - Time periodic driving: simplify with Floquet's theorem
  - Memoryless bath: reduce to a Lindblad master equation
- Can these two conditions be consistent?
  - 1. Relaxation processes in the bath are much faster than the drive and only the system and not the bath is driven
  - 2. The Floquet time evolution superoperator is Lindbladian, but the instantaneous time evolution superoperator may not be
- References covered in this journal club presentation
  - PRB 101, 100301 (2020)
  - Ann. Rev. Cond. Mat. Phys. 14, 35 (2023)
  - arXiv:2401.00131

Lindblad Master Equation (Non-Driven)

- Time evolution in the limit of continuous measurement by a memoryless bath $i\dot{\rho} = \mathcal{L}[\rho]$

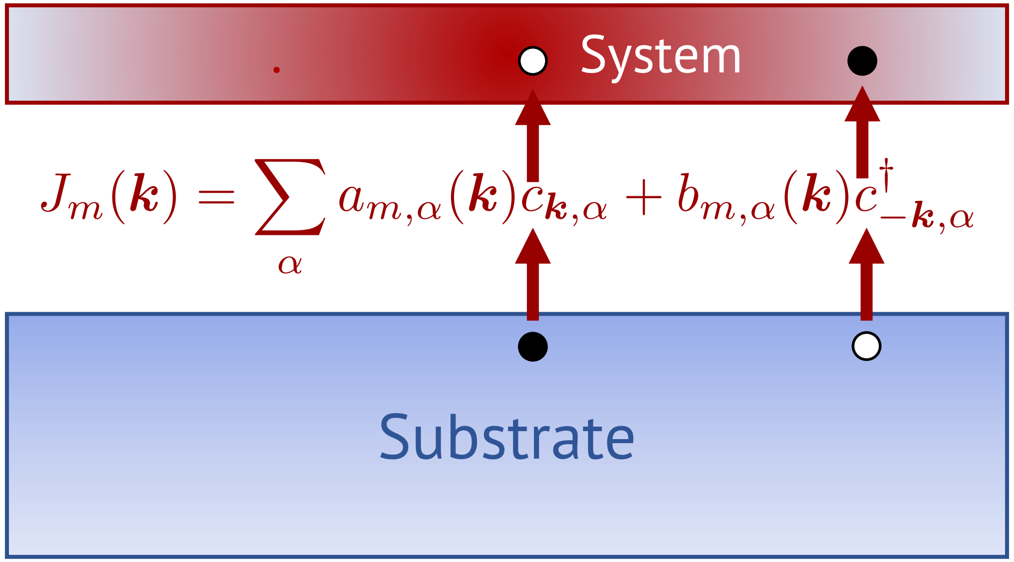
$$i\dot{\rho} = (\mathcal{L}^{\text{coh.}} + \mathcal{L}^{\text{n.h.}} + \mathcal{L}^{\text{jump}})[\rho]$$

- Expressed as

$$\mathcal{L}^{\text{coh.}}[\rho] = [H, \rho]$$

$$\mathcal{L}^{\text{n.h.}}[\rho] = -i\frac{\Gamma}{2}\sum_m \{J_m^\dagger J_m, \rho\}$$

$$\mathcal{L}^{\text{jump}}[\rho] = i\frac{\Gamma}{2}\sum_m 2J_m \rho J_m^\dagger$$



Jump operators linear in fermions ensure the action is quadratic.
J_m tunnels in a superposition of particles and holes in bands α.

Floquet's Theorem (Closed Systems)

- We are interested in periodic drives $\mathcal{H}(\tau) = \mathcal{H}(\tau + T)$
- Where generically we have

$$\mathcal{U}(\tau_1, \tau_0) = \mathcal{T} \exp\left[-\frac{i}{\hbar}\int_{\tau_0}^{\tau_1} \mathcal{H}(\tau)\,\mathrm{d}\tau\right]$$

- Which for the Floquet problem reduces to

$$\mathcal{U}(\tau_0 + T, \tau_0) \equiv e^{-i\mathcal{H}_{\mathrm{F}} \times T/\hbar}$$

With time-independent H_F

- Where for inter-cycle motion one constructs a "kick operator"
- We then have the Floquet modes and quasienergies $U_F \left|u_j\right\rangle = e^{-i\epsilon_j T} \left|u_j\right\rangle$

Konrad Viebahn's Boulder School notes

What One Can Say *When* A F.L. Exists 1

- Like closed systems one can consider the construction

$$\mathcal{U}(t,t_0)=\mathcal{T}e^{\int_{t_0}^{t_0+\tau}\mathcal{L}(t')dt'}(\mathcal{U}_F)^m=\mathcal{U}(t_0+\tau,t_0)(\mathcal{U}_F)^m$$

- Which also permits a decomposition in terms of Floquet modes and quasienergies $\mathcal{U}_F\rho_j=e^{\lambda_j T}\rho_j$, except the modes are matrices and the quasienergies are complex
- The modes *are not* necessarily physical (Hermitian, positive, Tr=1)
- But, for Lindbladian we are guaranteed at least one steady state
  - Hence the same holds if we are given a Floquet Lindbladian

2401.00131

What One Can Say When A F.L. Exists 2

- Key results in 2401.00131

1. There always exists at least one λ_j such that $|e^{\lambda_j T}| = 1$
   - Note: This can be thought of as the *definition* of existence of the F.L.
2. The space of such λ_j is known as the "steady state" space
3. Any physical state has support on the steady state space
4. After the non-steady state modes decay the steady state will be given by the stroboscopic evolution in the steady state space

So When Does a F.L. Exist?

- To my knowledge the *general* conditions under which a time-periodic superoperator evolution remains CPTP and permits a time-independent Floquet superoperator is unknown
- Diagnostics:
  - Spectrum: if there are any positive imaginary (negative real in other conventions) eigenvalues the evolution is *not* CPTP
  - Distance from a CPTP map: strength of noise terms (Commun. Math. Phys. 310, 383 (2012))
  - Branches of the log of *U* (Phys. Rev. Lett. 101, 150402)
  - Positivity of maximally entangled state under Choi-Jamiołkowski isomorphism (Phys. Rev. Lett. 105, 050403 (2010))

PRB 101, 100301 (2020)

Bonus: how does one write a F.L. ?

- Go through T. Mori Section 3