Mechanical Equilibrium

We say that mechanical equilibrium occurs when an object is at rest and not accelerating. This is the situation where there are no forces acting on the object, or the forces exactly balance. Equivalently, mechanical equilibrium is a critical point of the energy (since $F = -\nabla U$).

Example: hanging spring block system

Consider a spring with spring constant k and unstretched length ℓ_0 connected to the ceiling with a mass m attached so that the new equilibrium length is ℓ. What is $\ell - \ell_0$?

Method 1: Forces sum to zero

$$F_{\text{spring}} + F_{\text{gravity}} = 0$$

Or:

$$kx - mg = 0$$

Thus, with $x \equiv \ell - \ell_0$:

$$\ell - \ell_0 = \frac{mg}{k}$$

Method 2: Stationary point of the energy

$$\frac{\partial E}{\partial v} = \frac{\partial E}{\partial x} = 0$$

Where:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 - mgx$$

Evidently $\partial_v E = 0$ when $v = 0$, and we see:

$$\frac{\partial E}{\partial x} = kx - mg = 0$$

Which is identical to the above, so:

$$\ell - \ell_0 = \frac{mg}{k}$$