Dissipative Ising Model

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Transverse Field XY Model

spins

$$H = H_{XY} + H_{\perp}$$

$$H_{XY} = \sum_{n=1}^{N-1} J_n^x S_n^x S_{n+1}^x + J_n^y S_n^y S_{n+1}^y.$$

$$H_{\perp} = \sum_{n=1}^{N} h_n S_n^z,$$

Jordan-Wigner

$$S_n^x = \tfrac{1}{2}(S_n^+ + S_n^-)$$

$$S_n^y = \tfrac{1}{2i}(S_n^+ - S_n^-)$$

$$S_n^+ = e^{-i\pi \sum_{m<n} c_m^\dagger c_m} c_n^\dagger$$

$$S_n^- = e^{i\pi \sum_{m<n} c_m^\dagger c_m} c_n$$

$$S_n^z = c_n^\dagger c_n - \tfrac{1}{2},$$

fermions

$$H_{XY} = \frac{1}{4} \sum_{n=1}^{N-1} (J_n^x - J_n^y) c_n^\dagger c_{n+1}^\dagger + (J_n^x + J_n^y) c_n^\dagger c_{n+1} + h.c.$$

$$H_{\perp} = \sum_{n=1}^{N} h_n (c_n^\dagger c_n - \tfrac{1}{2}).$$

TFIM

Jx=1, Jy=0, h=const

Density from Keldysh Green's Function

definitions

$$G^<_{\alpha\beta}(t,t') = -ie^{i\phi}\langle c^\dagger_\beta(t')c_\alpha(t)\rangle$$
$$G^>_{\alpha\beta}(t,t') = -i\langle c_\alpha(t)c^\dagger_\beta(t')\rangle,$$

$$G^R_{\alpha\beta}(t,t') = +\theta(t-t')[G^>_{\alpha\beta}(t,t') - G^<_{\alpha\beta}(t,t')]$$
$$G^A_{\alpha\beta}(t,t') = -\theta(t'-t)[G^>_{\alpha\beta}(t,t') - G^<_{\alpha\beta}(t,t')]$$
$$G^K_{\alpha\beta}(t,t') = G^>_{\alpha\beta}(t,t') + G^<_{\alpha\beta}(t,t').$$

density

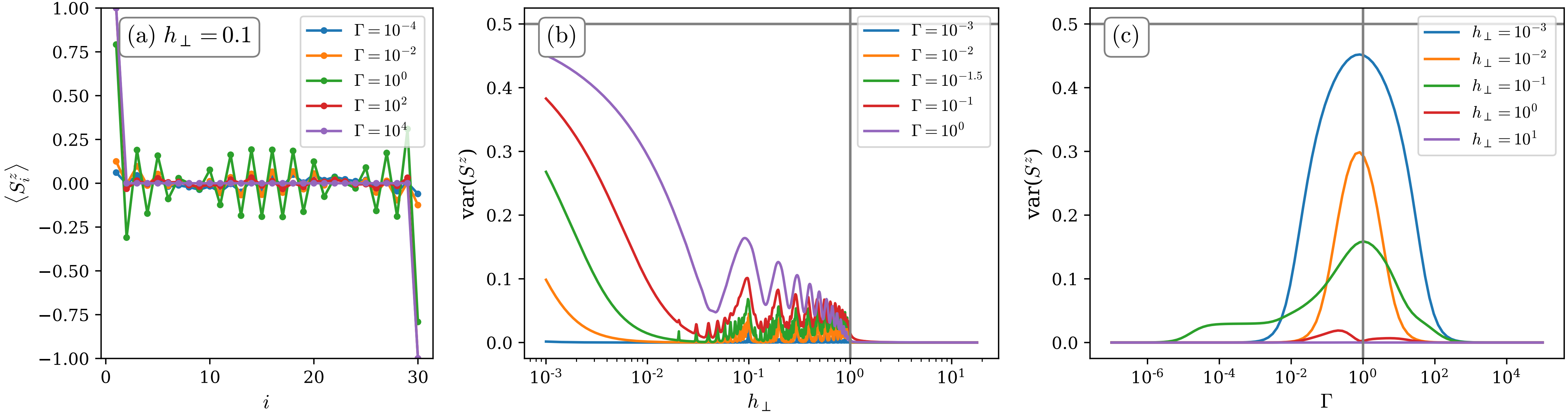
$$\begin{aligned}\langle c^\dagger_\alpha(t)c_\beta(t)\rangle &= \frac{i}{2}e^{i\phi}(G^K - G^R + G^A)_{\beta\alpha}(t,t)\\ &= \frac{1}{2}e^{i\phi}\left(\delta_{\beta\alpha} - iG^K_{\beta\alpha}(t,t)\right),\end{aligned}$$

$$\langle n_\alpha\rangle = \frac{e^{i\phi}}{2}\left(1 + \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega\ G^K_{\alpha\alpha}(\omega)\right)$$

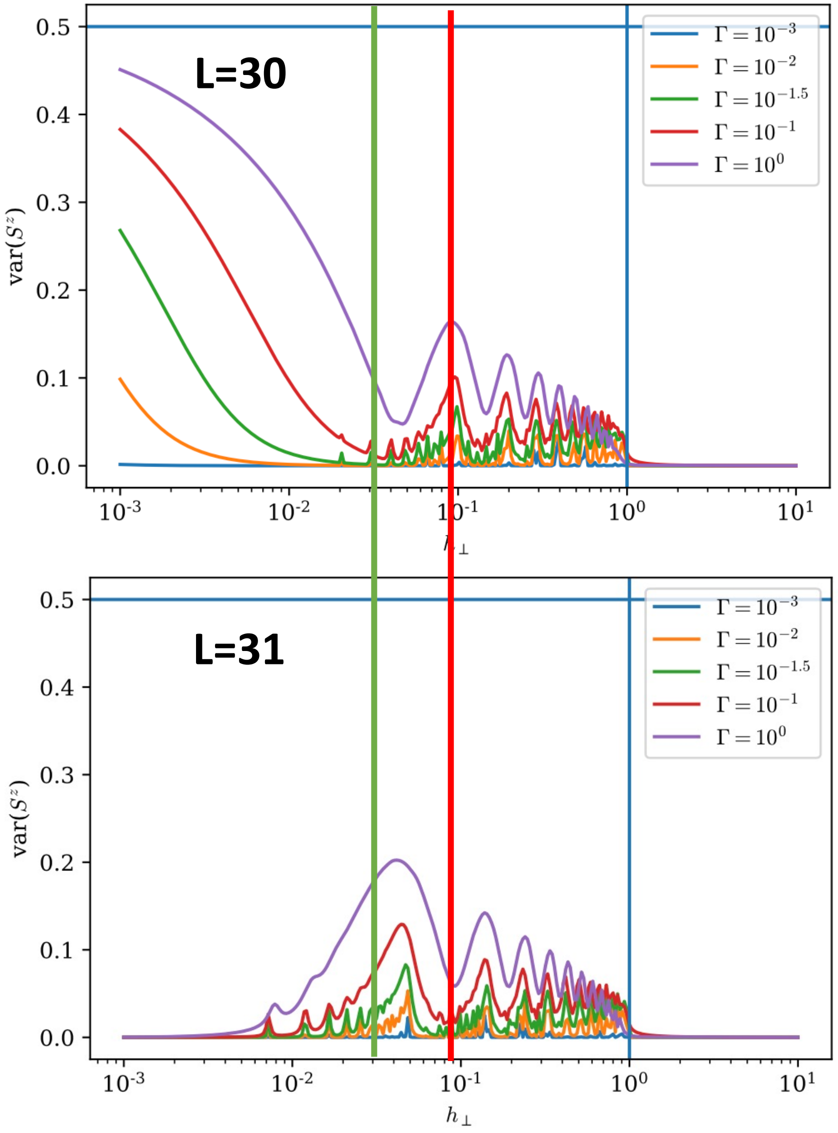
Calculate using Lehmann representation for GFs using a single particle matrix

Boundary Dissipation S^+/S^-

- Spin oscillations in steady state—2nd order perturbation theory
- Zero mean magnetization: because an equal superposition of stable and unstable configurations
- L=30 chain

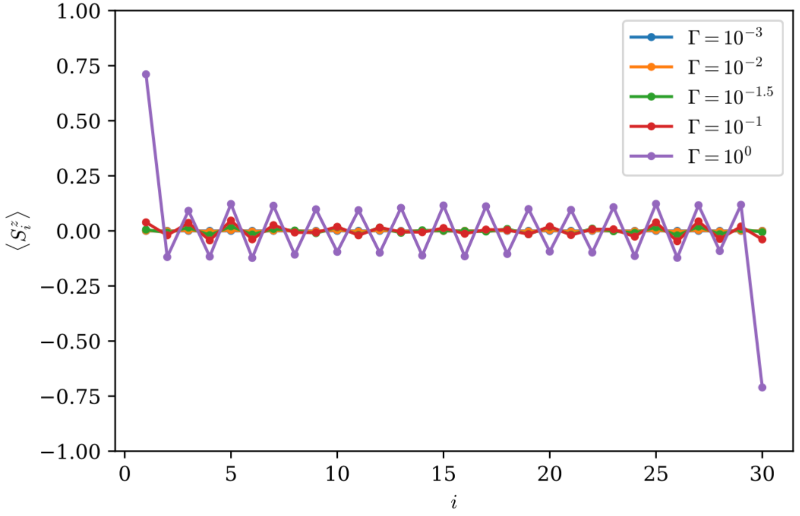


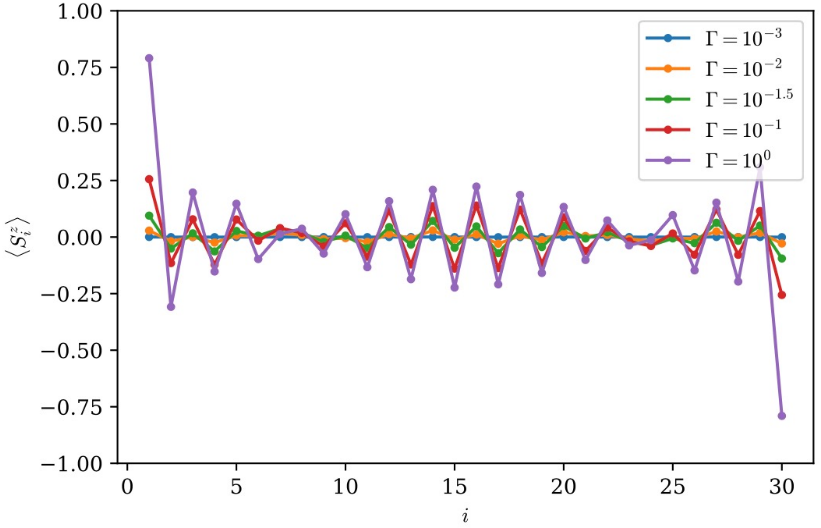
Varying Length

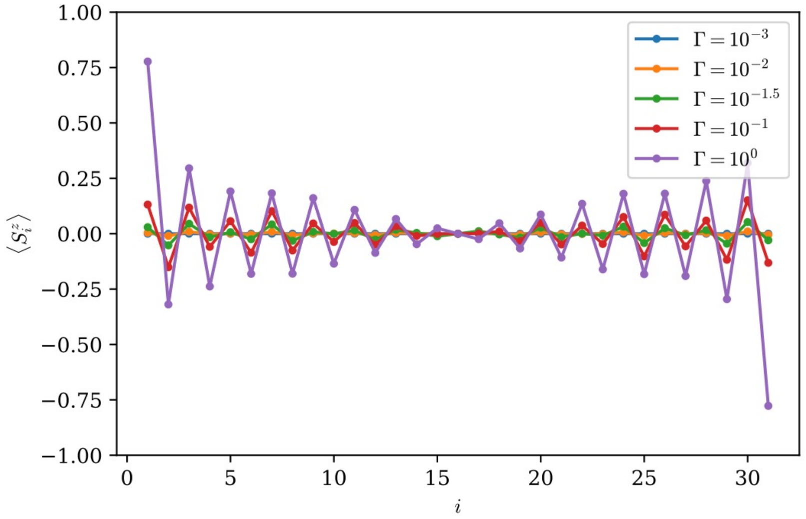


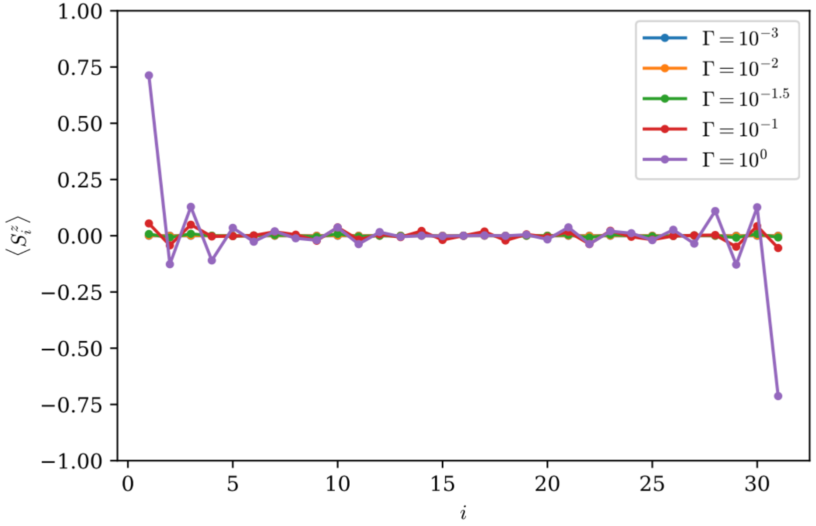
h=0.03

h=0.09



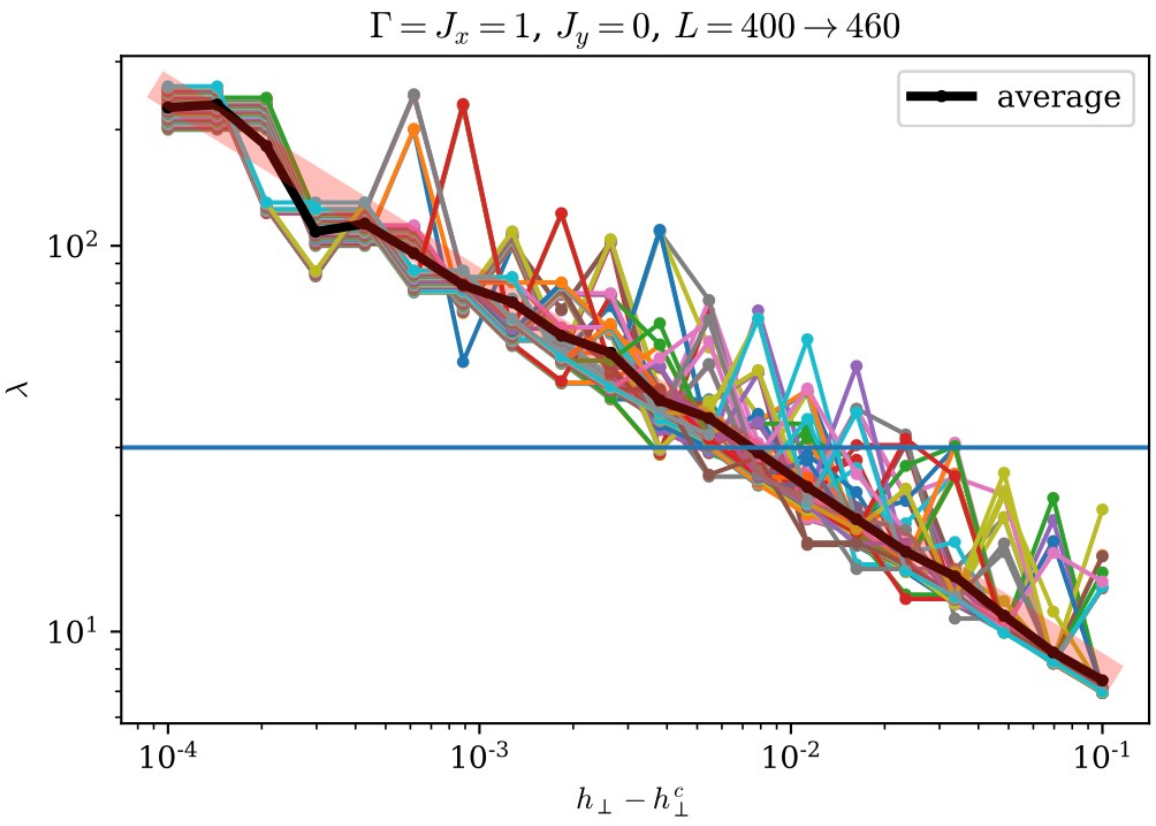


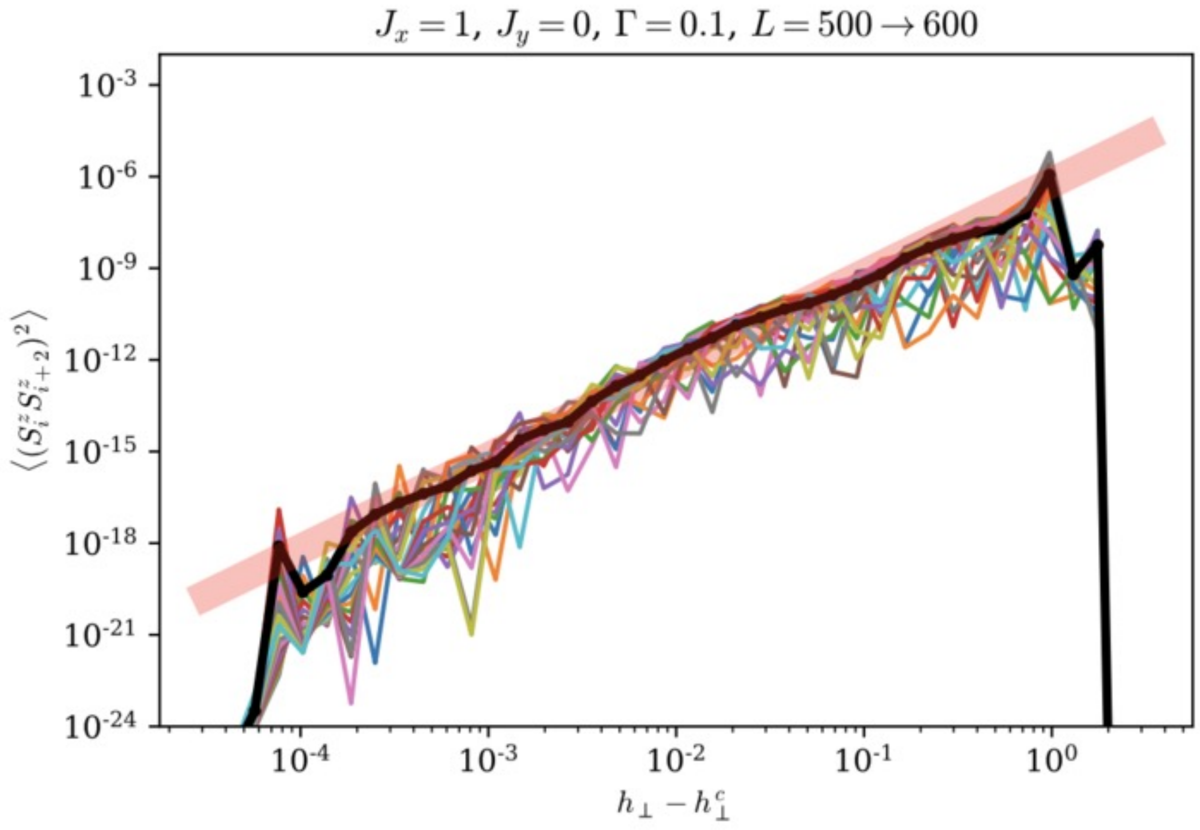




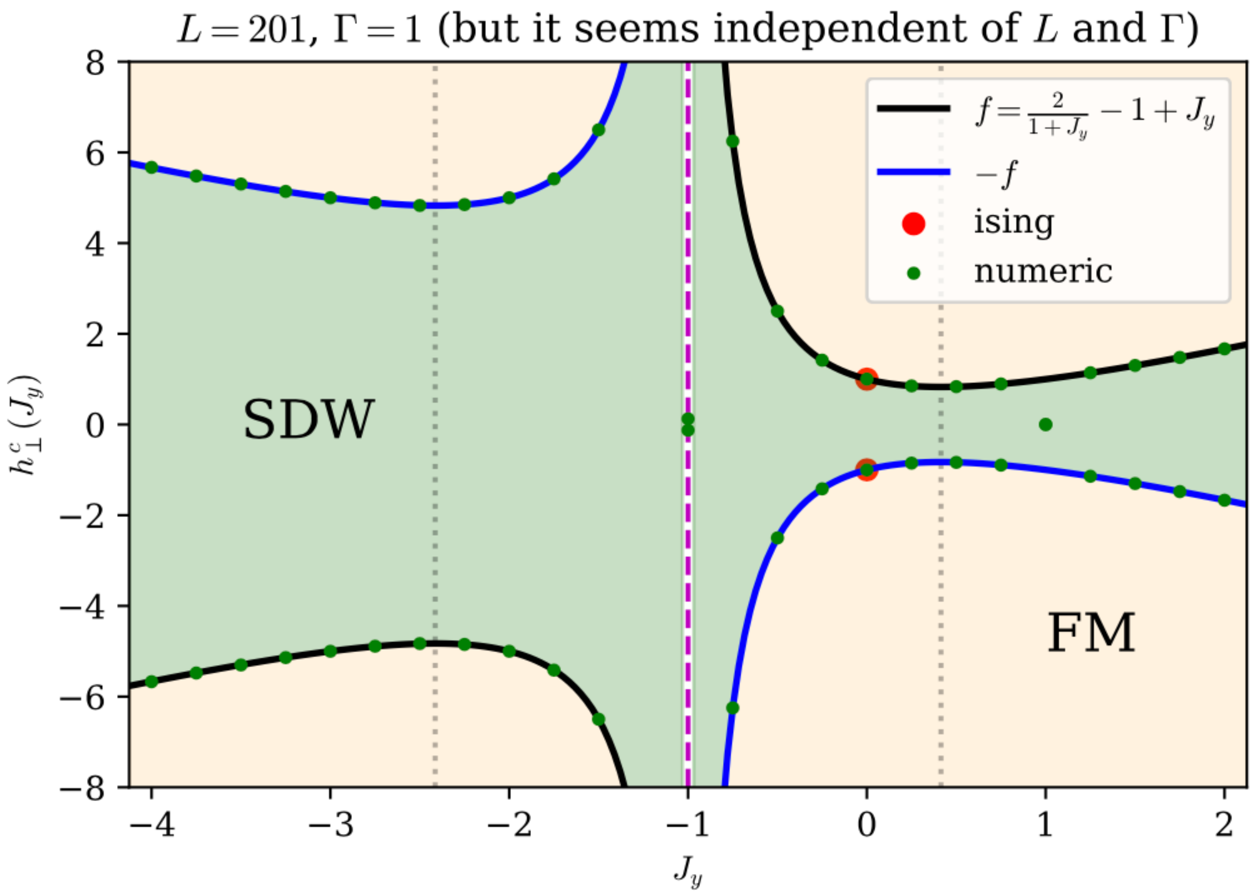
Updates

Scaling in the dissipative Ising model near hc

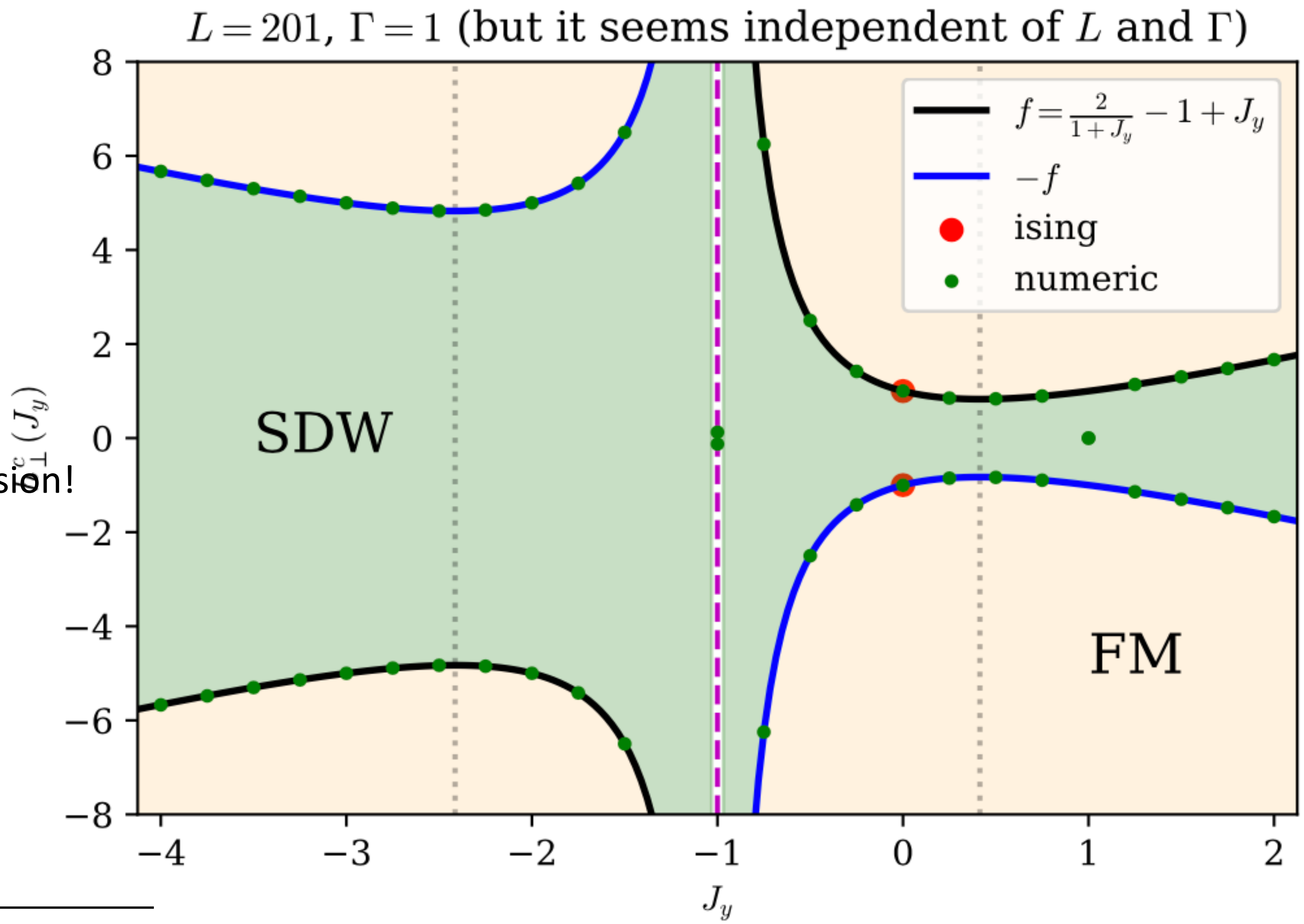




Phase diagram



Dualities?



Hyperbolas: pi rotation and reflection: inversion!

Hc->-Hc : z mirror symmetry

Jy-Jyc -> -Jy+Jyc : glide symmetry

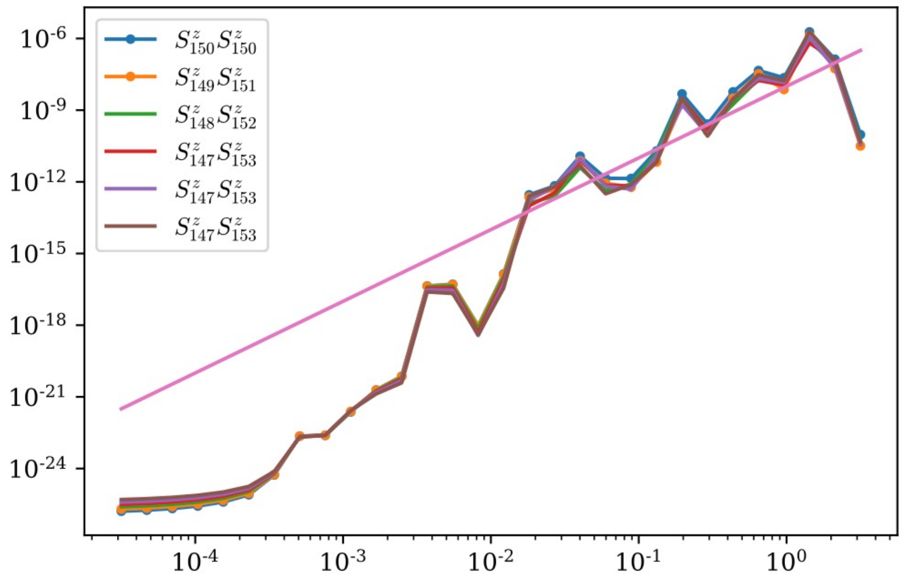
Same up to additive const. of 4

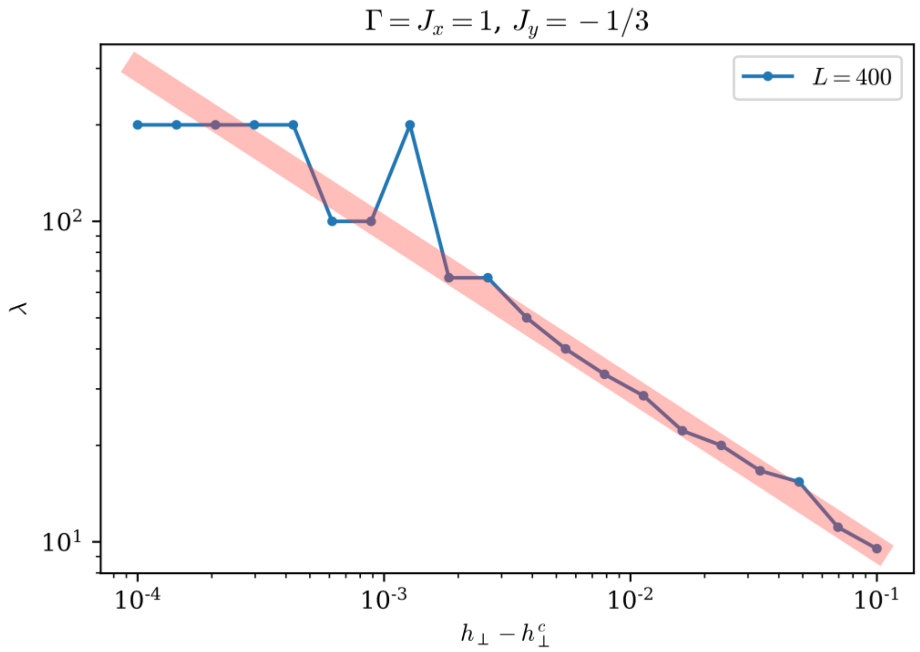
Dilation

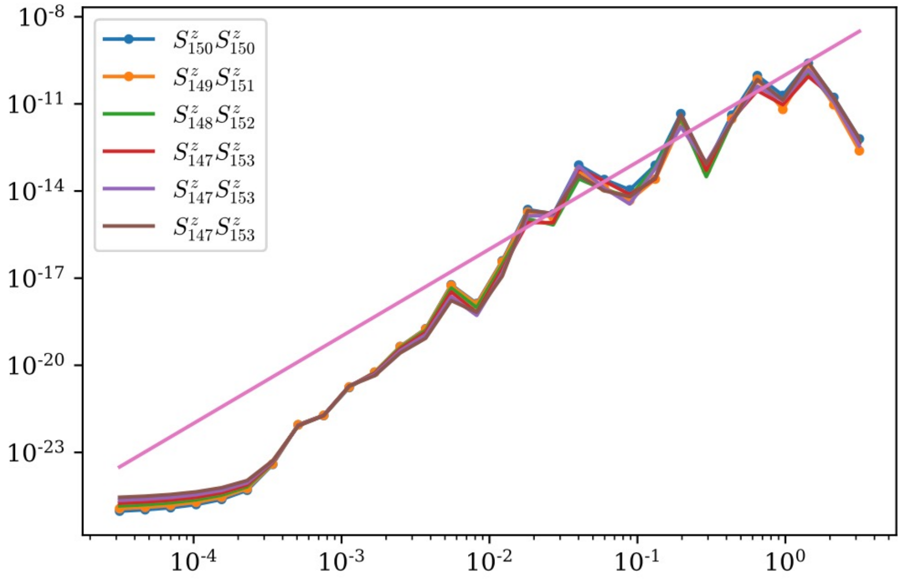
$$J_y^{\pm} = J_y^* + \frac{h-h^*}{2} \pm \sqrt{\frac{h-h^*}{2}\left(2\left(1+J_y^*\right) + \frac{h-h^*}{2}\right)}$$

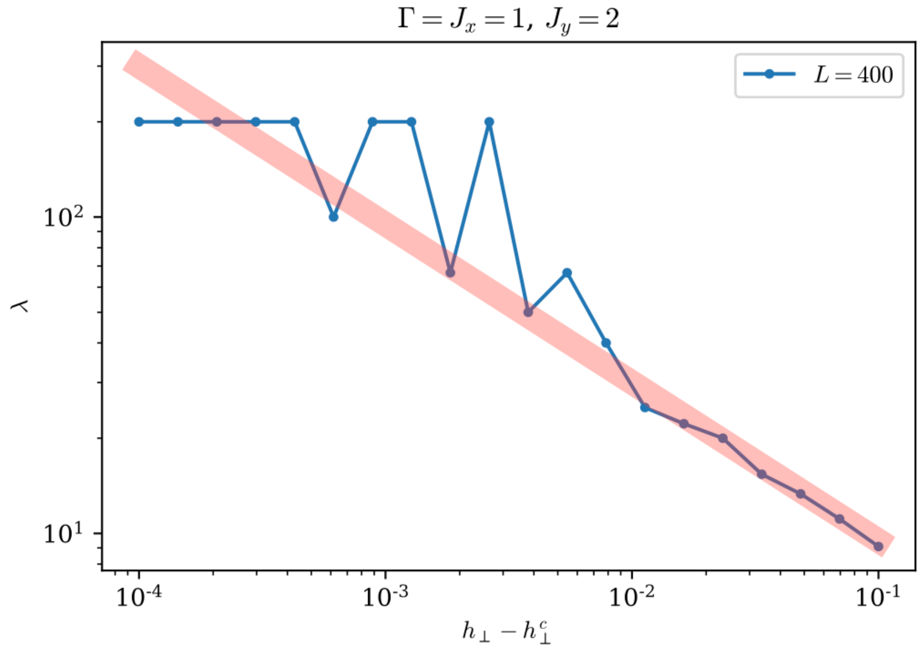
Hc = 5/3

- Jy=-1/3
- Jy=2

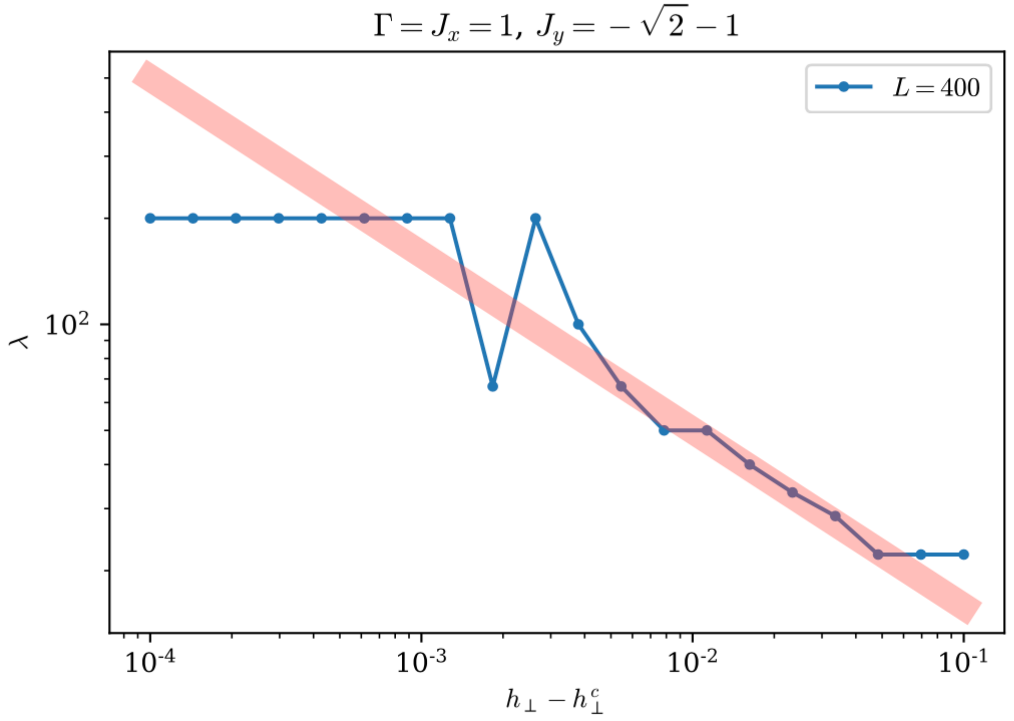


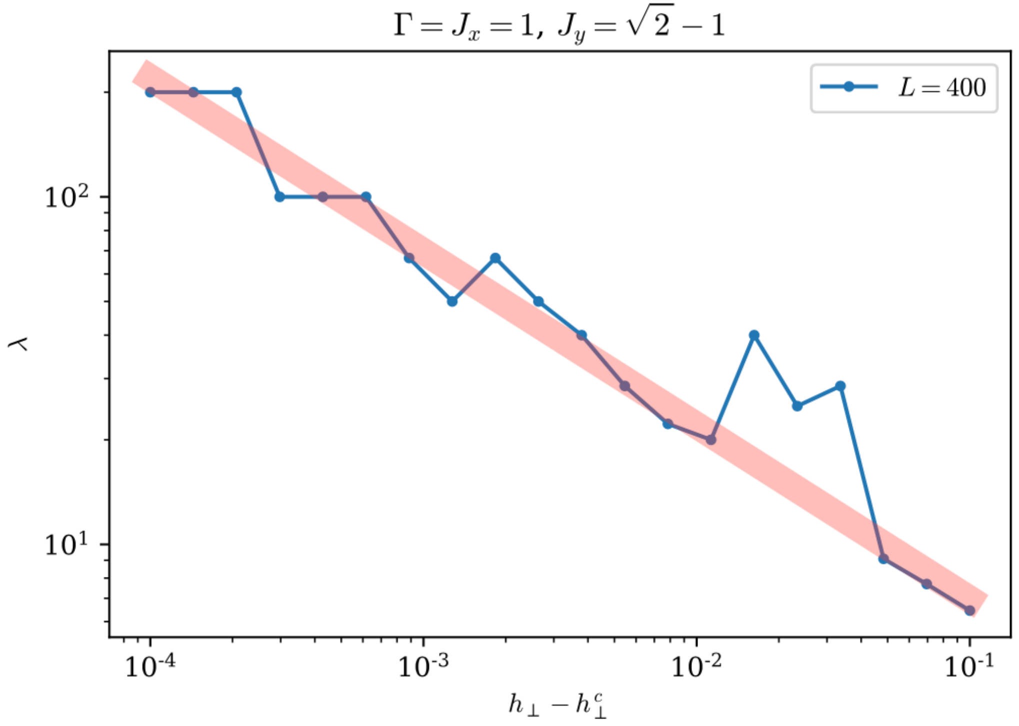






Jy = +-Sqrt(2)-1


$\Gamma = J_x = 1,\ J_y = -\sqrt{2} - 1$
$L = 400$
10^2
λ
10^{-4}
10^{-3}
10^{-2}
10^{-1}
$h_\perp - h_\perp^c$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10^2
10^1
λ
10^{-4}
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10^{-2}
10^{-1}
$h_\perp - h_\perp^c$

Near the Jx=Jy point