Consider $\rho(t) = \frac{1}{2}(\sigma_0 + \boldsymbol{m} \cdot \boldsymbol{\sigma})$ where $\boldsymbol{m} = (m_x, m_y, m_z)$ and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ for Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

Note that $\mathrm{Tr}[\rho] = 1$ and $\rho = \rho^\dagger$, and for $\boldsymbol{m} \cdot \boldsymbol{m} \leq 1$ that $\rho \succeq 0$ so ρ is a valid density matrix. Now let us consider

$$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}(\sigma_x + i\sigma_y), \qquad \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \frac{1}{2}(\sigma_x + i\sigma_y) \tag{2}$$

noting that $\sigma_+ = \sigma_-^\dagger$ and $\sigma_- = \sigma_+^\dagger$. Let us consider time evolution of the density matrix given by the Lindblad master equation

$$\frac{\partial}{\partial t}\rho = i[\rho, H] + \sum_m \gamma_m (2J_m \rho J_m^\dagger - \{J_m^\dagger J_m, \rho\}) \tag{3}$$

where we take that Hamiltonian to be $H = \boldsymbol{B} \cdot \boldsymbol{\sigma}$ and consider the jump operators $J_+ = \sigma_+$ and $\gamma_+ = \kappa n_B$ and $J_- = \sigma_-$ and $\gamma_- = \kappa(n_B + 1)$. We can then write

$$\begin{aligned} \frac{\partial}{\partial t}\rho &= i[\rho, \boldsymbol{B} \cdot \boldsymbol{\sigma}] + \kappa n_B (2\sigma_+ \rho \sigma_+^\dagger - \{\sigma_+^\dagger \sigma_+, \rho\}) + \kappa(n_B + 1)(2\sigma_- \rho \sigma_-^\dagger - \{\sigma_-^\dagger \sigma_-, \rho\}) && (4) \\ &= i[\rho, \boldsymbol{B} \cdot \boldsymbol{\sigma}] + \kappa n_B (2\sigma_+ \rho \sigma_- - \{\sigma_- \sigma_+, \rho\}) + \kappa(n_B + 1)(2\sigma_- \rho \sigma_+ - \{\sigma_+ \sigma_-, \rho\}) && (5) \end{aligned}$$

Now let us consider the time-evolution of the magnetization. We have

$$\begin{aligned} \frac{\partial}{\partial t} m_i &= \frac{\partial}{\partial t}\mathrm{Tr}[\rho \sigma_i] && (6) \\ &= \mathrm{Tr}[(\frac{\partial}{\partial t}\rho)\sigma_i] + \mathrm{Tr}[\rho(\frac{\partial}{\partial t}\sigma_i)] && (7) \\ &= \mathrm{Tr}[(\frac{\partial}{\partial t}\rho)\sigma_i] && (8) \\ &= \mathrm{Tr}[i[\rho, \boldsymbol{B} \cdot \boldsymbol{\sigma}]\sigma_i] \\ &\quad + \mathrm{Tr}[\kappa n_B (2\sigma_+ \rho \sigma_- - \{\sigma_- \sigma_+, \rho\})\sigma_i] && (9) \\ &\quad + \mathrm{Tr}[\kappa(n_B + 1)(2\sigma_- \rho \sigma_+ - \{\sigma_+ \sigma_-, \rho\})\sigma_i] \end{aligned}$$

where $i = x, y, z$. Now let us analyze each of these terms individually (where we do the matrix product and trace in Mathematica)

$$\begin{aligned} \mathrm{Tr}[i[\rho, \boldsymbol{B} \cdot \boldsymbol{\sigma}]\sigma_x] &= i\mathrm{Tr}[\tfrac{1}{2}(\sigma_0 + \boldsymbol{m} \cdot \boldsymbol{\sigma})\boldsymbol{B} \cdot \boldsymbol{\sigma}\sigma_x - \boldsymbol{B} \cdot \boldsymbol{\sigma}\tfrac{1}{2}(\sigma_0 + \boldsymbol{m} \cdot \boldsymbol{\sigma})\sigma_x] = 2(B_y m_z - B_z m_y) && (10) \\ \mathrm{Tr}[i[\rho, \boldsymbol{B} \cdot \boldsymbol{\sigma}]\sigma_y] &= i\mathrm{Tr}[\tfrac{1}{2}(\sigma_0 + \boldsymbol{m} \cdot \boldsymbol{\sigma})\boldsymbol{B} \cdot \boldsymbol{\sigma}\sigma_y - \boldsymbol{B} \cdot \boldsymbol{\sigma}\tfrac{1}{2}(\sigma_0 + \boldsymbol{m} \cdot \boldsymbol{\sigma})\sigma_y] = 2(B_z m_x - B_x m_z) && (11) \\ \mathrm{Tr}[i[\rho, \boldsymbol{B} \cdot \boldsymbol{\sigma}]\sigma_z] &= i\mathrm{Tr}[\tfrac{1}{2}(\sigma_0 + \boldsymbol{m} \cdot \boldsymbol{\sigma})\boldsymbol{B} \cdot \boldsymbol{\sigma}\sigma_z - \boldsymbol{B} \cdot \boldsymbol{\sigma}\tfrac{1}{2}(\sigma_0 + \boldsymbol{m} \cdot \boldsymbol{\sigma})\sigma_z] = 2(B_x m_y - B_y m_x) && (12) \end{aligned}$$

where we now note that

$$(B_x, B_y, B_z) \times (m_x, m_y, m_z) = (B_y m_z - B_z m_y, B_z m_x - B_x m_z, B_x m_y - B_y m_x) \tag{13}$$

so we see that

$$\mathrm{Tr}[i[\rho, \boldsymbol{B} \cdot \boldsymbol{\sigma}]\sigma_i] = 2(\boldsymbol{B} \times \boldsymbol{m})_i \tag{14}$$

Let us move on to the next term, where as above we complete the product and trace in Mathematica

$$\mathrm{Tr}[\kappa n_B(2\sigma_+\rho\sigma_- - \{\sigma_-\sigma_+,\rho\})\sigma_x] = -m_x\kappa n_B \tag{15}$$
$$\mathrm{Tr}[\kappa n_B(2\sigma_+\rho\sigma_- - \{\sigma_-\sigma_+,\rho\})\sigma_y] = -m_y\kappa n_B \tag{16}$$
$$\mathrm{Tr}[\kappa n_B(2\sigma_+\rho\sigma_- - \{\sigma_-\sigma_+,\rho\})\sigma_z] = 2(1-m_z)\kappa n_B \tag{17}$$

and

$$\mathrm{Tr}[\kappa(n_B+1)(2\sigma_-\rho\sigma_+ - \{\sigma_+\sigma_-,\rho\})\sigma_x] = -m_x\kappa(n_B+1) \tag{18}$$
$$\mathrm{Tr}[\kappa(n_B+1)(2\sigma_-\rho\sigma_+ - \{\sigma_+\sigma_-,\rho\})\sigma_y] = -m_y\kappa(n_B+1) \tag{19}$$
$$\mathrm{Tr}[\kappa(n_B+1)(2\sigma_-\rho\sigma_+ - \{\sigma_+\sigma_-,\rho\})\sigma_z] = -2(1+m_z)\kappa(n_B+1) \tag{20}$$

So combining everything we find

$$\frac{\partial}{\partial t}m_x = 2(\boldsymbol{B}\times\boldsymbol{m})_x - m_x\kappa(2n_B+1) \tag{21}$$
$$\frac{\partial}{\partial t}m_y = 2(\boldsymbol{B}\times\boldsymbol{m})_y - m_y\kappa(2n_B+1) \tag{22}$$
$$\frac{\partial}{\partial t}m_z = 2(\boldsymbol{B}\times\boldsymbol{m})_z - 2\kappa(m_z(2n_B+1)+1) \tag{23}$$

Now let us introduce $T_2 = 1/(2n_B+1)$, $T_1 = 1/2\kappa(2n_B+1) = T_2/2\kappa$, $\langle m_z\rangle = -1/(2n_B+1)$ from which we see that

$$\frac{\partial}{\partial t}m_x = 2(\boldsymbol{B}\times\boldsymbol{m})_x - \frac{m_x}{T_2} \tag{24}$$
$$\frac{\partial}{\partial t}m_y = 2(\boldsymbol{B}\times\boldsymbol{m})_y - \frac{m_y}{T_2} \tag{25}$$
$$\frac{\partial}{\partial t}m_z = 2(\boldsymbol{B}\times\boldsymbol{m})_z - \frac{m_z-\langle m_z\rangle}{T_1} \tag{26}$$

which are the Bloch equations. In the last line we used that

$$-2\kappa(m_z(2n_B+1)+1) = -\frac{1}{T_1}\frac{2\kappa(m_z(2n_B+1)+1)}{2\kappa(2n_B+1)} = -\frac{1}{T_1}(m_z+\frac{1}{2n_B+1}) = -\frac{m_z-\langle m_z\rangle}{T_1} \tag{27}$$