

DRL to HRL:

Tensor Network Approaches to Nuclear Noise in Semiconducting Qubits

Spenser Talkington

29 August 2024

Outside of academia

- There is a world outside academia
- Some very interesting science is being done by govt/companies, especially with quantum devices
- Academia
 - Smaller scale experiments/funding
 - Very transparent with results sharing
- Industry/Government
 - Larger scale experiments/funding
 - Less freedom of direction and less freedom to share research results

| Date | Milestone |
|------------|--|
| October 1 | Started internship applications |
| November 9 | Applied to HRL (others from Oct – Dec) |
| Nov-Dec | Other interviews (IBM/Quantinuum/Leidos) |
| March 7 | Interview with HRL at March Meeting |
| March 13 | Zoom interview with HRL |
| April 3 | Offer letter from HRL |
| June 3 | Internship start |
| August 23 | Internship end |

Most asked interview question:

“What experience do you have writing and maintaining collaborative code?”

I gather that mentioning using GitHub/GitLab for any real-world example/project is a good answer

What is HRL?



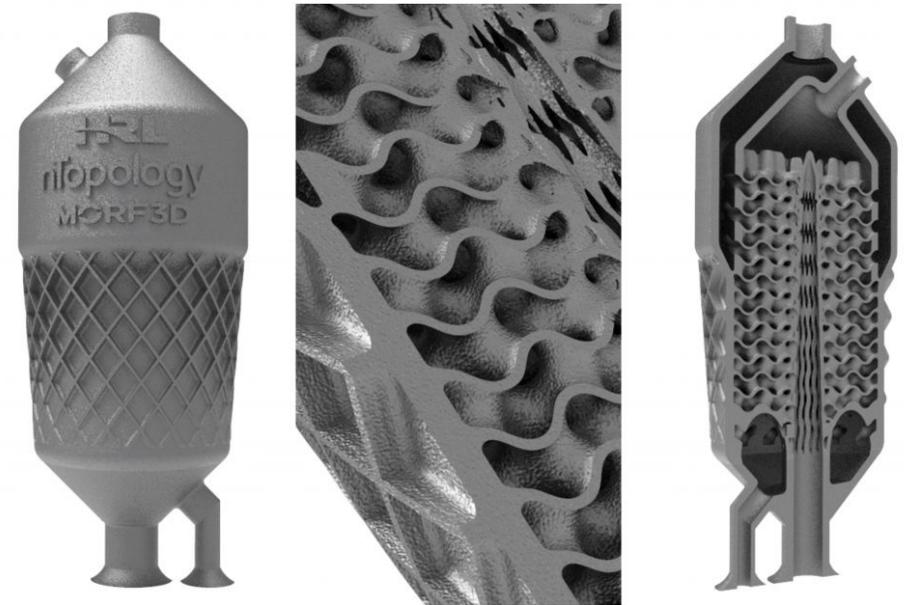
- Bell Labs like, but government focused
- Research and development laboratory
 - ~1000 scientists + support staff
 - Owned jointly by Boeing and GM
 - 10% of funding from owners
 - 90% from US government contracts
 - Based in Southern California / LA area
- Role: intellectual property (IP) generation
- Results, technologies, and initiatives are often compartmentalized / not publicly shared



Areas of research

- Multiple laboratories

- Microfabrication technology (MTL)
 - Uses in-house semiconductor fab to build state of the art electronic devices
 - Quantum computation + quantum programming + quantum theory groups
- Materials and Microsystems (MML)
 - Sensors and Electronics (SEL)
 - Vision Systems (VSL)
 - <https://www.hrl.com/laboratories>



Accomplishments: first laser (1960), first LCD wristwatch (1975), self-driving car across US (1984), ion propulsion for spacecraft (1997), ...

Folks I worked with

- PI of quantum theory:
Thaddeus (15 years at HRL)
- Stephen (1 year at HRL)
 - Previously: postdoc at Harvard
focus on DFT of moiré materials
- Lucas (1 year at HRL)
 - Previously: grad student at UIUC
focus on tensor networks and
near term quantum algorithms



Thaddeus D. Ladd

Principal Scientist, HRL Laboratories, LLC
Verified email at alumni.stanford.edu - [Homepage](#)

[Quantum Information Techn...](#)



Stephen Carr

[HRL Laboratories](#)
Verified email at hrl.com - [Homepage](#)

[Condensed Matter](#) [2D Materials](#)
[Density Functional Theory](#) [Magnetic Resonance](#)



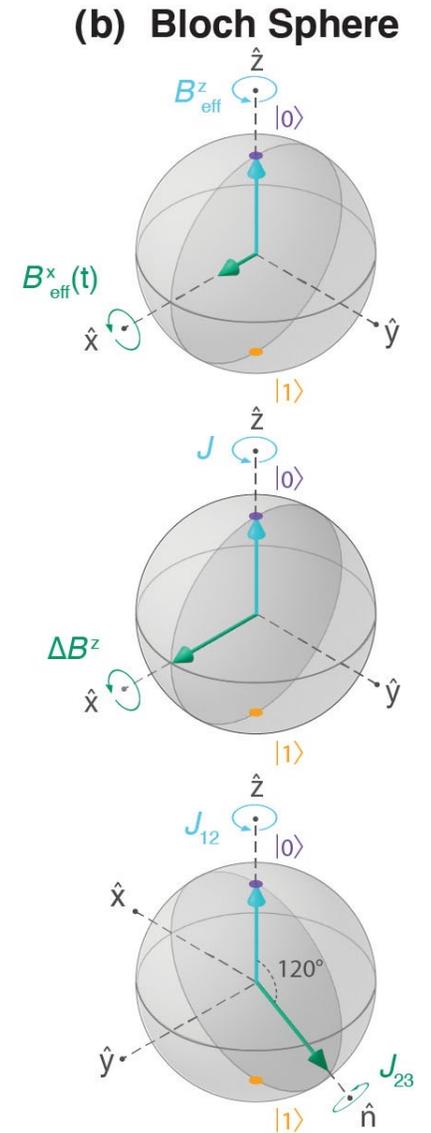
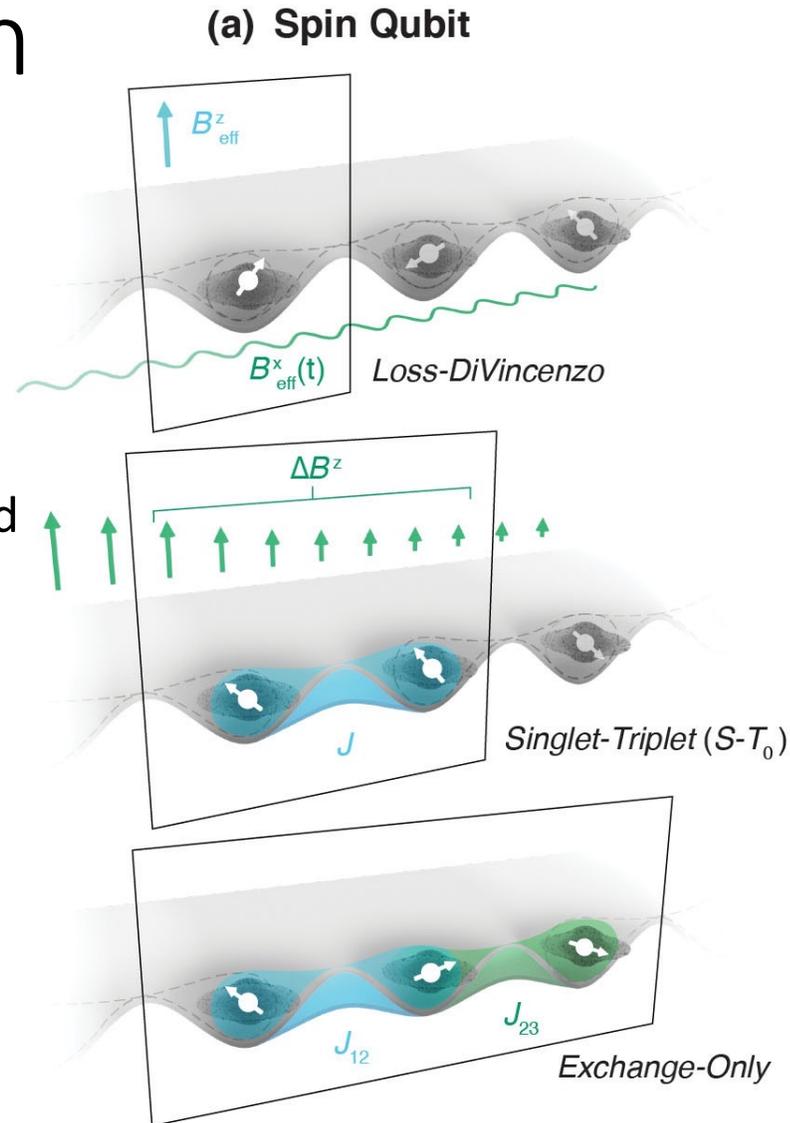
Lucas Slattery

Research Scientist, HRL Laboratories
Verified email at illinois.edu

[Quantum Physics](#)

Quantum computation

- Why quantum computation?
 - Simulate interesting many-body physics
 - New algorithms: if we can do an NP problem in P time then we can do other NP problems in P time (e.g. factorization)
- What do we need to get it?
 - Let's listen to Gene's former student David DiVincenzo
 1. Scalability
 2. Initialization
 3. Coherence
 4. Universal gates
 5. Measurement
- How to do it in semiconductors?
 - Loss-DiVincenzo: control with B fields
 - Singlet-Triplet: control with B and E field
 - Exchange only: control with E fields only



Width and Depth

- Widths of 10s for trapped ions to ~ 1000 for superconductors reported
- Circuit depths are much more limiting than widths
 - DiVincenzo's 3rd criterion: coherence, is a limiting factor for *all* platforms
- What can one do at different depths?
- March meeting talk by Google (Ryan Babbush)
 - $10^3 - 10^5$ gates: scrambling, OTOCs, Floquet dynamics
 - $10^4 - 10^7$ gates: magnets, superconductors, coarse grained chemistry
 - $10^7 - 10^{10}$ gates: chemistry
 - $10^{10} - 10^{14}$ gates: ab initio materials

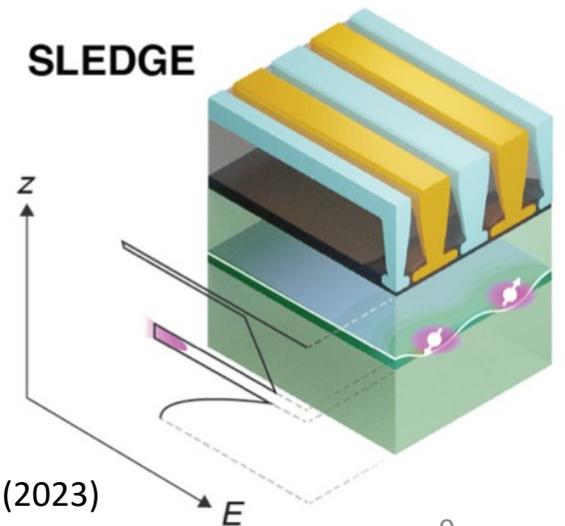
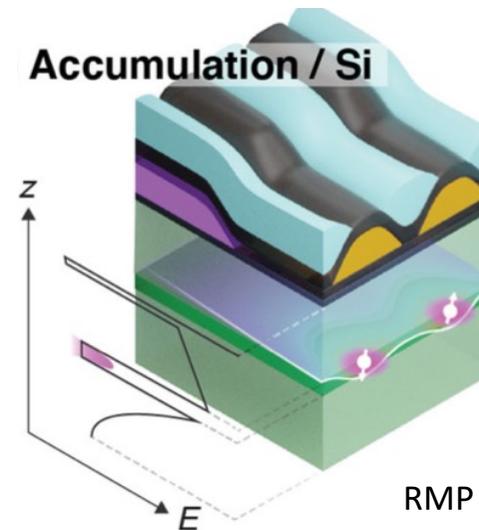
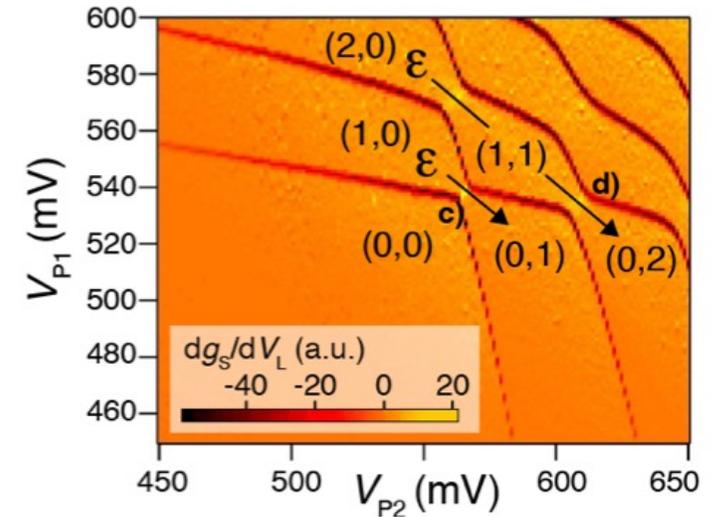
The (publicly known) state of the art

- Width: number of qubits (10^1 - 10^3)
- Depth: number of 2-qubit gates (above a specified fidelity)

| Platform | Gate | Author | Error rate | Depth (90%) | Limiting issue |
|-----------------|------|----------------------------------|--------------------|-------------|-------------------------|
| Trapped ions | ? | Quantinuum, press release (2024) | 9×10^{-4} | 122 | Ion loss/gradient noise |
| Superconductors | ? | IQM, press release (2024) | 1×10^{-3} | 105 | Frequency collisions |
| Superconductors | fSim | Google, PRL 125, 120504 (2020) | 4×10^{-3} | 26 | Frequency collisions |
| Trapped ions | ? | IonQ (2022), SEC filing | 4×10^{-3} | 26 | Ion loss/gradient noise |
| Trapped atoms | ? | QuEra, Nature 626, 58 (2024) | 5×10^{-3} | 21 | Atom loss |
| Semiconductors | CZ | Intel, Nature 627, 80 (2024) | 5×10^{-3} | 21 | Electromagnetic noise |
| Semiconductors | SWAP | HRL, Nature 615, 817 (2023) | 7×10^{-3} | 15 | Electromagnetic noise |
| Superconductors | CZ | IBM, press release (2022) | 8×10^{-3} | 13 | Frequency collisions |
| Superconductors | ? | Rigetti, press release (2022) | 8×10^{-3} | 13 | Frequency collisions |

Exchange-only semiconductor qubits

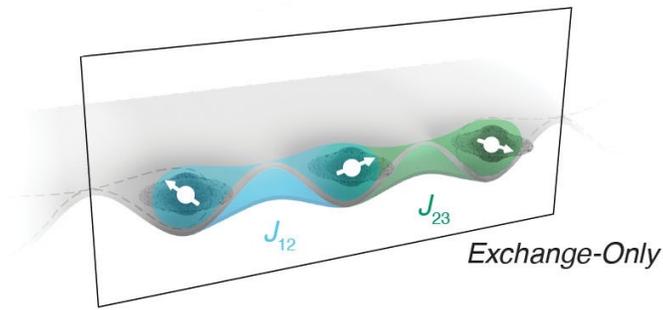
- Single excess electrons statically confined in quantum wells
 - SiO₂/Si/SiO₂ wells
 - SiGe/Si/SiGe wells (better materials)
- Manipulate with electric fields only
- Benefits:
 - Scalability
 - Fast control
- Detriments:
 - Mediocre coherence
 - Slow measurement



RMP 95, 025003 (2023)

Logical qubits

- “Decoherence free subspace” (DFS) qubits (Nature 408, 339 (2000))
 - Can be controlled using static electric fields: exchange only
- Encode qubit in three physical spins
 - Total spin S
 - Spin on electrons 1 and 2: $S_{1,2}$
 - Total spin-z S_z
- Readout: state, and if leaked (5-8)
- This encoding is invariant under total magnetic field changes



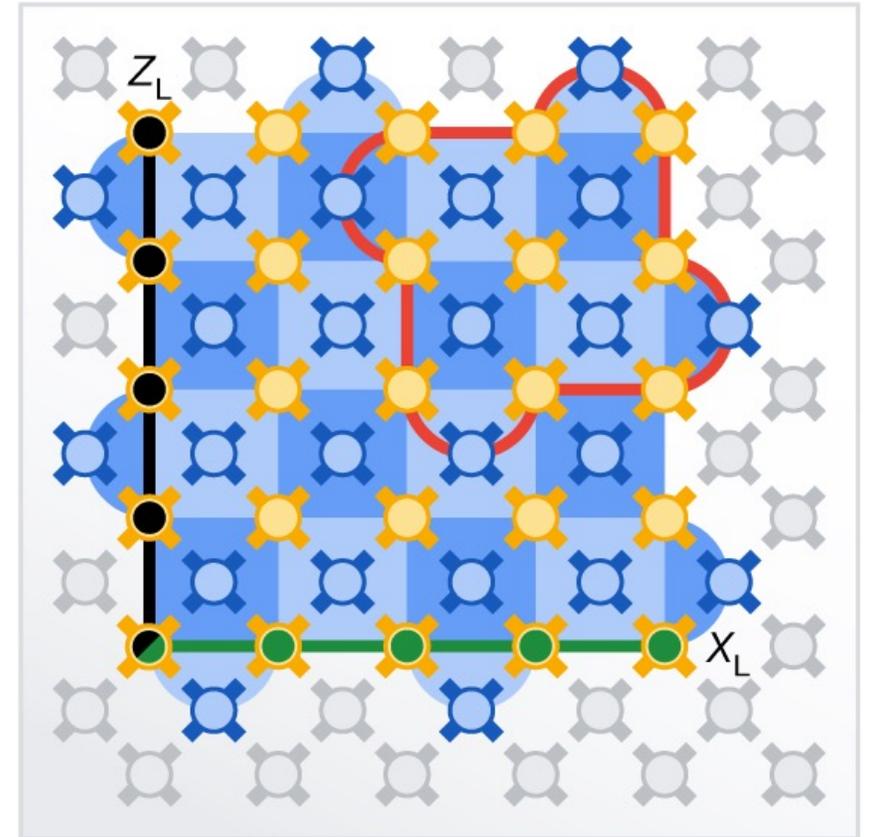
$$\begin{aligned}
 |S_0\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \\
 |T_+\rangle &= |00\rangle \\
 |T_0\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\
 |T_-\rangle &= |11\rangle
 \end{aligned}$$

$$\begin{aligned}
 |1\rangle &= \frac{1}{\sqrt{2}}(|010\rangle - |100\rangle) &= |S_0\rangle |0\rangle \\
 |2\rangle &= \frac{1}{\sqrt{2}}(|011\rangle - |101\rangle) &= |S_0\rangle |1\rangle \\
 |3\rangle &= \sqrt{\frac{2}{3}}|001\rangle - \frac{1}{\sqrt{6}}|010\rangle - \frac{1}{\sqrt{6}}|100\rangle &= \frac{1}{\sqrt{3}}(\sqrt{2}|T_+\rangle |1\rangle - |T_0\rangle |0\rangle) \\
 |4\rangle &= \frac{1}{\sqrt{6}}|011\rangle + \frac{1}{\sqrt{6}}|101\rangle - \sqrt{\frac{2}{3}}|110\rangle &= \frac{1}{\sqrt{3}}(|T_0\rangle |1\rangle - \sqrt{2}|T_-\rangle |0\rangle) \\
 |5\rangle &= |000\rangle &= |T_+\rangle |0\rangle \\
 |6\rangle &= \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle) &= \frac{1}{\sqrt{3}}(|T_+\rangle |1\rangle + \sqrt{2}|T_0\rangle |0\rangle) \\
 |7\rangle &= \frac{1}{\sqrt{3}}(|011\rangle + |101\rangle + |110\rangle) &= \frac{1}{\sqrt{3}}(\sqrt{2}|T_0\rangle |1\rangle + |T_-\rangle |0\rangle) \\
 |8\rangle &= |111\rangle &= |T_-\rangle |1\rangle
 \end{aligned}$$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-----------|---------------|----------------|---------------|----------------|---------------|---------------|----------------|----------------|
| S | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{3}{2}$ | $\frac{3}{2}$ | $\frac{3}{2}$ | $\frac{3}{2}$ |
| $S_{1,2}$ | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| S_z | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{3}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{3}{2}$ |

Aside: surface codes

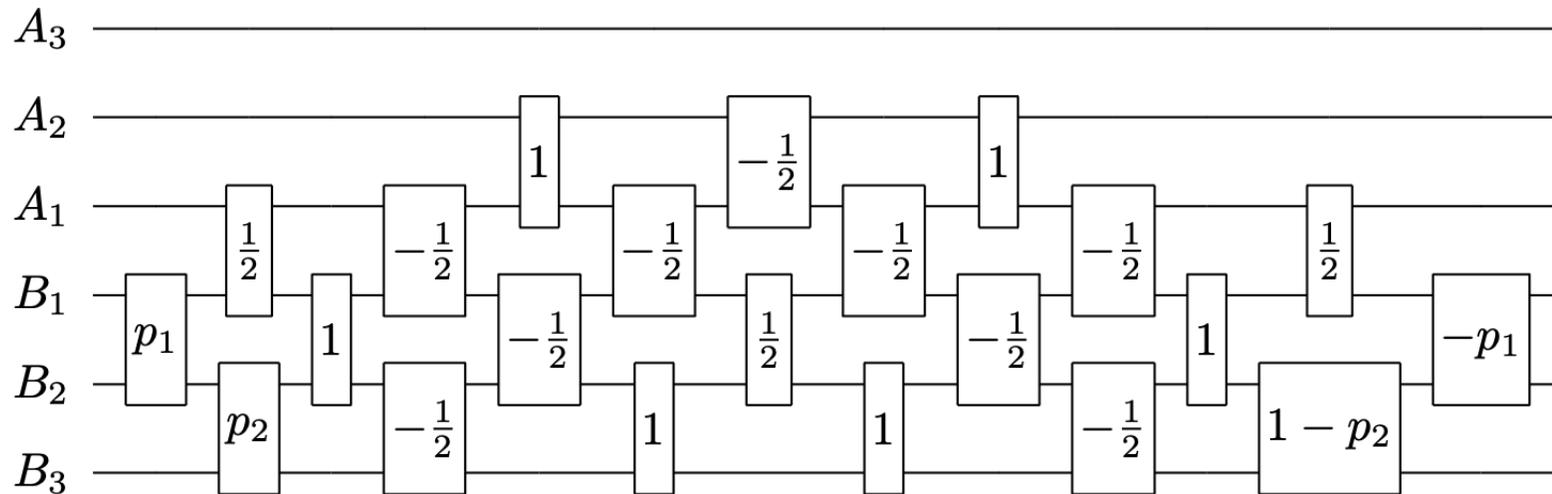
- The logical qubits mentioned above are distinct from the stabilizer codes/surface codes often discussed for logical qubits
- Key idea: logical information is encoded in non-local degrees of freedom (winding in the toric code), and until local errors spread across the whole system (with a string) the information is protected



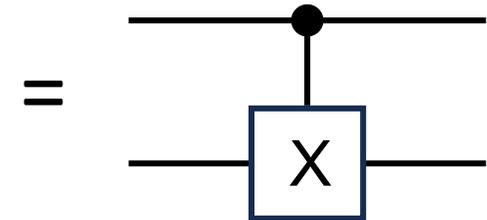
Nature **614**, 676 (2023)

A first logical gate: Fong-Wandzura

- Logical gates are rather longer than physical gates, but total time can still be $\sim 10\text{-}100$ ns per logical gate
- But: free from one error source--total magnetic field fluctuations

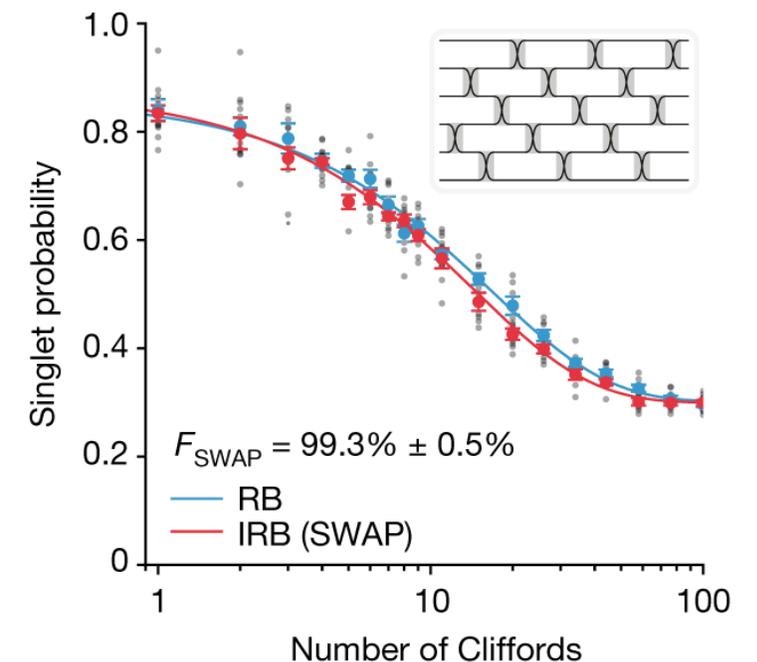


arXiv 1102.2909



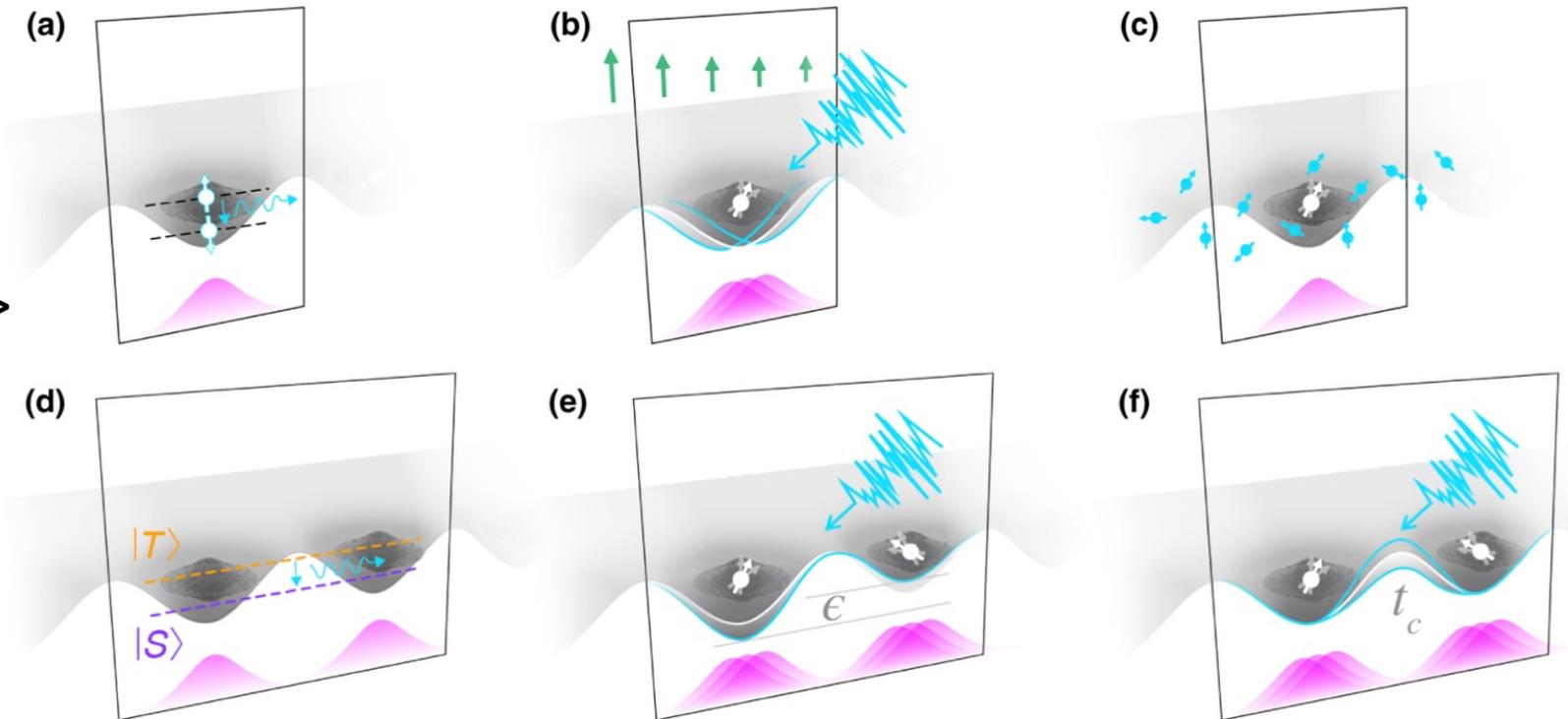
Universal logic

- Cliffords (e.g. CX, H, $S = \sqrt{Z}$) combined with $T = \sqrt{S}$ are universal
- Realize these and benchmark them for DFS qubits
 - Nature 615, 817 (2023)
 - Error 29×10^{-3} for all 2-qubit Cliffords
 - Better for select gates, e.g. SWAP
- Randomized benchmarking (RB)
 - Sequence that compiles to the identity
- Interleaved randomized benchmarking (IRB)
 - RB with a specific gate repeated frequently
- Some other errors: encoding, leakage, readout



Limits on coherence

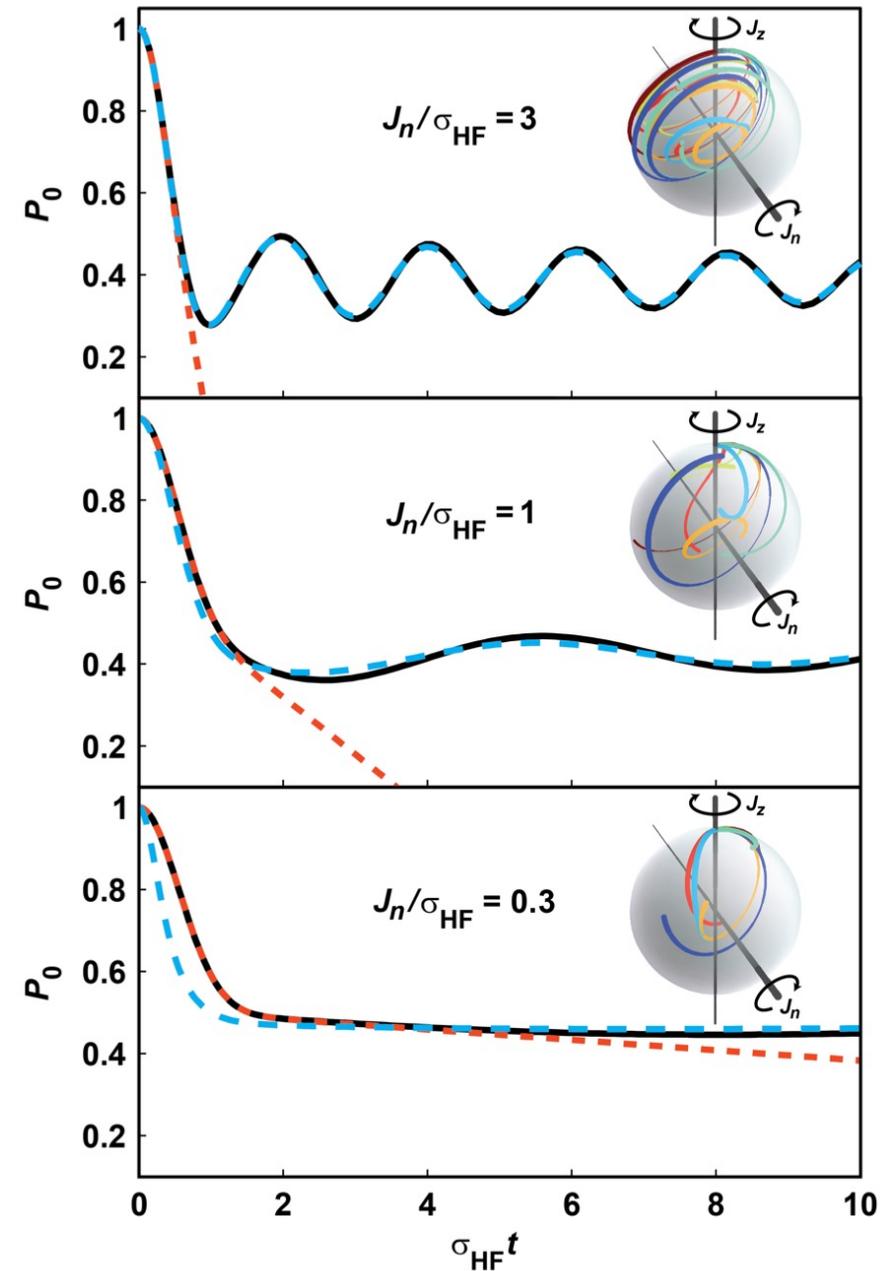
- T_1 : decay timescale from $|1\rangle$ to $|0\rangle$
- T_2 : decay timescale from $|1\rangle$ to classical superposition $|0\rangle + |1\rangle$
- Contributions
 - (a,d) Radiation/relaxation
 - (b,e,f) Charge noise
 - (c) Nuclear noise
 - Phonons
 - ...
- Make T_1, T_2 better \rightarrow everything gets better



RMP 95, 025003 (2023)

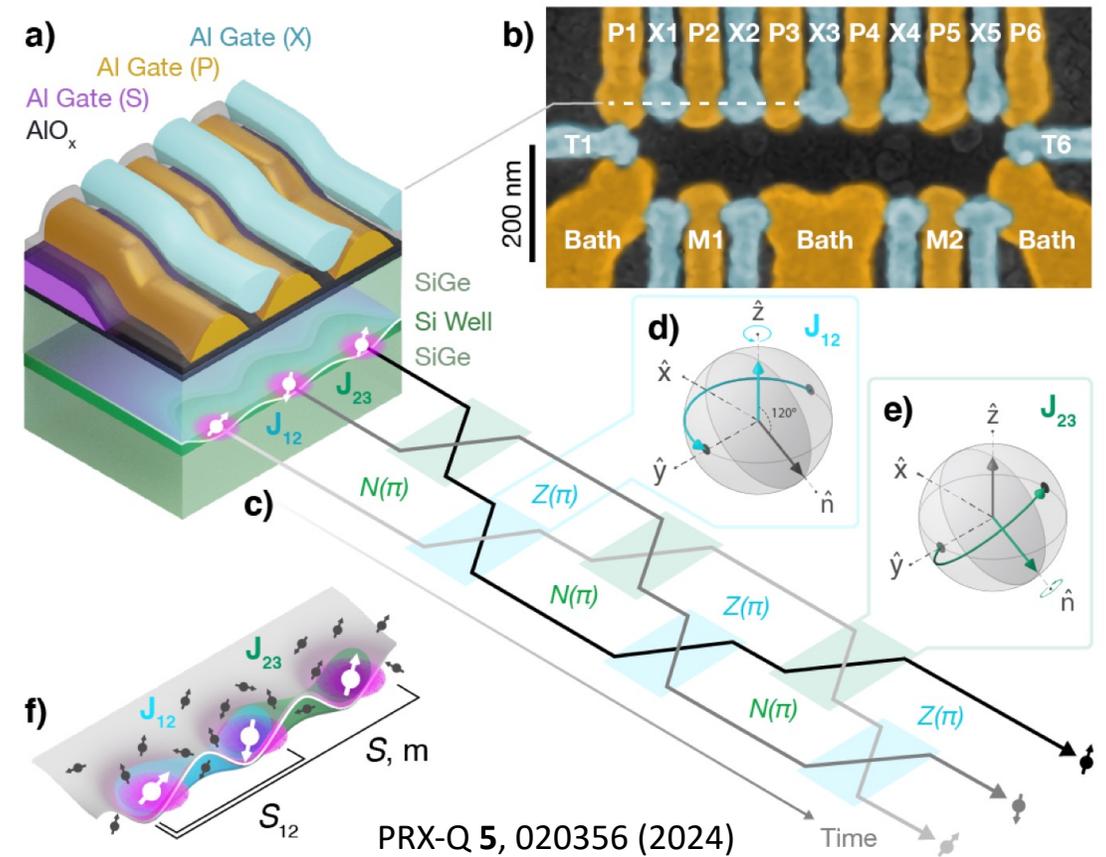
Idling is a big issue

- Where do errors occur?
 - One qubit gates and even two qubit gates have reasonably low error rates
 - Some qubits will idle a while in many algorithms
 - Ex. Readout of one qubit: readout is slow: $\sim 1-10 \mu\text{s}$, and others idle during this
 - Don't want to lose their information
- Without proactive steps information disappears with T_2
 - Free Inductive Decay (FID) as a Gaussian—right
 - Slow this down: active dynamical decoupling



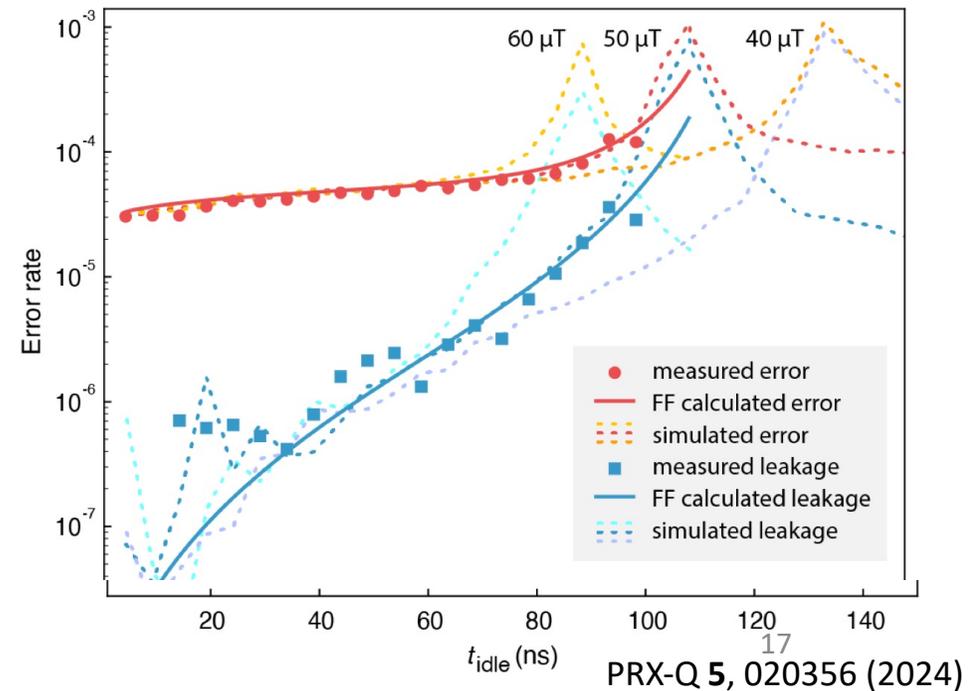
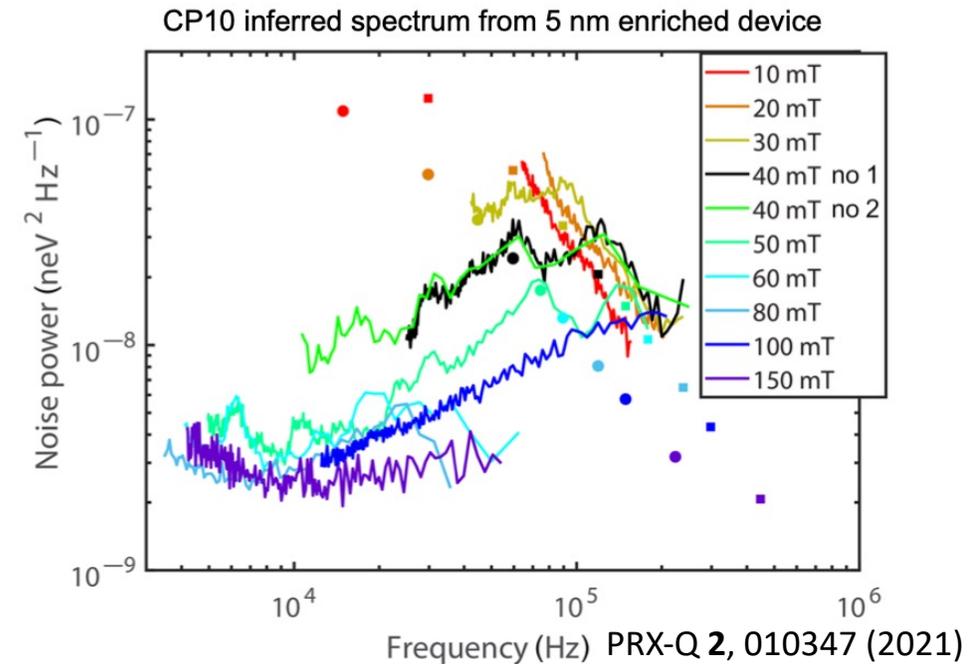
Dynamical decoupling: NZ1

- Now that we have eliminated the total magnetic field the next terms that enter the energy are gradients of the magnetic field
 - Different B fields on each dot of the DFS qubit
- Get rid of this error too: dynamical decoupling (DD)—swap the electrons around so they all feel the same field on average
 - One possible DD sequence is NZ1
- T_2 from $2 \mu\text{s}$ (FID) to $720 \mu\text{s}$ (NZ1)



The issue with resonances

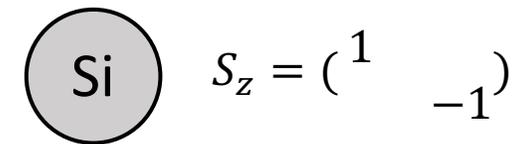
- Periodic sequences can hit resonances
 - System degrees of freedom couple to some other degree of freedom which provides a “virtual” pathway to decohere in a way that would not be possible with system-only dynamics, e.g. transition to leakage states
 - Hitting a resonance can make error rates exponentially worse: try to avoid them
- How long to idle between N and Z pulses?
 - N and Z pulses are very fast ($\sim 1-10$ ns)
 - The total time idling is at most the readout of a nearby qubit ($\sim 1-10$ μ s)



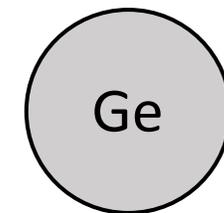
Resonances can come from nuclear levels

- Nuclear physics is generally quite hard
 - Let's simplify to an effective model that gets the resonances right
 - Nuclei with even numbers of protons + neutrons have spin-0 and no magnetic resonances
 - Nuclei with odd numbers of protons + neutrons have spin-(n-p)/2 and have resonances
- Now the devices we consider are made from Si and Ge
 - The only odd isotopes that are abundant are ^{29}Si and ^{73}Ge

• Si is element 14 so the ^{29}Si nucleus has spin $(15-14)/2 = 1/2$



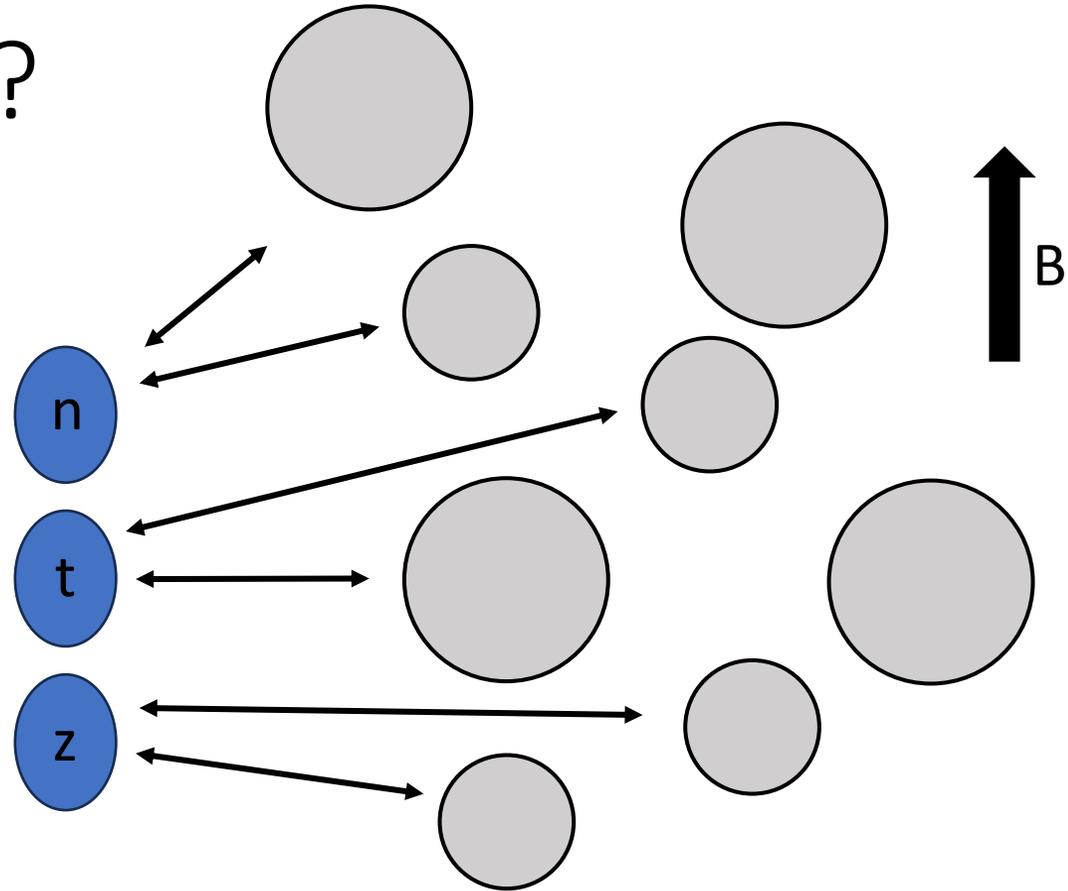
• Ge is element 32 so the ^{73}Ge nucleus has spin $(41-32)/2 = 9/2$



$$S_z = \text{diag}(9, 7, 5, 3, 1, -1, -3, -5, -7, -9)$$

So, what is the model?

- Central spin model
 - DFS qubit (8x8) coupled to nuclei
- Pulses
- Zeeman $g_{\text{atom}} B \cdot \sigma$
- Hyperfine A
- Quadrupole $\sim I_z^2$



$$\begin{aligned}
 H = & H_{\text{pulse}} + \sum_{d=n,t,z} g_{\text{el}} B \cdot \sigma^d + \sum_{\text{Si}=1,\dots,N_{\text{Si}}} g_{\text{Si}} B \cdot I^{\text{Si}} + \\
 & \sum_{\text{Ge}=1,\dots,N_{\text{Ge}}} g_{\text{Ge}} B \cdot I^{\text{Ge}} + \sum_{d=n,t,z, \text{Si}=1,\dots,N_{\text{Si}}} A_{ij}^{d,\text{Si}} \sigma_i^d I_j^{\text{Si}} + \\
 & \sum_{d=n,t,z, \text{Ge}=1,\dots,N_{\text{Ge}}} A_{ij}^{d,\text{Ge}} \sigma_i^d I_j^{\text{Ge}} + \sum_{\text{Ge}=1,\dots,N_{\text{Ge}}} \xi_{\text{Ge}} I_z^2
 \end{aligned}$$

How to simulate it

- Exact calculations: Kronecker tensor product
 - $d_{\text{DFS}} = 8, d_{\text{Si}} = 2, d_{\text{Ge}} = 10$
 - Diagonalizing greater than $\sim 1000 \times 1000$ is infeasible
 - Can't go beyond 2 Ge nuclei / 7 Si nuclei
 - There are often more than this in the Si wells
- Alternative: use tensor networks!
 - Compress representation to an MPS
 - But this is a density matrix: need to vectorize
 - Also, it is a 0D cluster rather than a 1D chain
 - Different algorithm: “automated compression of environments”

Automated Compression of Environments (ACE)

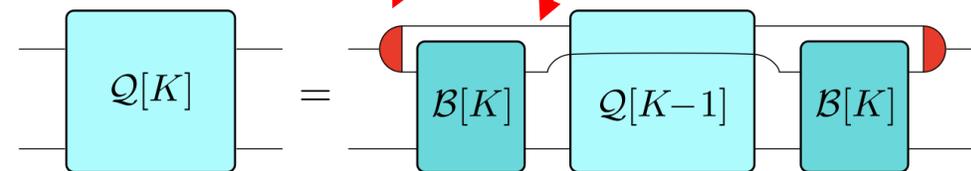
- Method by Moritz Cygorek (TU Dortmund), et al

- C++ implementation freely available at <https://github.com/mcygorek/ACE>

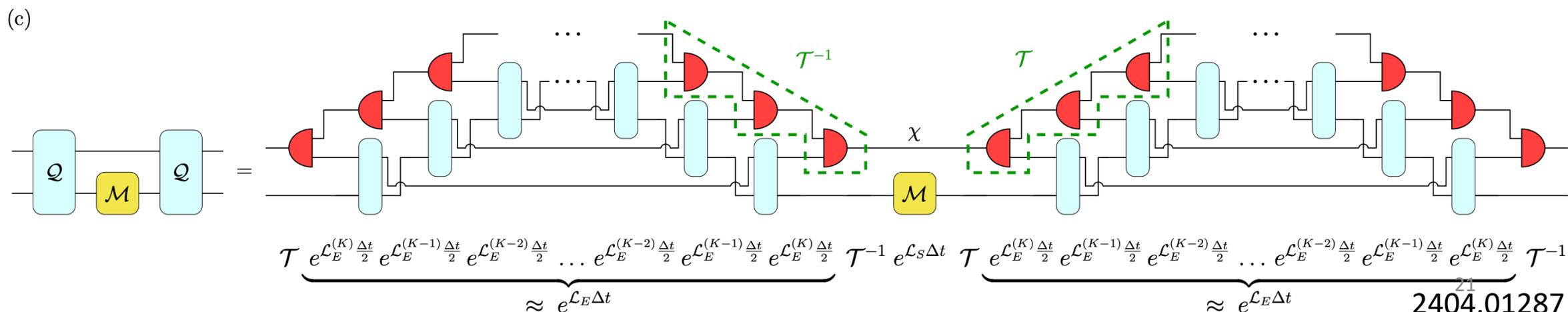
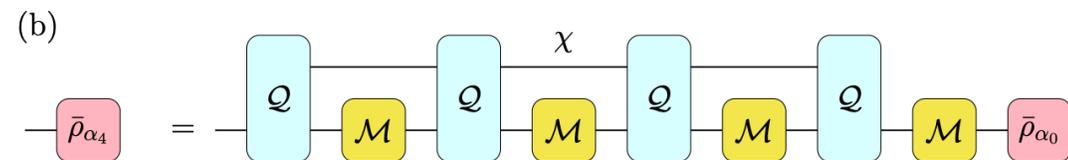
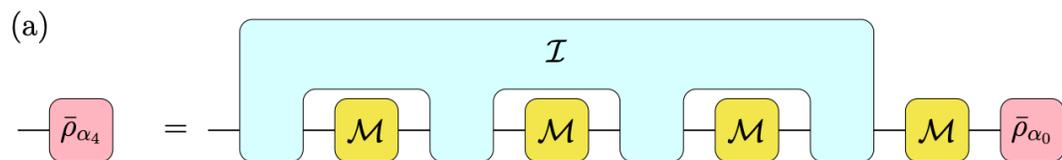
- Trotterized time evolution; successive addition of K environment modes (build the pyramid from the inside out)

SVD: the expensive part!

Environment modes are decoupled



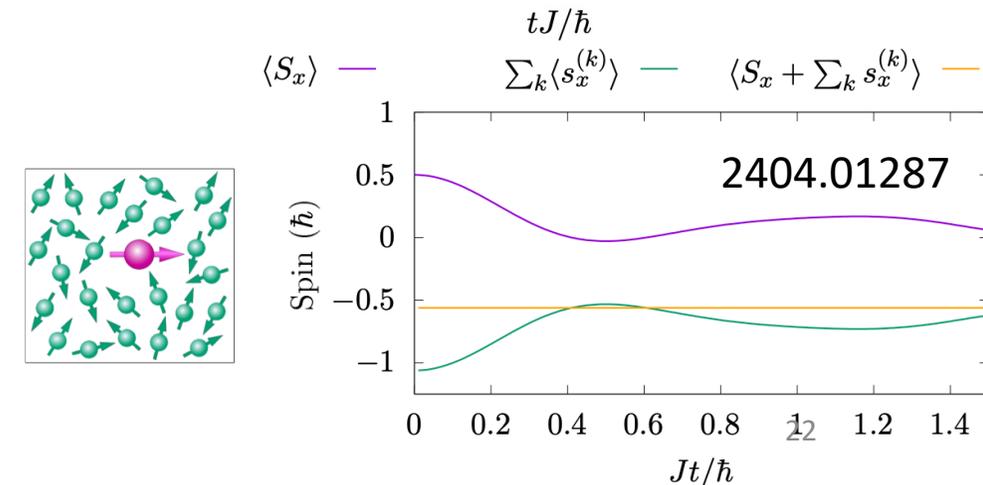
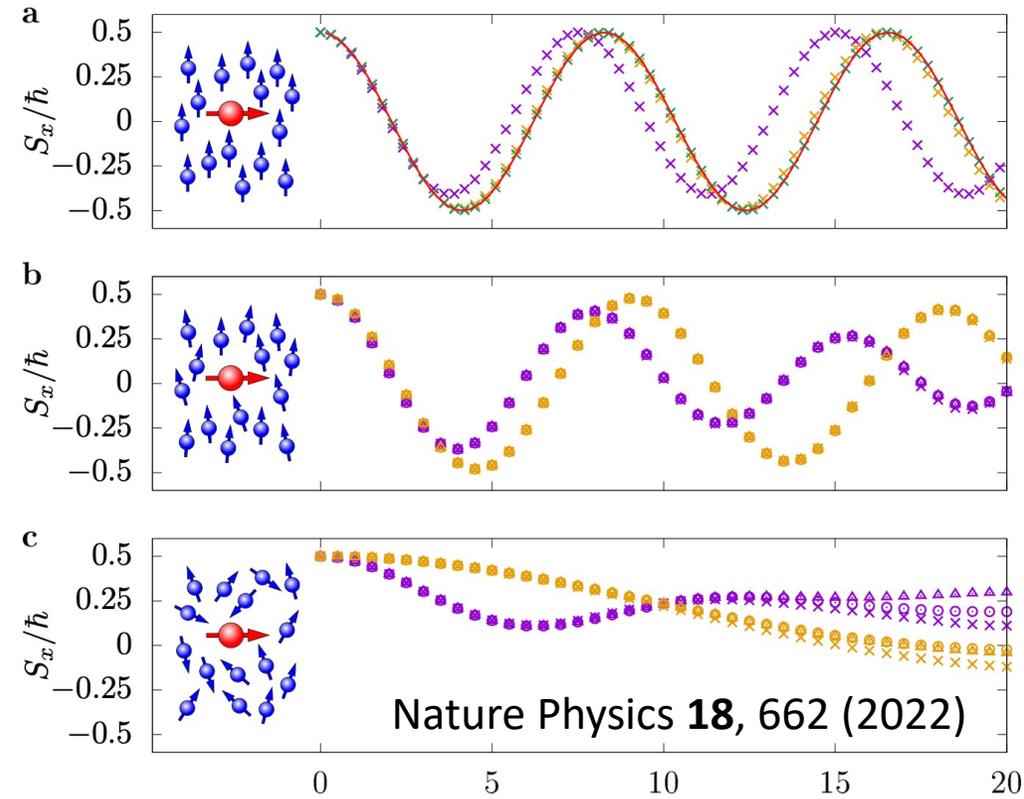
Nature Physics **18**, 662 (2022)



ACE for Central Spin Models

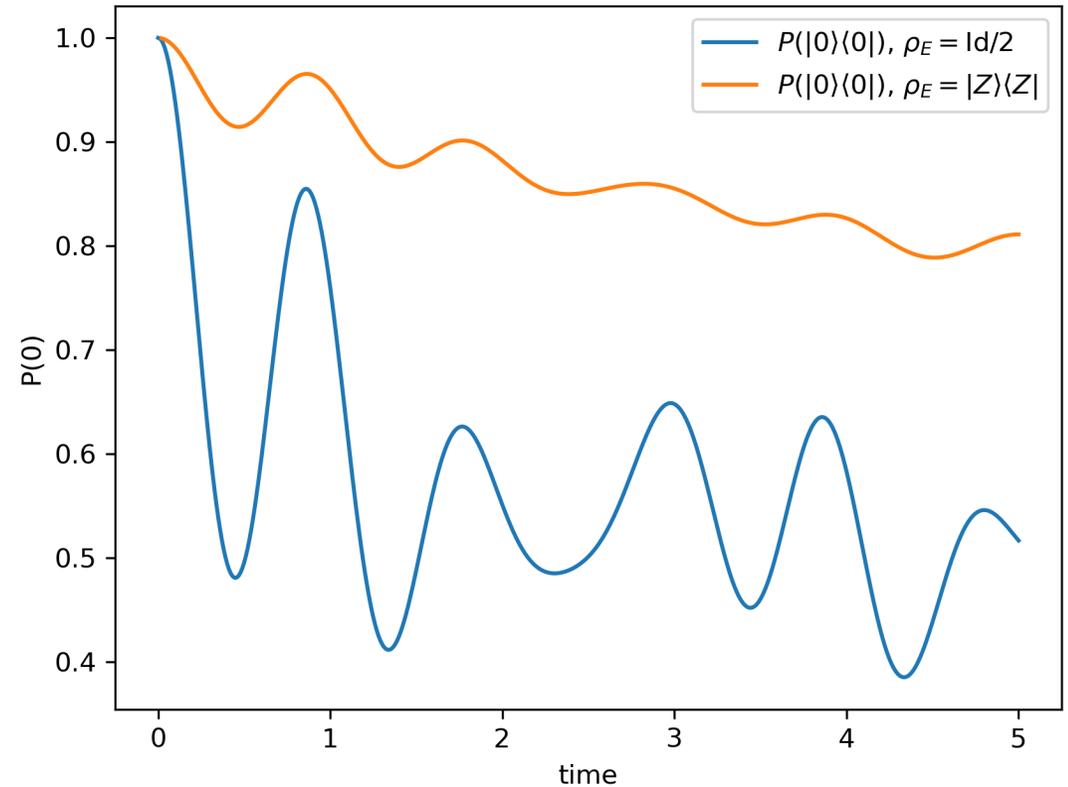
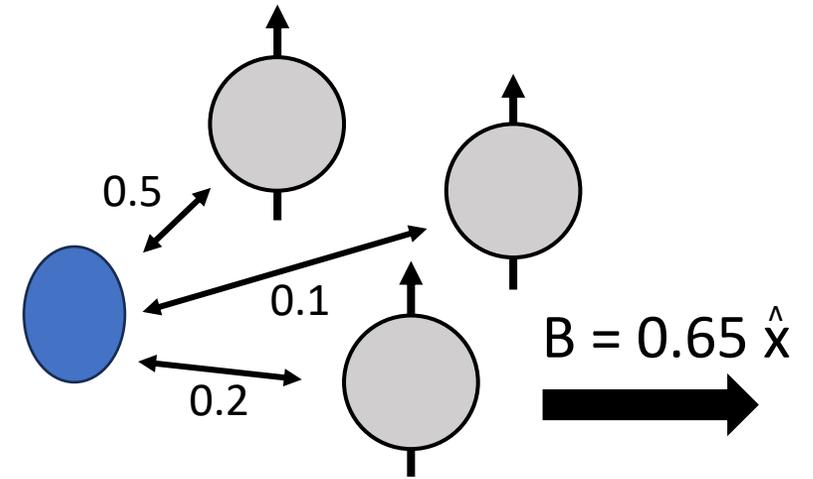
- Algorithm is quite efficient for small system sizes and small bath dimensions
- Can calculate for many (1000s) spin-1/2 environment modes (recall $1000 \gg 7$)
- Can also calculate for fermions, bosons with truncated mode number, etc
- Here
 - 2x2 system \rightarrow 8x8 system,
 - 2x2 bath modes \rightarrow 2x2, 10x10 bath modes
 - Much slower, but better than exact

$N = 10$: $\epsilon = 10^{-10}$ \times $\epsilon = 10^{-13}$ \circ $\epsilon = 10^{-16}$ \triangle
 $N = 100$: $\epsilon = 10^{-10}$ \times $\epsilon = 10^{-13}$ \circ $\epsilon = 10^{-16}$ \triangle
 $N = 1000$: $\epsilon = 10^{-10}$ \times analytic —



Polarization dependence

- Here: consider one spin-1/2 prepared in the $|0\rangle = +1/2$ Z eigenstate coupled to three spin-1/2 bath modes via $A(\sigma_+ I_- + \sigma_- I_+)$, B field in x, no pulses
- In the case of a depolarized bath ($\text{Id}/2$) the decay is rapid (but not coherent)
- In the case of a polarized bath ($|Z\rangle\langle Z|$) the decay is slower as the system spin precesses about the effective B field
- Very suggestive (and I did more work):
 - Pre-priming sequences for nuclei may improve T_2 , specifics not covered here



Decay at zero effective field

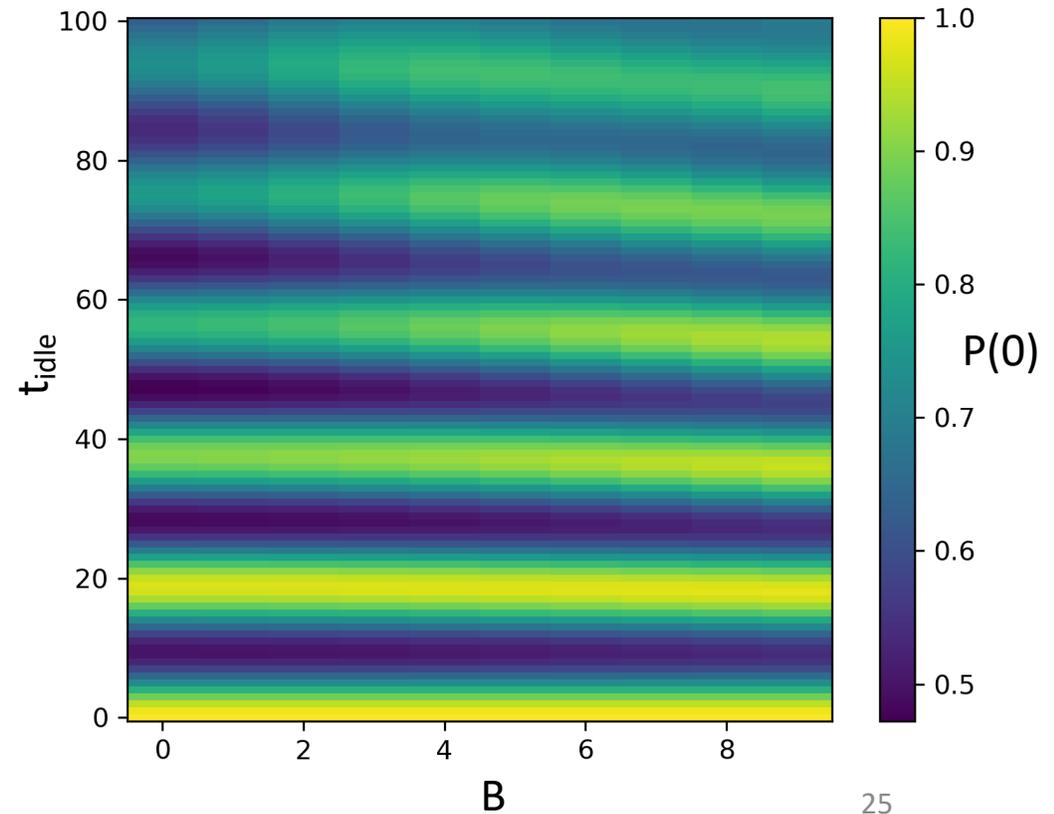
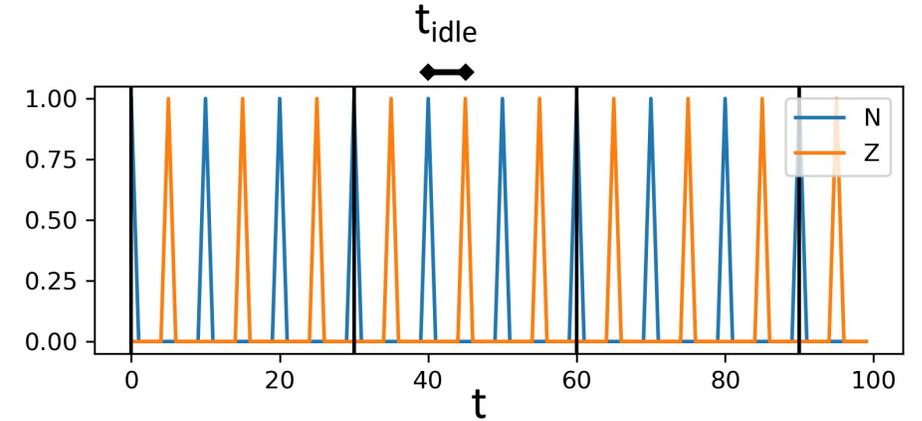
- As we are attempting to eliminate noise sources, we may think that the nuclei provide an effective magnetic field so with zero nuclear magnetization there will be no decay
 - This is not correct
- What is the actual reason for loss of coherence?
 - Becoming a mixed state (classical superposition of quantum states)
 - Quantify using purity $\text{Tr}[\rho^2]$ and bipartite entanglement entropy

No figure here, sorry!

Imagine a decay in the singlet probability which is well matched by the decay in the purity of the state. This can be understood through the fact that there is no term in the system Hamiltonian that couples singlet and triplet, so it is through hybridization with the environment that the triplet state obtains finite (classical) amplitude

Optimal decoupling pulses

- Now we can average the magnetic noise over the dots using N and Z SWAPs
 - Here we do the Carr-Purcell π pulse instead
- How long should we idle?
 - Avoid the bad resonances
 - Sample ensemble of nuclear parameters
 - Extracted from DFT+relaxation and classical electromagnetic field calculations
- Right: qualitative heatmap of decoherence: some idle times are much better to operate at than others
- Optimal regimes of operation (yellow)



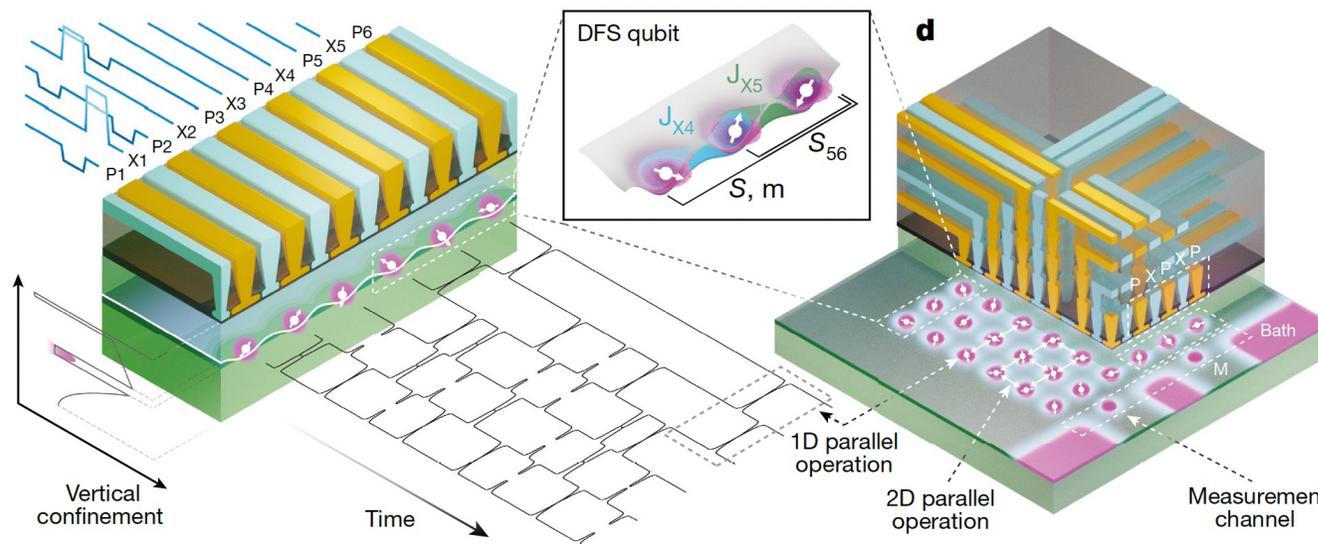
Where to go from here?

- Larger depths

- Improve T_1 and T_2
 - Avoid resonances, better pulse sequences
 - Better materials quality (isotopic purification)
 - Faster electronics—from 100 MHz to 1 GHz

- Larger widths

- SLEDGE (single-layer etch-defined gate electrode)
- Electron-photon coupling and waveguides



Nature **615**, 817 (2023)

Further reading

Silicon Quantum Electronics, RMP 85, 961 (2013)

Semiconductor qubits in practice, Nature Reviews Physics 3, 157 (2021)

Semiconductor Spin Qubits, RMP 93, 025003 (2023)

Thank you!