# Dissipation Induced Fermionic Flat Bands

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# Outline

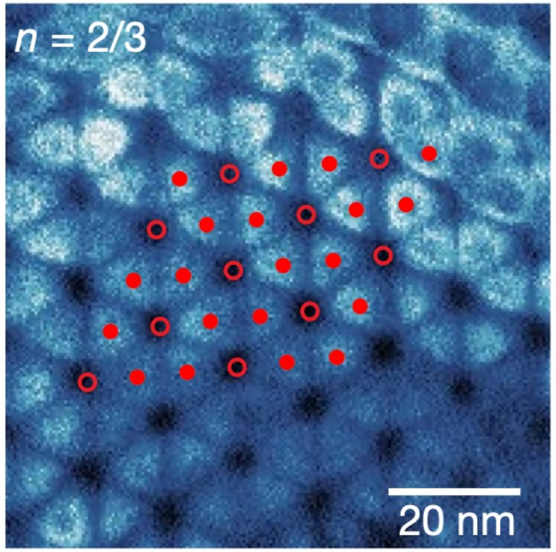
- Context
- Dissipative System-Substrate Coupling
- Emergence of Flat Band Zero Modes from Dissipative Coupling
- Symmetry Protected "Dark Space" Mechanism for Flat Band Creation
- Experimentally Relevant Situation
- Open Questions

# Dominant (Energy) Scales

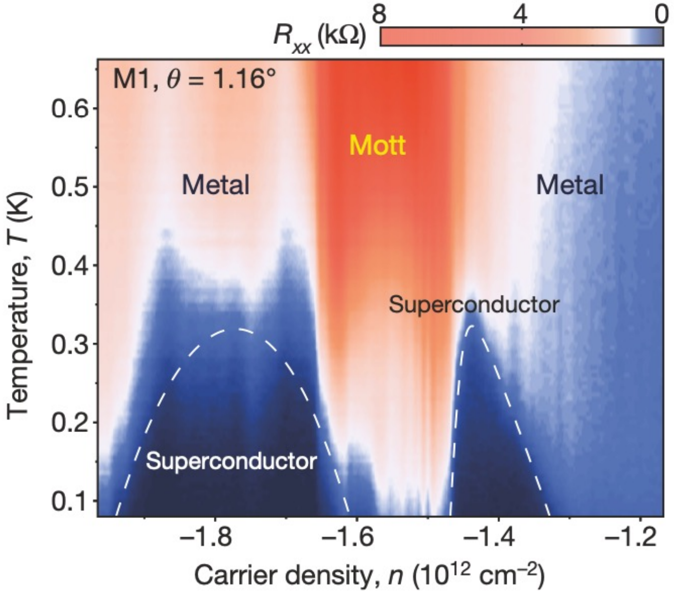
- Physics works because we are (usually) able to separate scales
  - Only address the scales relevant to the problem at hand
- In Condensed Matter we have many mechanisms across energy scales
  - μeV ex. NMR, fine structure, microwave probes
  - meV ex. superconducting gaps, moiré bandwidths, THz probes
  - eV ex. semiconductor gaps, optics
  - keV ex. X-ray spectroscopies
- Competing energy scales and competing phases
  - ex. Coulomb vs kinetic term in small angle twisted bilayer graphene

# Flat Bands: Dominant Coulomb Interaction

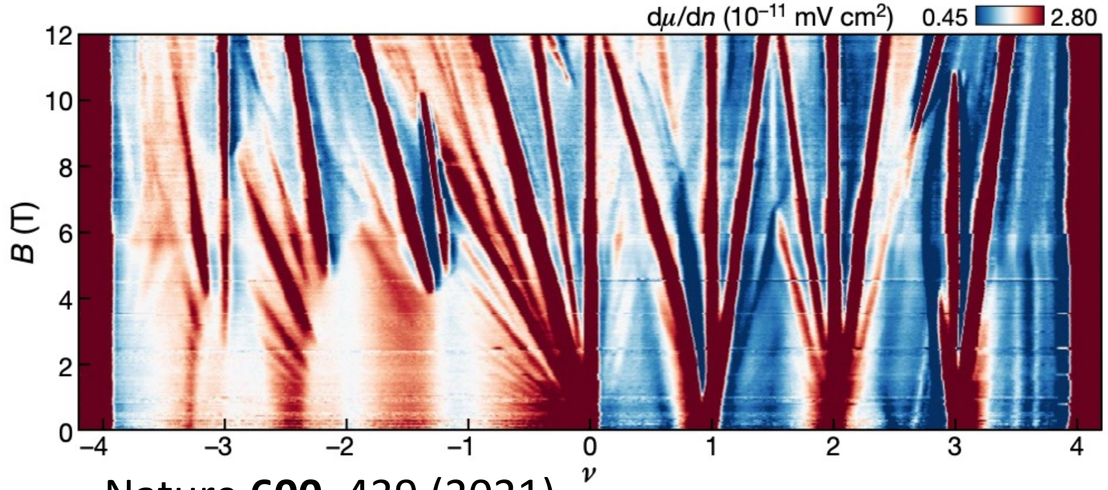
- Interesting stuff happens here!
  - Superconductivity
  - Charge orders
  - Magnetic orders
  - Fractionalized states
  - And more!




Nature **597**, 650 (2021)



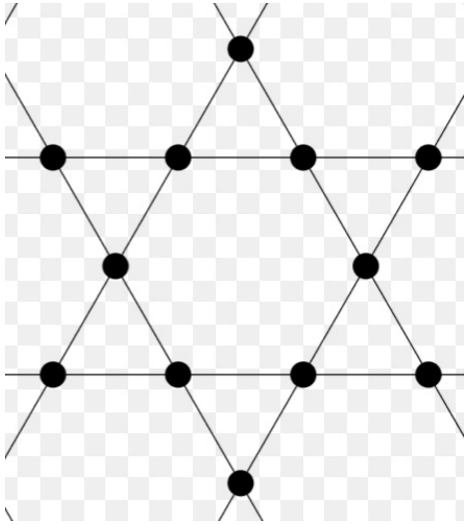

Nature **556**, 43 (2018)



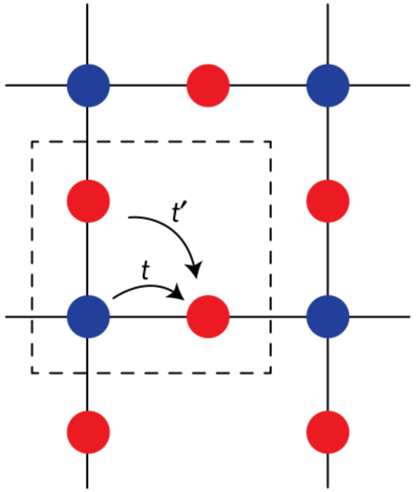

Nature **600**, 439 (2021)

# Ways to Realize Flat Bands

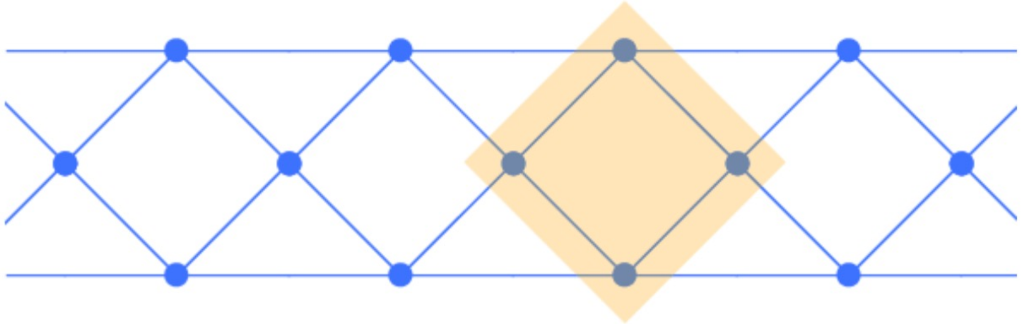
- Atomic-Like Insulators
  - Nobel gas crystals, dimers, etc
- Kinetic Interference
  - Kagomé and Lieb lattices
- Quantum Interference
  - Diamond chain, Magic Angle TBLG
- What all these have in common
  - Symmetry -> Rank-Nullity
- Here: substrate engineering



Wikipedia

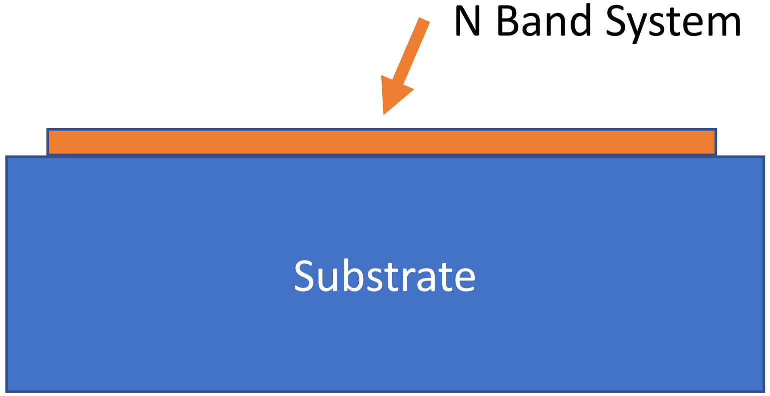



Nat. Phys. **13**, 672 (2017)

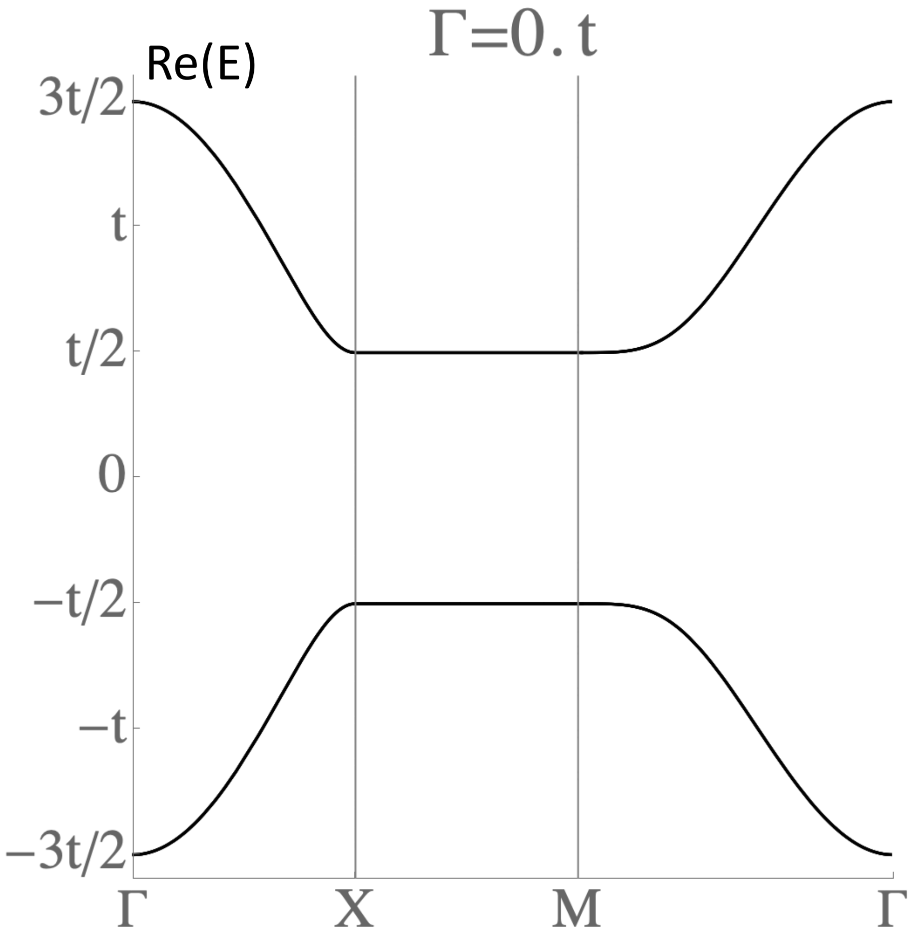


Phys. Rev. Research **3**, 023210 (2021)

# A Non-Inert Substrate




Band Flattening in QWZ Model



# Approaches to Coupled Systems

- Partial trace over density matrices

$$H_{\text{tot}} = H_{\text{sys}} + H_{\text{sub}} + H_{\text{int}}$$
$$\rho_{\text{sys}} = \text{Tr}_{\text{sub}}(\rho_{\text{tot}})$$
$$\dot{\rho} = -i[H, \rho]$$

- Integrating out in a path integral to arrive at an effective action

$$S_{\text{tot}} = S_{\text{sys}} + S_{\text{sub}} + S_{\text{int}}$$
$$S_{\text{eff}}(\text{system d.o.f.}) = S_{\text{sys}} + \int_{\text{substrate d.o.f.}} (S_{\text{sub}} + S_{\text{int}})$$
$$Z = e^{iS}$$

# Density Matrix Formalism

Environment operators

$$M_{\alpha\beta} = \int_{-\infty}^{\infty} dt\, e^{i\omega t} \mathrm{Tr}_{\mathrm{sub}}\left(e^{-iH_{\mathrm{sub}}t} E_\alpha^\dagger e^{iH_{\mathrm{sub}}t} E_\beta \rho_{\mathrm{sub}}(0)\right)$$

Over DOFs in substrate that aren't in system, ex. $k_\perp$

Thermal density matrix

$$H_{\mathrm{int}} = \sum_m S_m \otimes E_m$$

$$\dot{\rho} = -i[H, \rho]$$

Many Assumptions (particular to density matrix)

Coupling strength (real #)

$$i\dot{\rho}_{\mathrm{sys}} = [H_{\mathrm{sys}} + H_{\mathrm{lamb}}, \rho_{\mathrm{sys}}] - i\frac{\Gamma}{2} \sum_{\omega, k_\parallel, \alpha, \beta} M_{\alpha\beta}\left(\{S_\alpha^\dagger S_\beta, \rho_{\mathrm{sys}}\} - 2S_\beta \rho_{\mathrm{sys}} S_\alpha^\dagger\right)$$

Coherent shift from coupling to substrate

Encodes $H_{\mathrm{sub}}$ and $H_{\mathrm{int}}$

System operators act on left *and* right

# The Lindbladian

- Introduce a superoperator $\mathcal{L}$

$$i\dot{\rho} = \mathcal{L}[\rho]$$

- The diagonalization of M

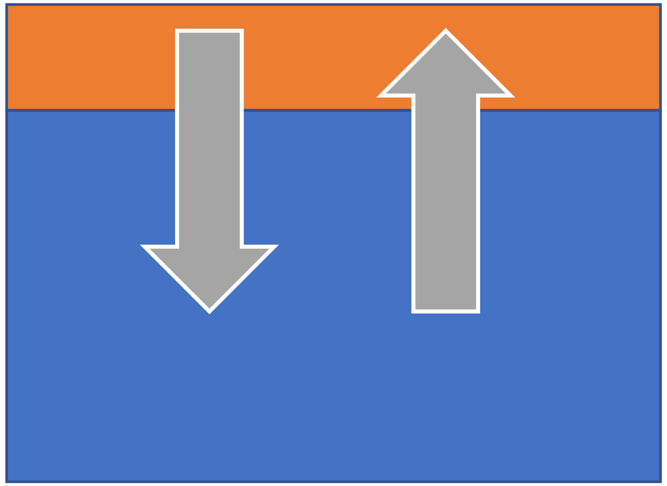
$$J_m(\boldsymbol{k}) = \sum_\alpha a_{m,\alpha}(\boldsymbol{k})c_{\boldsymbol{k},\alpha} + b_{m,\alpha}(\boldsymbol{k})c^\dagger_{-\boldsymbol{k},\alpha}$$

$$\mathcal{L}[\rho] = [H, \rho] - i\frac{\Gamma}{2}\sum_m \left(\{J_m^\dagger J_m, \rho\} - 2J_m\rho J_m^\dagger\right)$$

Non-Hermitian jump operators

- Normal modes of $\mathcal{L}$
  - Particle-like
  - Hole-like
  - Generalized band structure

# What Does Dissipation Mean?



- Mathematically: quantum jump operators
  - Act on both left and right on density matrices
  - Non-Hermitian!
    - Evolution is not unitary
    - Particle number is not conserved
- Intuitively: scrambling degrees of freedom
  - Particles can tunnel between the system and a bath
  - Particles in a band have a finite lifetime/“memory”
  - Information that existed in the system is lost to local measurements as it is stored in non-local degrees of freedom

# The Single Particle Lindbladian I

- We want the normal modes of $\mathcal{L}$
- Express in terms of "left" and "right" superfermions

$$\ell_{\boldsymbol{k},\alpha}\rho = c_{\boldsymbol{k},\alpha}\rho\mathcal{P} \qquad r_{\boldsymbol{k},\alpha}\rho = \rho c^{\dagger}_{\boldsymbol{k},\alpha}\mathcal{P}$$

Fermion parity operator $(-1)^N$

  - Generalization of Prosen's "third quantization" *New J. Phys.* **10**, 043026 (2008)
- With $\mathcal{L} = \boldsymbol{\Phi}^{\dagger}[L_{\mathrm{coh}}(\boldsymbol{k}) - iL_{\mathrm{dis}}(\boldsymbol{k})]\boldsymbol{\Phi}$ for $\boldsymbol{\Phi}_{\boldsymbol{k}} = (\boldsymbol{\ell}_{\boldsymbol{k}}, \boldsymbol{r}_{\boldsymbol{k}}, \boldsymbol{\ell}^{\dagger}_{-\boldsymbol{k}}, \boldsymbol{r}^{\dagger}_{-\boldsymbol{k}})$

$$L_{\mathrm{dis}} = \frac{\Gamma}{4}\begin{pmatrix} A-B & -2B & C-C^{\top} & 2C^{\top} \\ -2A & B-A & -2C & C-C^{\top} \\ (C^{\top}-C)^{*} & -2C^{*} & (B-A)^{\top} & 2A^{\top} \\ 2C^{\dagger} & (C^{\top}-C)^{*} & 2B^{\top} & (A-B)^{\top} \end{pmatrix}$$

BdG form

# The Single Particle Lindbladian II

- Let

$$A_{\alpha,\beta} = \sum_m a^*_{m,\alpha} a_{m,\beta},$$

$$B_{\alpha,\beta} = \sum_m b_{m,\alpha} b^*_{m,\beta},$$

$$C_{\alpha,\beta} = \sum_m a^*_{m,\alpha} b_{m,\beta}$$

These are just defined for notational simplicity and have no deeper meaning

- And

$$L_{\text{coh}} = \frac{1}{2}\begin{pmatrix} H & 0 & 0 & 0 \\ 0 & H & 0 & 0 \\ 0 & 0 & -H^\top & 0 \\ 0 & 0 & 0 & -H^\top \end{pmatrix}$$

Also BdG form

# Symmetries of the Single Particle Lindbladian

- We can write in terms of pseudospins for particles/holes $\eta$ and left/right contours $\tau$ to make the symmetries manifest
- L has BdG form, so we expect
  - Charge conjugation symmetry

$$\mathcal{C}^{-1}L^{\top}\mathcal{C} = -L \qquad \mathcal{C} = \eta_1 \otimes \tau_0$$

  - Time reversal symmetry (here “contour-reversal symmetry”)

$$\mathcal{T}^{-1}(iL)^{*}\mathcal{T} = iL \qquad \mathcal{T} = \eta_2 \otimes \tau_2$$

  - Chiral symmetry

$$\mathcal{S}^{-1}(iL)^{\dagger}\mathcal{S} = -iL \qquad \mathcal{S} = i\eta_3 \otimes \tau_2$$

(anti)-commutation relations need to be generalized for non-Hermitian L, see Phys. Rev. X **8**, 031079 (2018)

# The Dark Space I

- Now, impose one additional symmetry that commutes with L

$$\mathcal{D} = \eta_3 \otimes \tau_1$$

- One then has that A=B and C=C$^T$
- The dissipative part of the Lindbladian must then be

$$L_{\text{dis}} = -\frac{\Gamma}{2}\left(\mathrm{Re}\,(A)\,\eta_3 \otimes \tau_1 + i\,\mathrm{Im}\,(A)\,\eta_0 \otimes \tau_1 + \mathrm{Im}\,(C)\,\eta_1 \otimes \tau_2 + \mathrm{Re}\,(C)\,\eta_2 \otimes \tau_2\right)$$

- Which is Hermitian!

# The Dark Space II

- This pseudospin representation can then be rotated so that

$$U L_{\text{dis}} U^\dagger = -\frac{\Gamma}{2}\tau_1 \otimes \begin{pmatrix} \boldsymbol{a}^\dagger \boldsymbol{a} & \boldsymbol{a}^\dagger \boldsymbol{b} \\ \boldsymbol{b}^\dagger \boldsymbol{a} & \boldsymbol{b}^\dagger \boldsymbol{b} \end{pmatrix}$$

$[\boldsymbol{a}]_{m,\alpha} = a_{m,\alpha},\ [\boldsymbol{b}]_{m,\alpha} = b_{m,\alpha}$

Eigenvalues ±1

N zero eigenvalues with eigenvectors

$|\phi_i^\pm\rangle = U^\dagger\left(\boldsymbol{u}_i,\ \boldsymbol{v}_i,\ \pm\boldsymbol{u}_i,\ \pm\boldsymbol{v}_i\right)$

$\boldsymbol{a}\boldsymbol{u}_i + \boldsymbol{b}\boldsymbol{v}_i = 0$

- $|\phi^\pm\rangle$ span a dissipationless “dark space”
- Generically the other modes are short lived, but there are more long-lived modes if there are fewer jump operators than orbitals
  - These “rank deficient” modes are a manifestation of the Rank-Nullity Theorem

# Order 1: Projecting Into the Dark Space

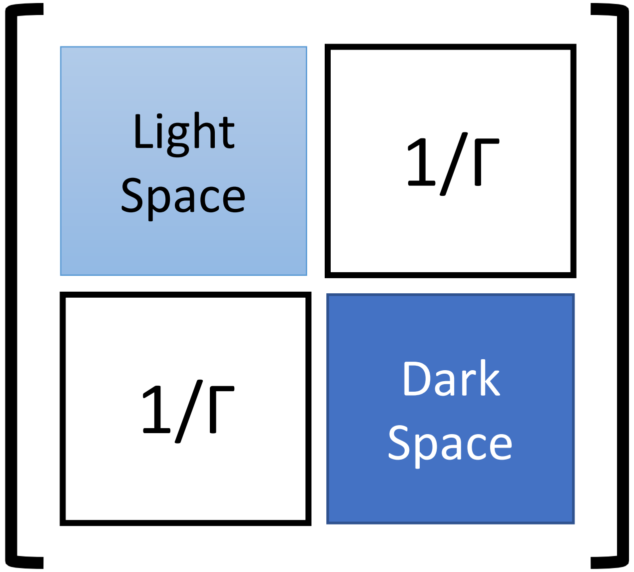
- Now if we consider the strong dissipation limit $\Gamma \gg t$ we can treat the coherent part of the evolution as a perturbation to the dissipative part
- To lowest order, just project into the dark space

$$\widetilde{L}_{ij} = \langle \phi_i | L_{\mathrm{coh}} | \phi_j \rangle$$

- One gets N (generically dispersive) bands with infinite lifetime
- Charge-conjugation symmetry: modes are paired $\epsilon(\boldsymbol{k}) \leftrightarrow -\epsilon(\boldsymbol{k})$
  - So $\mathrm{Tr}(\widetilde{L}) = 0$ : if N is odd then there must be a “dangling” zero mode $\epsilon = 0$
- So: odd number of bands and $[L, \mathcal{D}] = 0$, and $\Gamma$ strong enough $\implies$ long-lived flat band

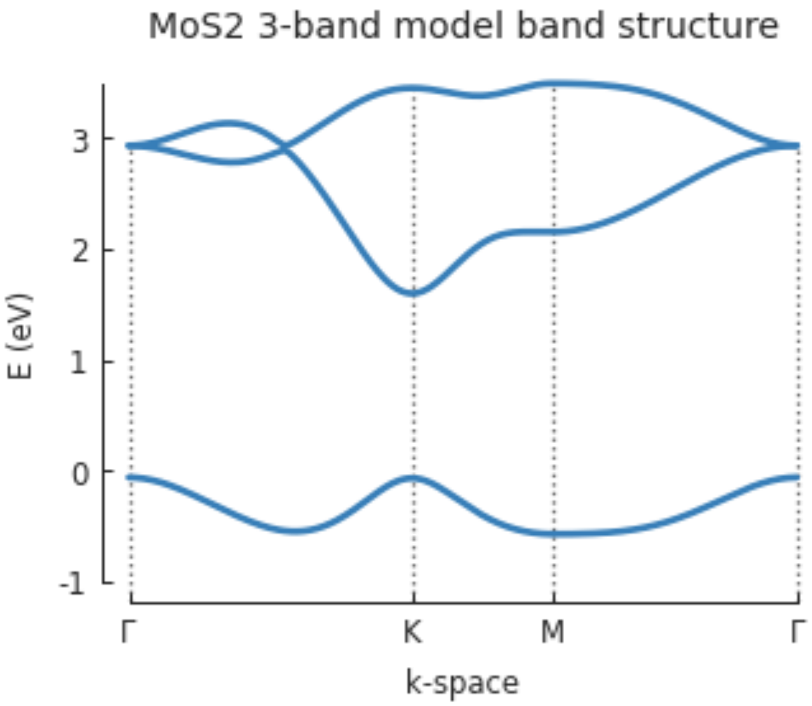
# Order 2: Finite Lifetime Corrections

- Second order corrections lead to finite lifetime
- But processes that couple the dark to the light spaces have lifetimes ~ $1/\Gamma$, so the flat band is counterintuitively long lived in the large $\Gamma$ limit
- Can rotate the Lindbladian to be block diagonal



# Example: A 3-Band Model ($MoS_2$)

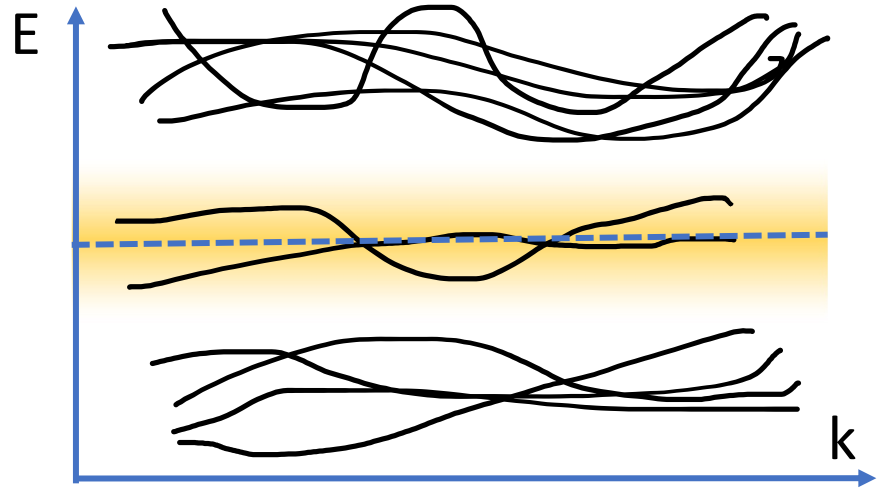
- An odd number of bands
  - Any jump operators with dark-space symmetry will lead to the formation of a flat band
- But this model isn't a very good idea!
  - Band flattening on this energy scale isn't experimentally plausible for coupling t o superconductor
  - Energy scales are *way* too big
    - 500 meV = 6000 K




Phys. Rev. B **88**, 085433 (2013) via PyBinding package

# Aside: What Does it Mean to Have an Odd #?

- Real systems often have a few bands near the Fermi energy and a spaghetti of bands at large positive and negative energies
  - How can we assign a system a number of bands?
- Only count the bands in an energy window
  - (1) The window for which $\Gamma$ is large to the bandwidths and energies
  - (2) The window for which the jump operators fulfill dark space symmetry

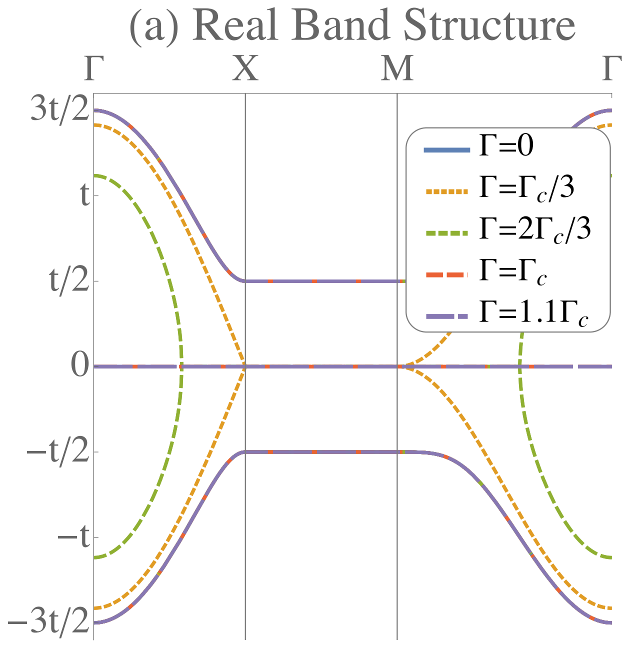


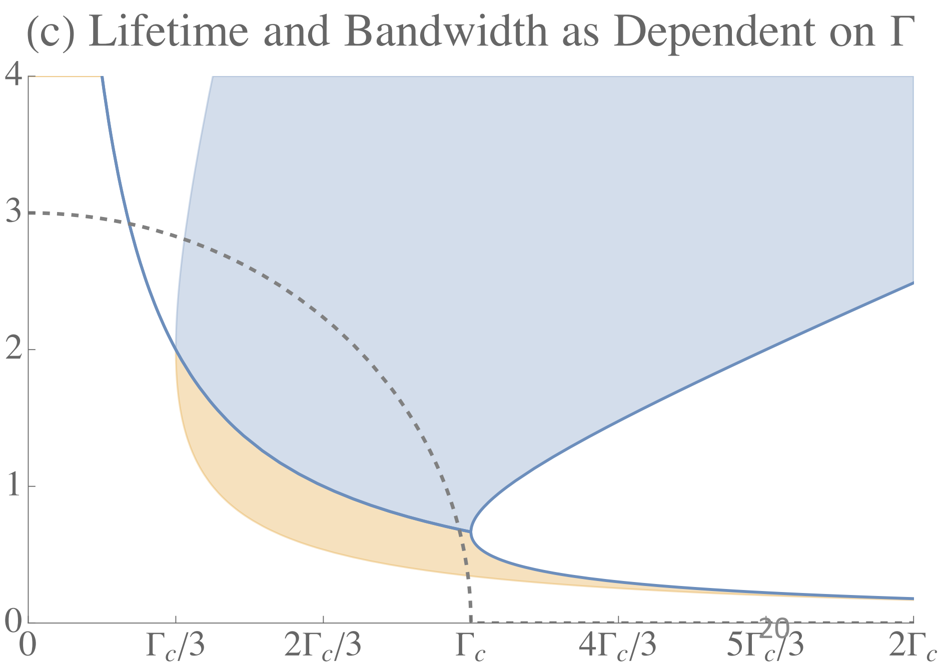
# Example: Qi-Wu-Zhang Model

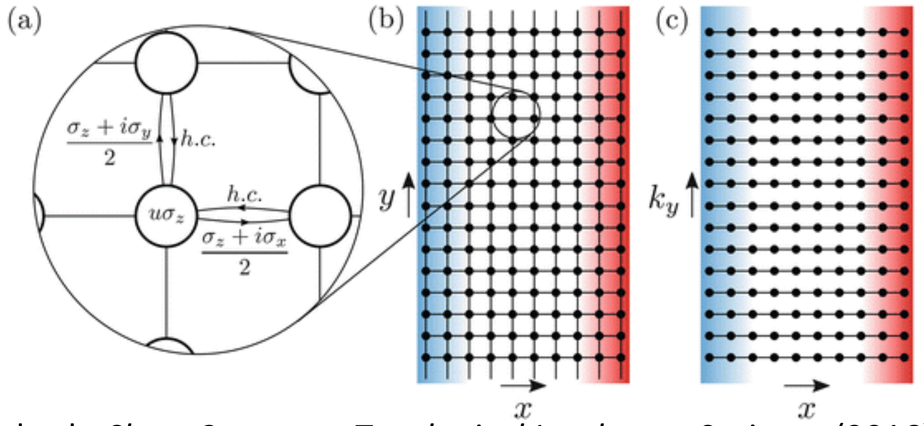
- Jump operator: couple to just one band

$$J = c_{\boldsymbol{k},m} + c^\dagger_{\boldsymbol{k},m}$$

- Rank nullity means that we get a flat band
- Critical dissipation rate corresponds to the maximum bandwidth
- Topology: Chern number zero




Asboth, *Short Course on Topological Insulators*, Springer (2016)

# Spinful Mechanism

- Same dark space operator (but with spin now too)

$$\mathcal{D} = \eta_3 \otimes \tau_1 \otimes \sigma_0$$

- We need two jump operators to remain TRS invariant

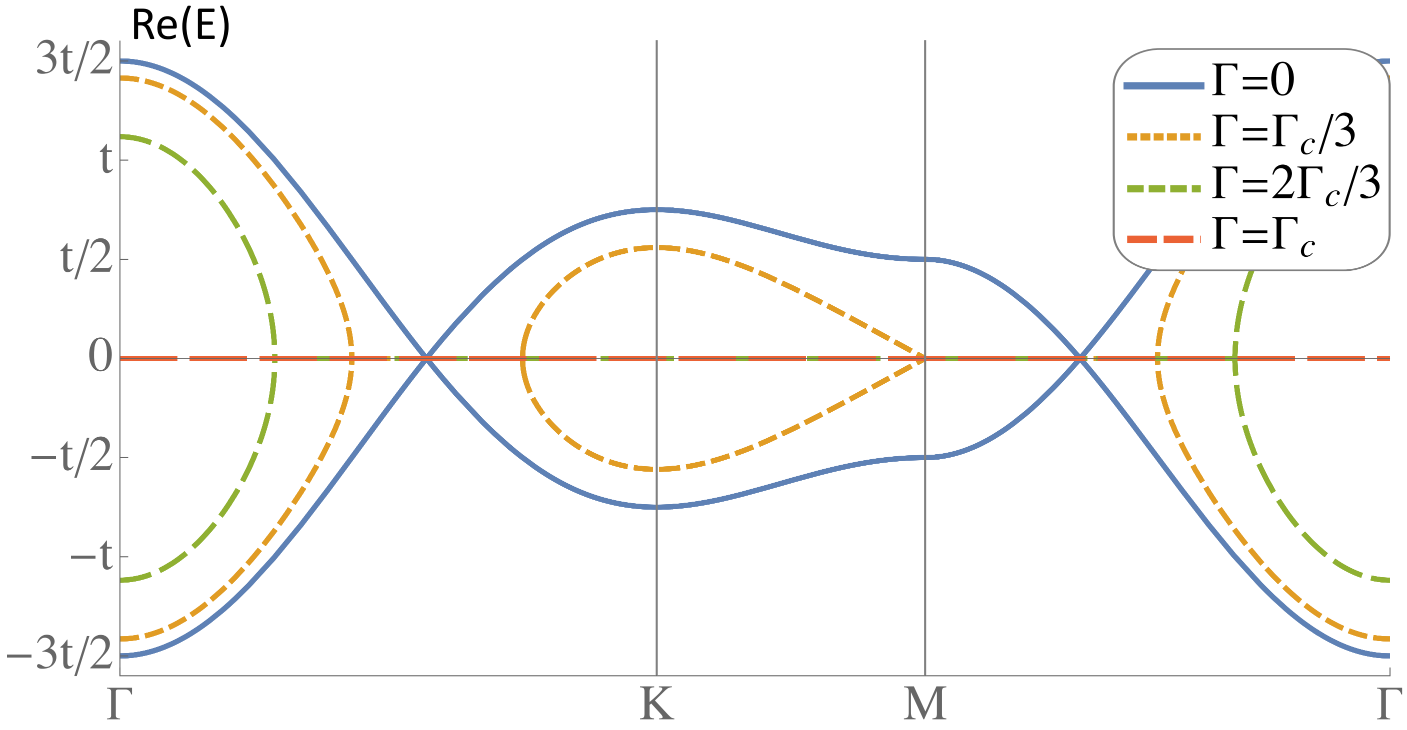
$$J_\uparrow = c_{\boldsymbol{k},\uparrow} + c^\dagger_{-\boldsymbol{k},\downarrow}$$

$$J_\downarrow = c_{\boldsymbol{k},\downarrow} + c^\dagger_{-\boldsymbol{k},\uparrow}$$

- Dark space argument is then the same
  - Project into dissipationless subspace and get a dangling mode
  - (or) Rank deficient

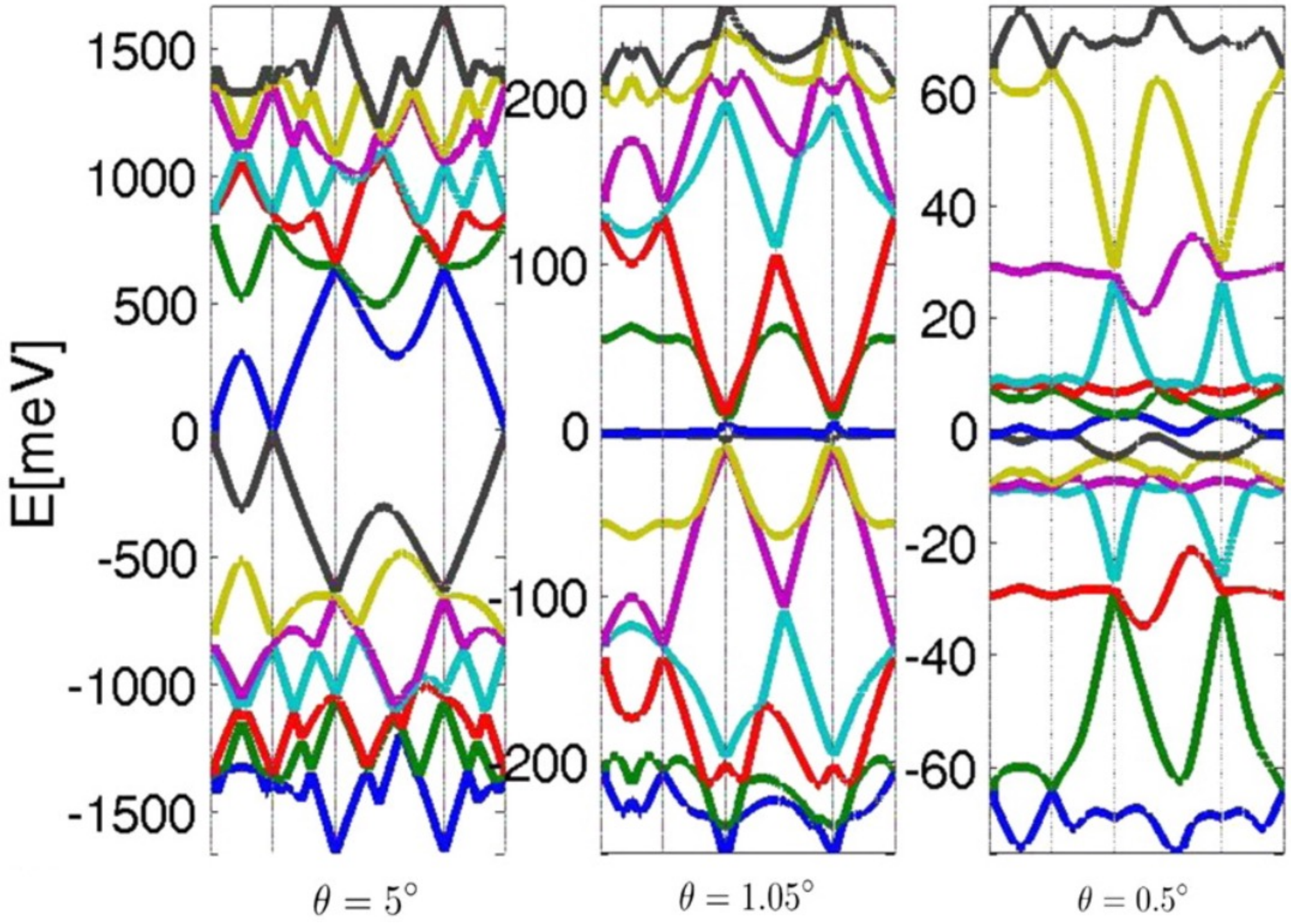
# Example: NN Spinful TRS One Band Model

- Let $H(\boldsymbol{k}) = d(\boldsymbol{k})\sigma_0 + \lambda(\boldsymbol{k})\sigma_3$ with $d(\boldsymbol{k})$ even, $\lambda(\boldsymbol{k})$ odd
- Choose both jump operators of the form $J_\sigma = c_{\boldsymbol{k},\sigma} + c^\dagger_{-\boldsymbol{k},-\sigma}$
  - Both leads to TRS
- Nearest neighbor hopping
- Triangular Lattice



# Magic-Angle Twisted Bilayer Graphene

- Isolated Flat Band
- Can we make it flatter?
  - Be less close to magic angle and still get interesting phases?

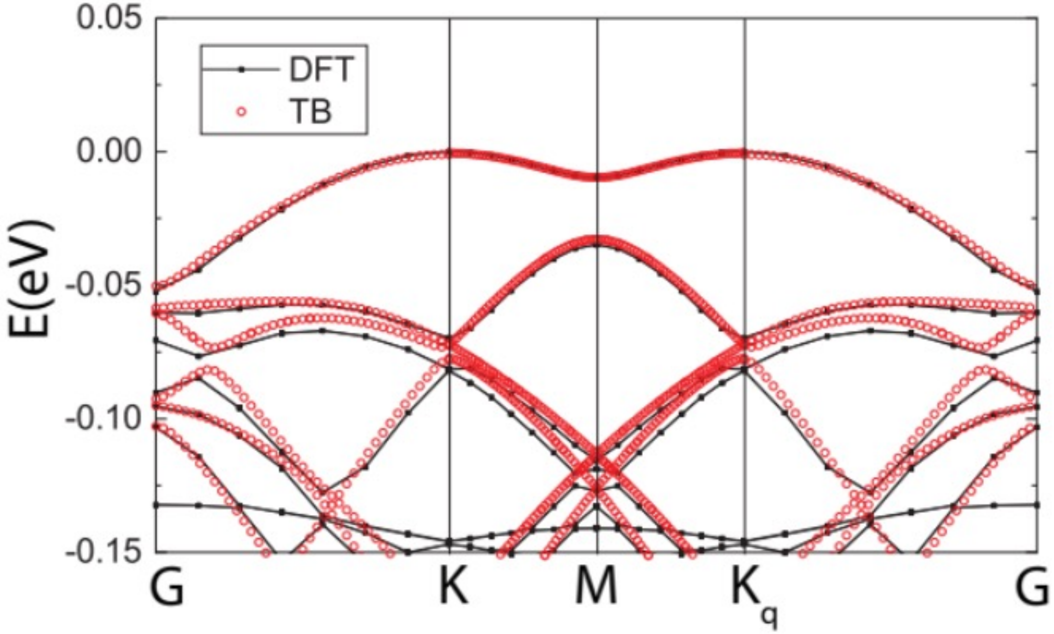



*PNAS* **108**, 30 (2011)

# Twisted TMD Homobilayers

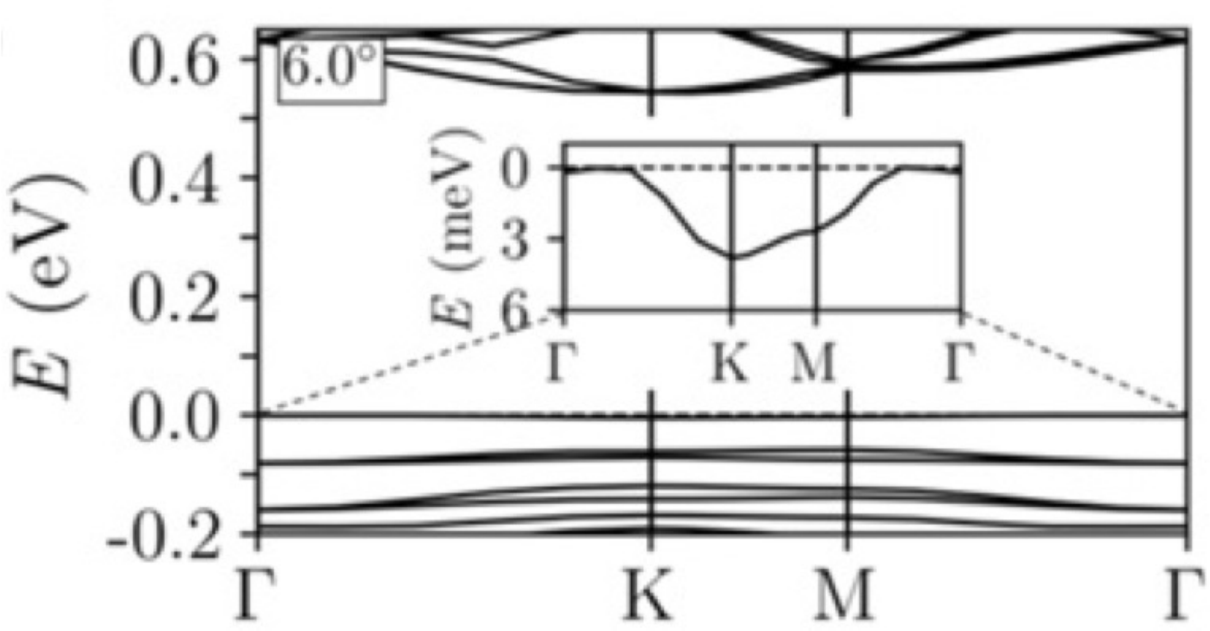
- Isolated almost flat band
- Likely some other TMD homo or hetero bilayers will have similar structures

$WSe_2$ @ 3.89 degrees




*Nat. Mater.* **19** 861 (2020)

$PtSe_2$ @ 6.0 degrees




*Electron. Struct.* **4** 014004 (2022)

# Mean-Field *s*-Wave Superconductor

- Generic form of jump operators

$$J_m(\boldsymbol{k}) = \sum_\alpha a_{m,\alpha}(\boldsymbol{k}) c_{\boldsymbol{k},\alpha} + b_{m,\alpha}(\boldsymbol{k}) c^\dagger_{-\boldsymbol{k},\alpha}$$

- The BdG quasiparticles look very similar

$$\begin{pmatrix} \alpha_{\boldsymbol{k}\uparrow} \\ \alpha^\dagger_{-\boldsymbol{k}\downarrow} \end{pmatrix} = \begin{pmatrix} \cos(\theta_{\boldsymbol{k}}) & \sin(\theta_{\boldsymbol{k}}) \\ \sin(\theta_{\boldsymbol{k}}) & -\cos(\theta_{\boldsymbol{k}}) \end{pmatrix} \begin{pmatrix} c_{\boldsymbol{k}\uparrow} \\ c^\dagger_{-\boldsymbol{k}\downarrow} \end{pmatrix}$$

- What happens to the symmetries of the Lindbladian if we are tunneling of Cooper pairs?

# Jump Operators of the *s*-Wave Superconductor

- Integrating and diagonalizing,

$$M_{\alpha\beta} = \int_{-\infty}^{\infty} dt\; e^{i\omega t}\mathrm{Tr}_{\mathrm{sub}}\big(e^{-iH_{\mathrm{sub}}t}E_\alpha^\dagger e^{iH_{\mathrm{sub}}t}E_\beta\rho_{\mathrm{sub}}(0)\big)$$

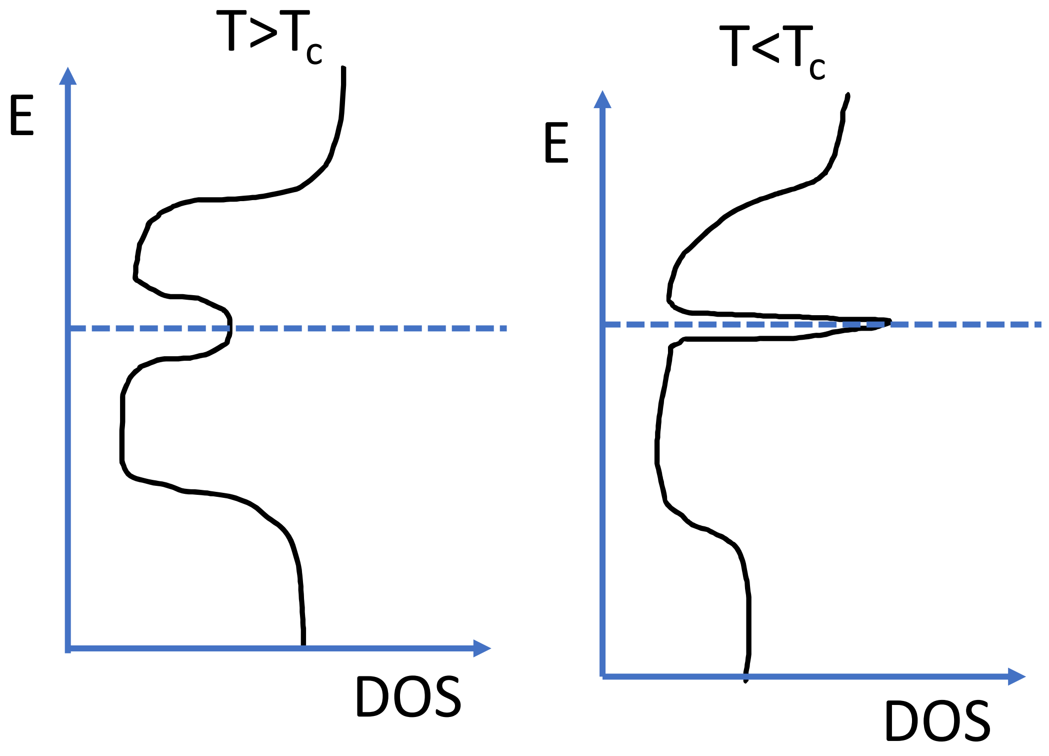
$$-\frac{(\omega+\sqrt{\omega^2+\Delta^2})}{\Delta}c^\dagger_{\boldsymbol{k},\sigma}+1\;c_{-\boldsymbol{k},-\sigma}$$

$$-\frac{(\omega+\sqrt{\omega^2+\Delta^2})}{\Delta}c_{\boldsymbol{k},\sigma}+1\;c^\dagger_{-\boldsymbol{k},-\sigma}$$

- Which for $\omega \ll \Delta$ are of the form we studied above
  - Band flattening occurs in this window while other states just become finite lived

# Experimental Proposal

- Twisted TMD bilayer
  - TBLG (near magic angle)
  - $WSe_2$ (~<4 deg)
  - $PtSe_2$ (~6 deg)
- On top of a *s*-wave superconductor
  - Or a high-$T_c$ superconductor
- T dependence of near-IR optics
  - Sharper absorption edge
  - Narrower Drude peak
  - Although there are issues with probing the superconductor vs probing the surface
- Could also do ARPES, pump-probe, or possibly even interferometry



# Summary

- Flat bands exhibit a panoply of fascinating strongly interacting phases
- Substrates need not be inert and can help to engineer flat bands
- We showed that this engineering can rely on the symmetries of the coupling between the system and the substrate
  - When a “dark space” symmetry holds, flat bands form above a critical dissipation rate
  - This does not rely on the crystalline properties of the system
- Dissipation: an rich new subfield with lots of “low hanging fruit”
  - Existing work: (1) driving systems to states; (2) almost flat Chern bands (M. Goldstein) ; (3) classifying Lindbladians (S. Lieu)

# Thank you!