Dynamic Response of Dissipative Spin Chains

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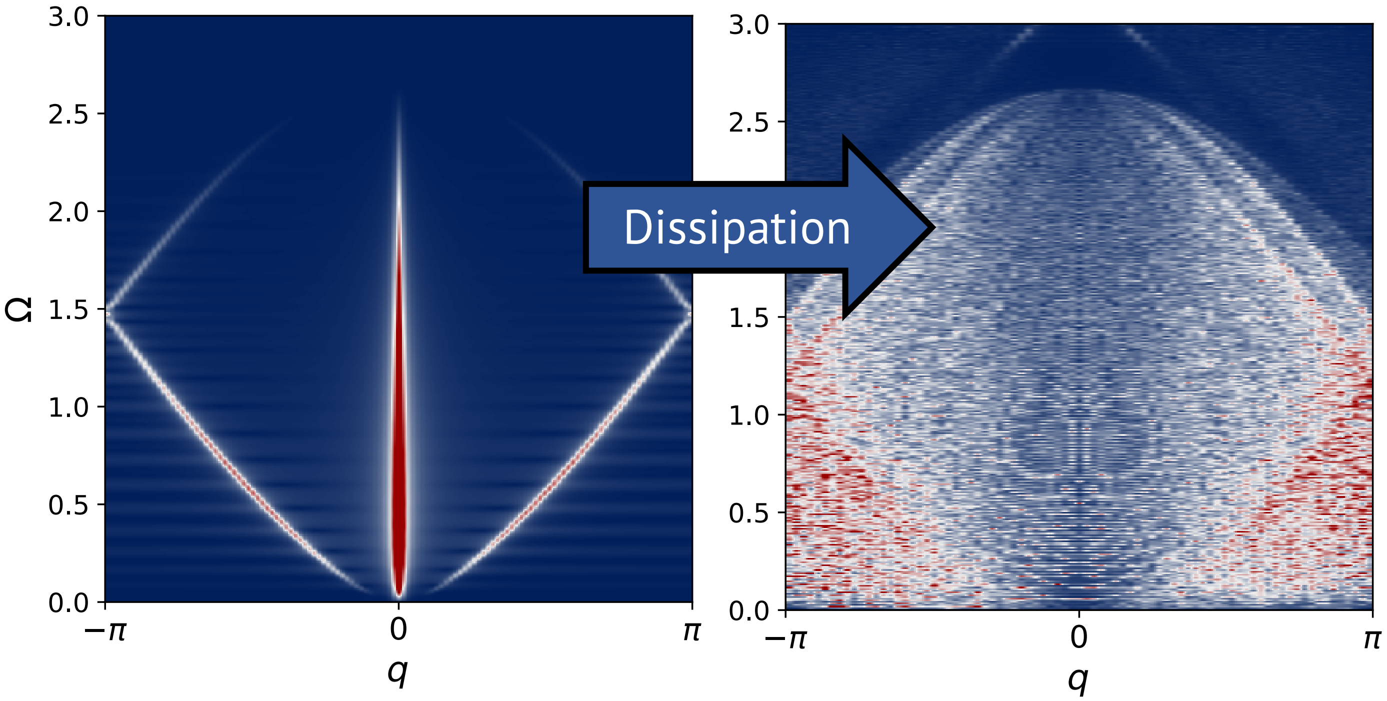
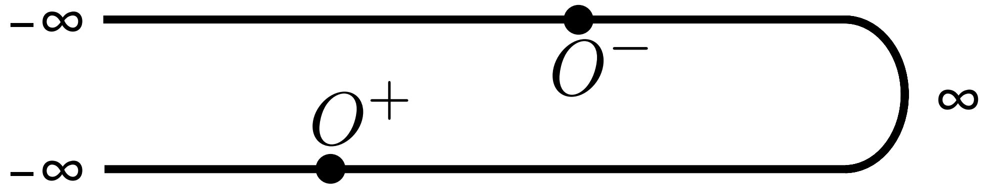
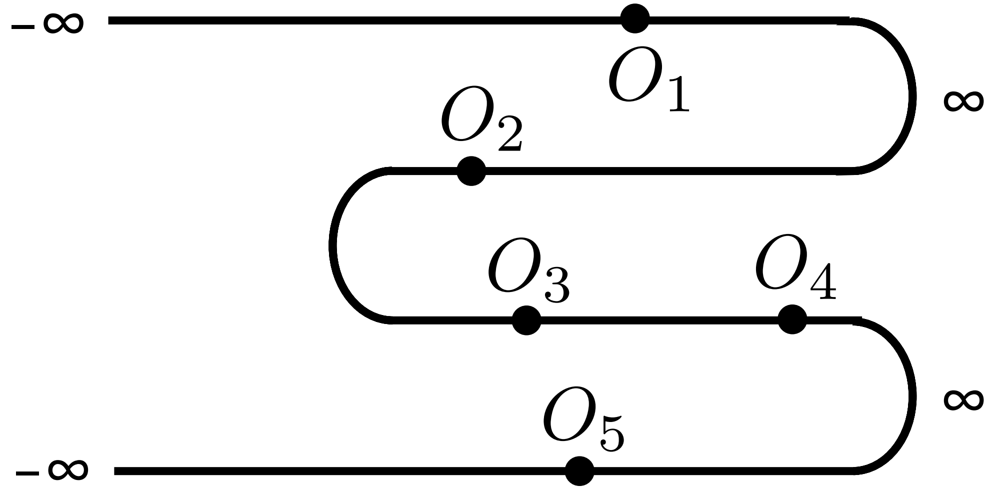


Contours and Correlators

- Take expectation values at times on the Keldysh contour



- Can express two-point correlators in terms of G^R, G^A and G^K
- Multiple contours is generalized by OTOCs



Lindblad Master Equation

- Time evolution in the limit of continuous measurement by a memoryless bath $i\dot{\rho} = \mathcal{L}[\rho]$

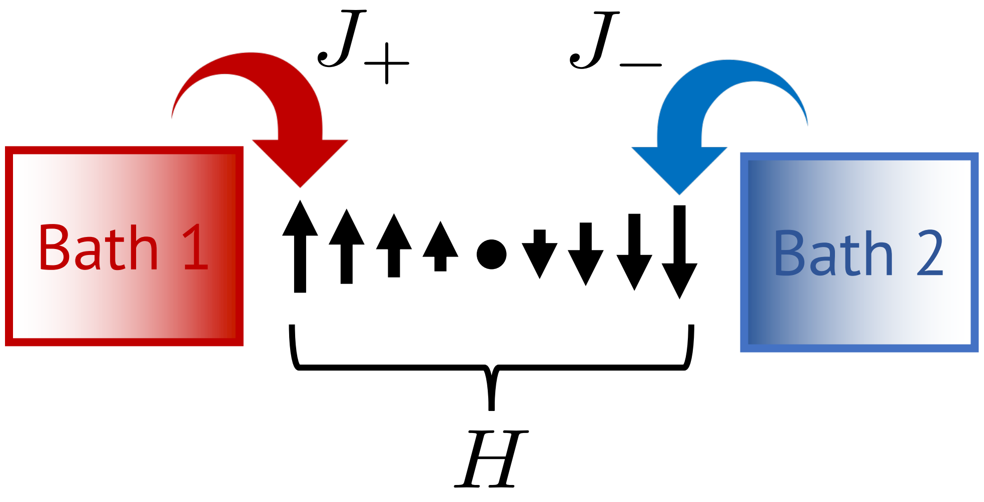
$$i\dot{\rho} = (\mathcal{L}^{\text{coh.}} + \mathcal{L}^{\text{n.h.}} + \mathcal{L}^{\text{jump}})[\rho]$$

where,

$$\mathcal{L}^{\text{coh.}}[\rho] = [H, \rho]$$

$$\mathcal{L}^{\text{n.h.}}[\rho] = -i\frac{\Gamma}{2}\sum_m \{J_m^\dagger J_m, \rho\}$$

$$\mathcal{L}^{\text{jump}}[\rho] = i\frac{\Gamma}{2}\sum_m 2J_m \rho J_m^\dagger$$

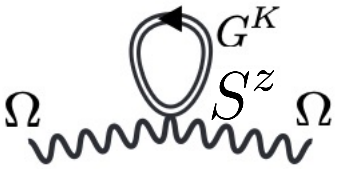


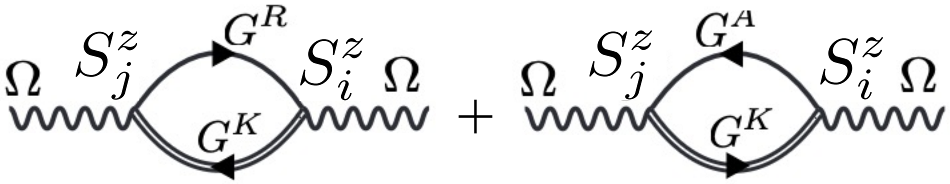
Spin chain with boundary dissipation. Jump operators linear in spins map to jump operators linear in fermions under Jordan-Wigner transformation.

Energy scales
J: spin-spin coupling
h: transverse field
Γ: dissipation strength

Green's Functions

- We want to probe the steady state with static density response $\langle S^z \rangle_{\text{ss}}$ and dynamic response $\mathcal{S}^z_{i,j}(\Omega) = \int dt\ e^{-i\Omega t} \langle [S^z_i(t), S^z_j(0)] \rangle_{\text{ss}} \theta(t)$

$\langle S^z \rangle_{\text{ss}} =$ 

$\mathcal{S}^z_{i,j}(\Omega) =$ 

- These can be expressed in terms of Green's functions
  - Closed system: G^R and G^A
  - Open system: G^R, G^A and a Keldysh Green's function G^K
- Jump terms lead to complex self energy!
- Wick's theorem can be used to reduce multi-point correlation functions to two-point correlation functions

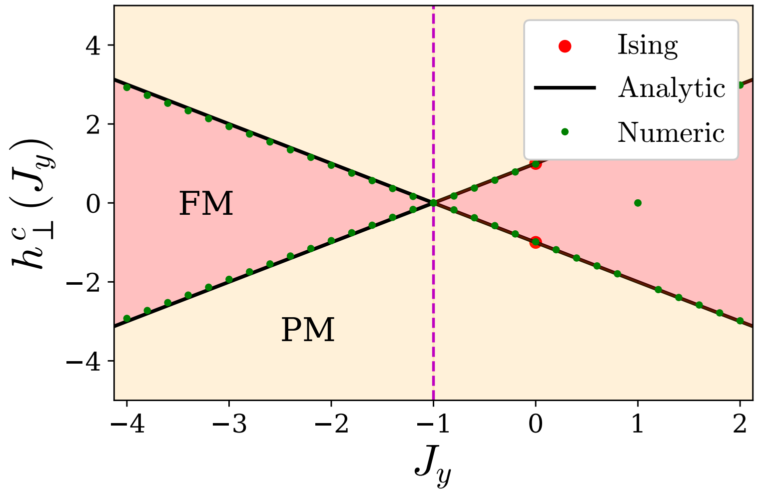
Lindblad-Keldysh GFs:
- Thompson and Kamenev, Ann. Phys. 455, 169385 (2023)
- McDonald and Clerk, Phys. Rev. Res. 5, 033107 (2023)

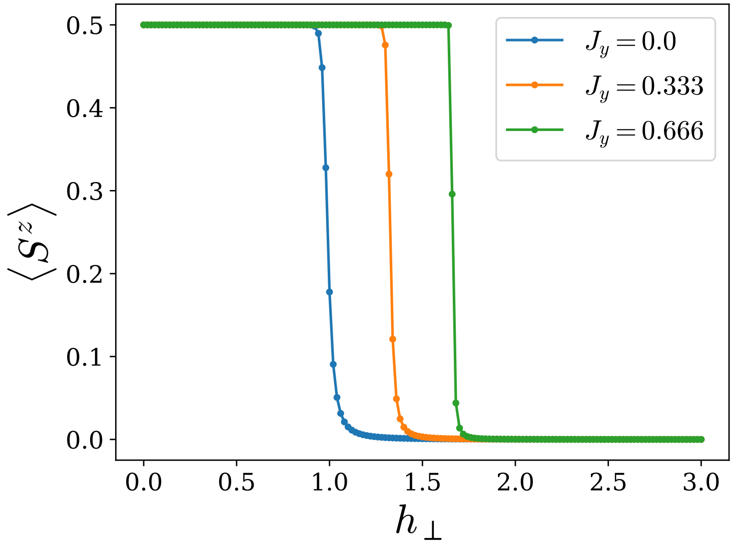
Ising and XY Models: Closed Systems I

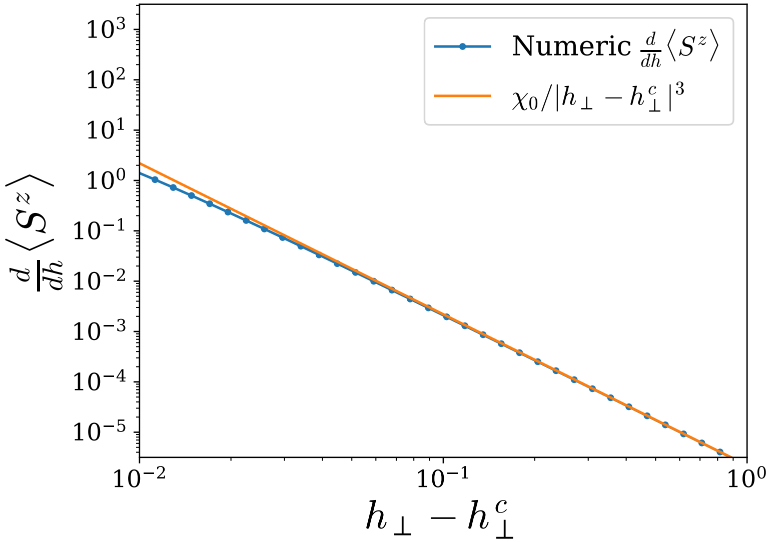
- Transverse field XY spin chain is paradigmatic example of gapped/gapless quantum matter

$$H_{XY} = \sum_{n=1}^{N-1} J_n^x S_n^x S_{n+1}^x + J_n^y S_n^y S_{n+1}^y + \sum_{n=1}^{N} h_n S_n^z$$

- Jordan-Wigner transformation to fermions
- Find magnetization
  - Paramagnetic to ferro-magnetic transition
- Susceptibility diverges with a power law





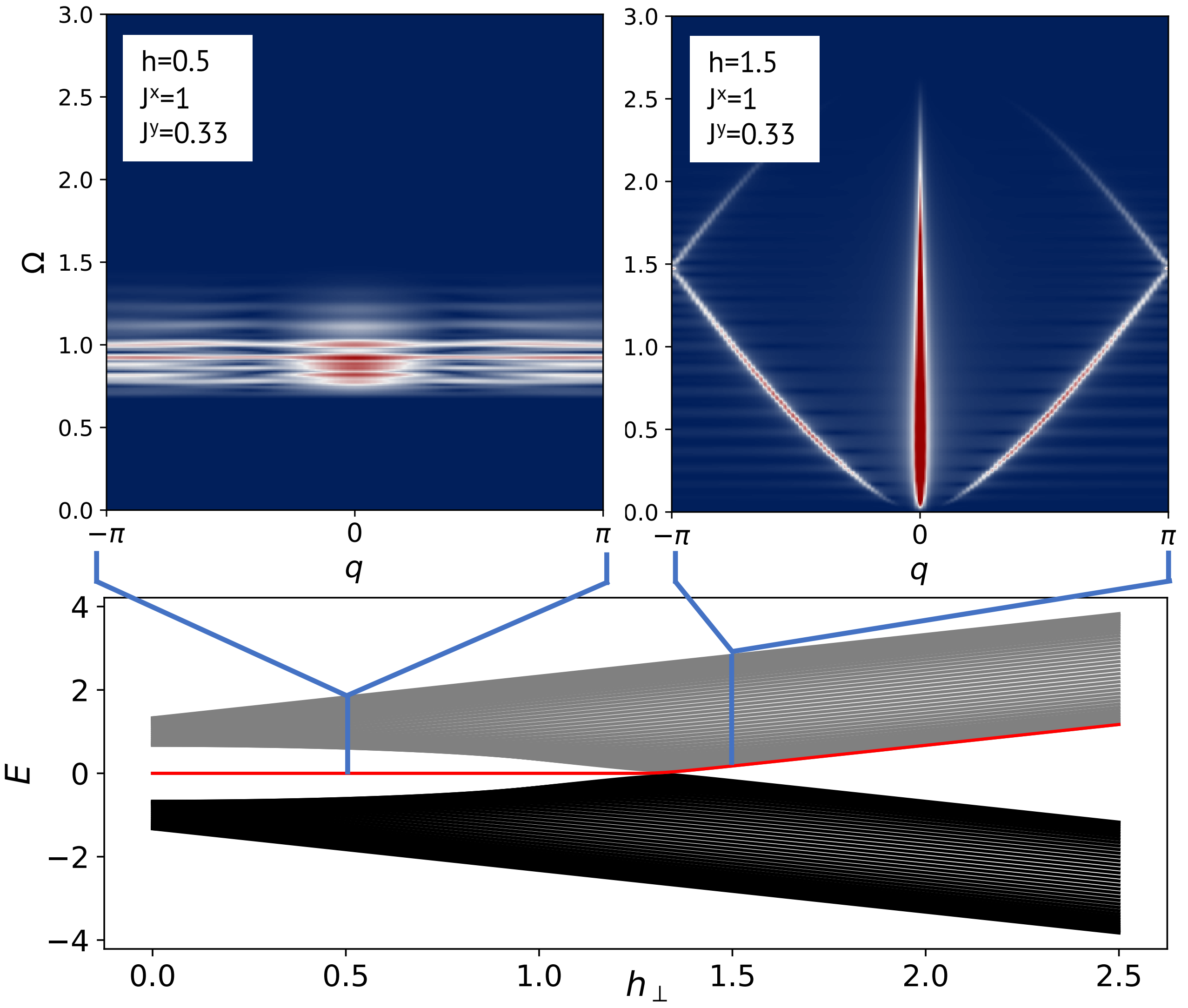


Ising and XY Models: Closed Systems II

- Dynamic correlation function

$$\chi_{ij}(\Omega) = \sum_{n>0} \frac{\langle u_0|S_i^z|u_n\rangle\langle u_n|S_j^z|u_0\rangle}{\Omega + i\eta - (\epsilon_n - \epsilon_0)}$$

- Probe excitations above $|u_0>$
- See the structure of gapped and gapless excitations
- All negative energy states are filled (think Pauli blocking)



Interlude

Closed Systems

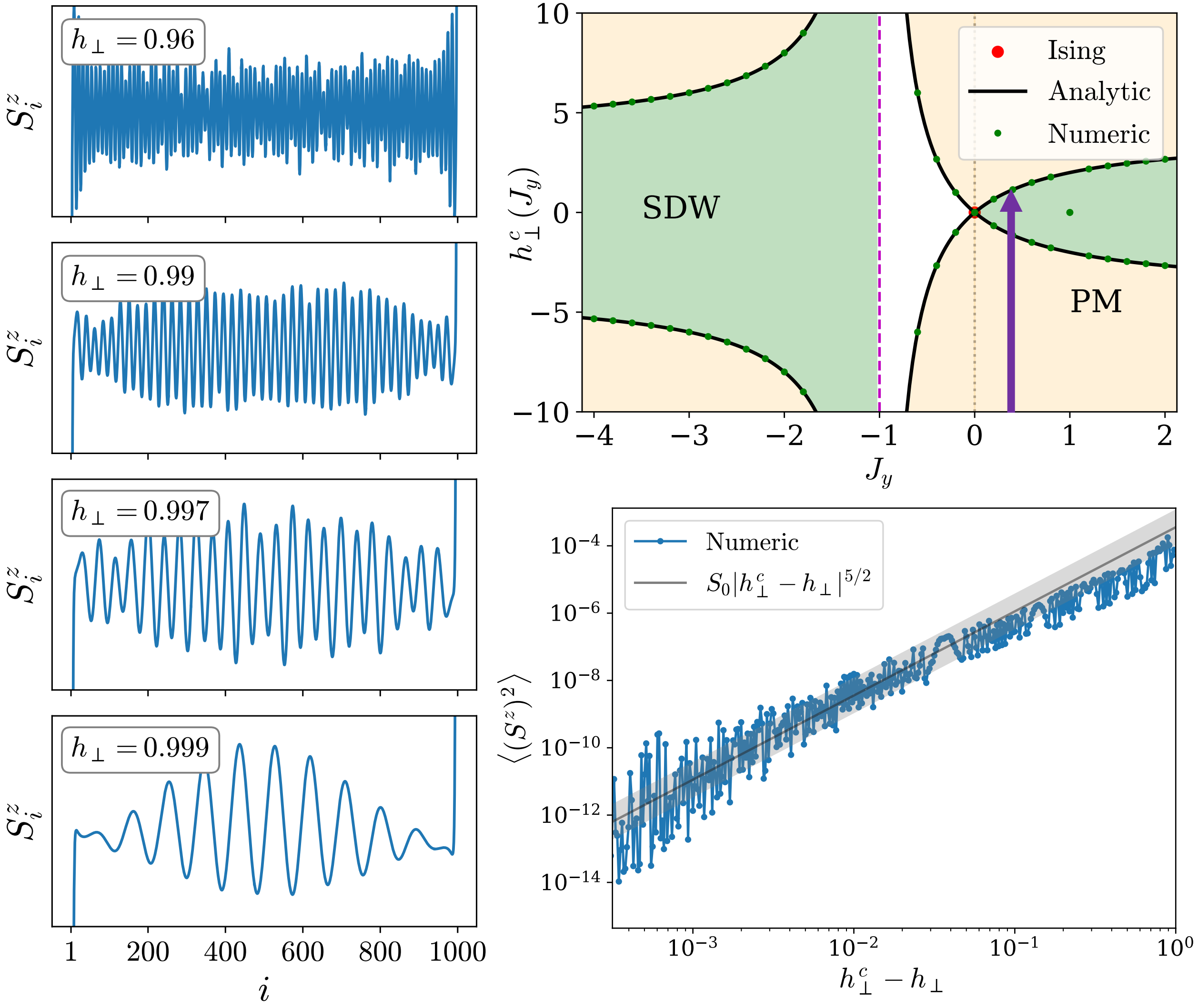
- Hermitian
- Real eigenvalues
- Orthogonal eigenstates
- Fermi-Dirac and Bose-Einstein distributions

Open Systems

- Non-Hermitian
- Complex eigenvalues
- Bi-orthogonal eigenstates
- Distribution function given by G^K
  - In Lindblad formalism this does not recover FD and BE distribution functions in the limit of weak dissipation

Ising and XY Models: Open Systems I

- Spin chain with boundary dissipation S^+ on left, S^- on right
- New phase: spin-density wave
  - Magnetization $\langle S^z \rangle$ vanishes, but its higher moments do not
  - Phase boundary is very different from equilibrium phase boundary
- Wavelength diverges and $\langle (S^z)^2 \rangle$ exhibits power-law scaling on near critical point h^c
- Choose model with $J^x=1$, $J^y=1/3$ so that $h^c = 1$

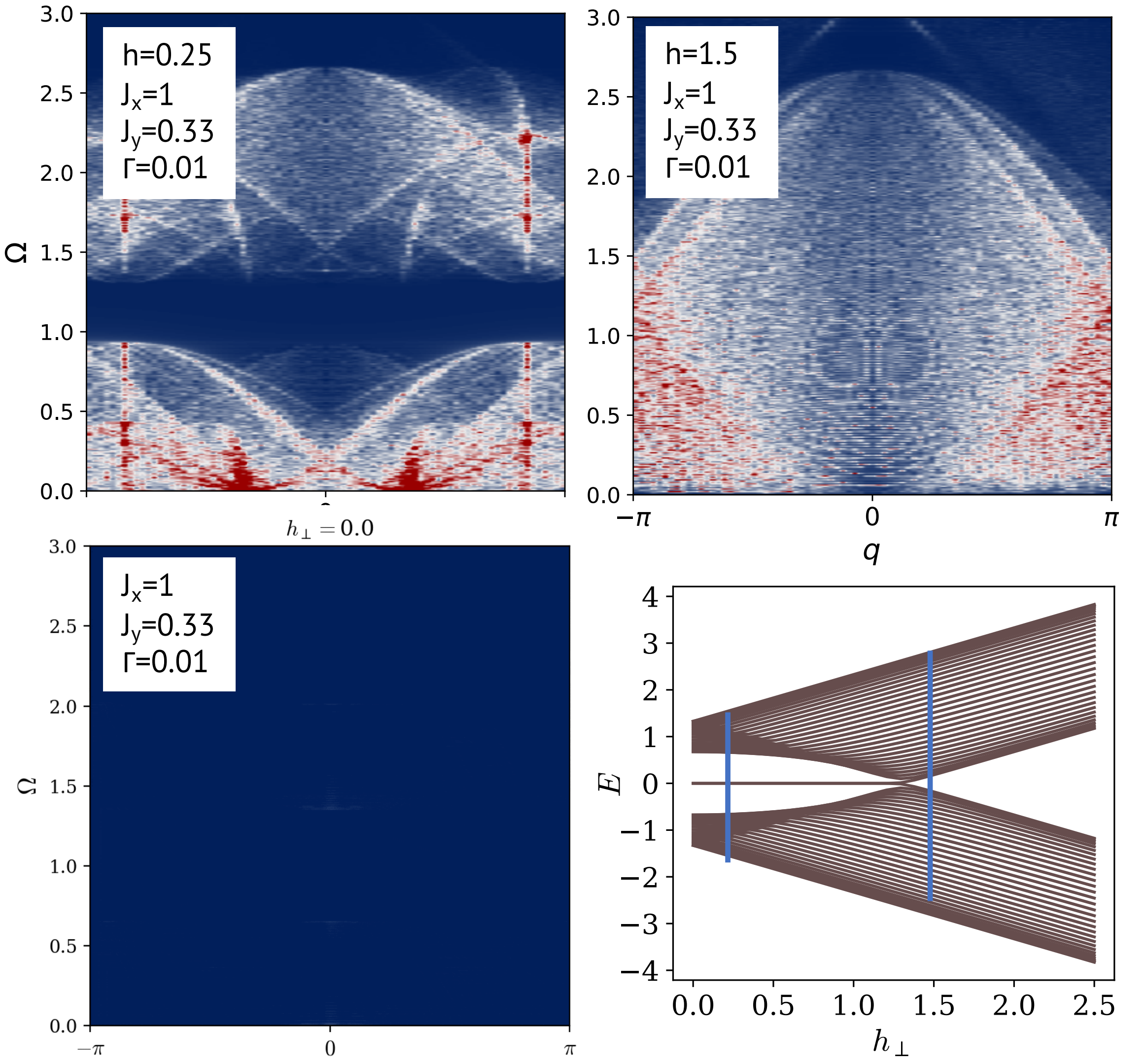


Ising and XY Models: Open Systems II

- Dynamic correlation function

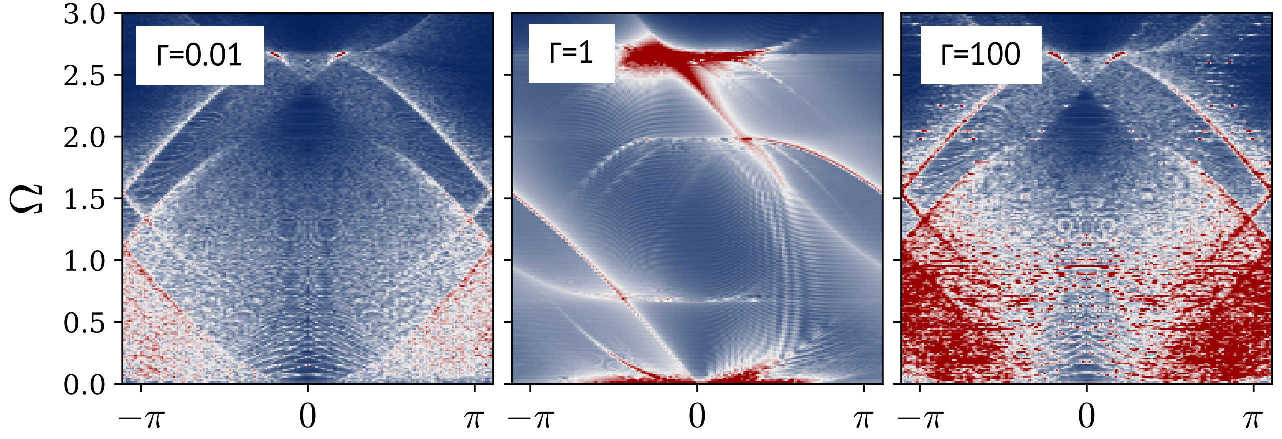
$$\mathcal{S}^z_{i,j}(\Omega) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \Big(\mathrm{Tr}[S^z_i G^R(\omega) S^z_j G^K(\omega+\Omega)] + \mathrm{Tr}[S^z_i G^K(\omega-\Omega) S^z_j G^A(\omega)] \Big)$$

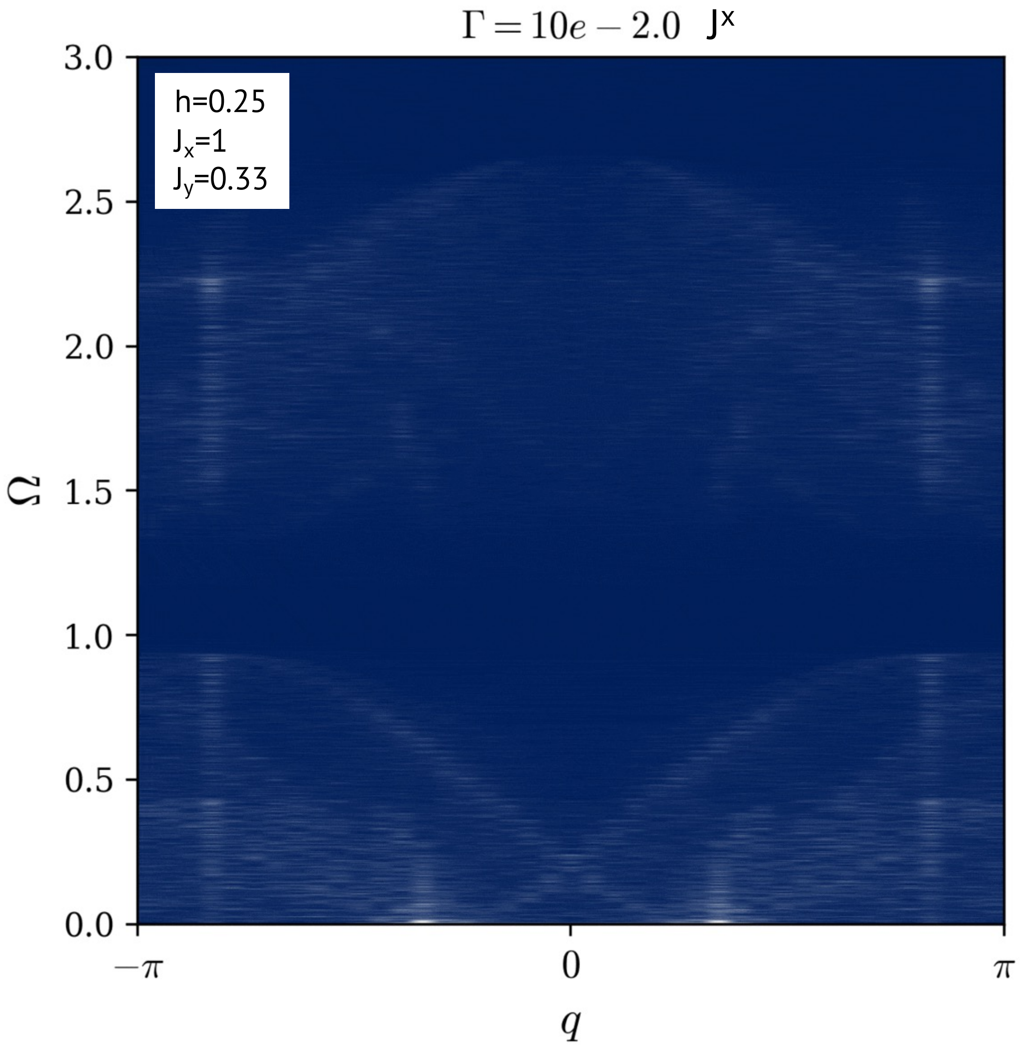
- Fractional occupation given by distribution function for ρ_{ss}
  - States at all energies contribute
  - Spectral gaplessness for $h < h^c$
- Vertical lines at q_{SDW} from SDW
- Dispersing modes from $2q_{\mathrm{SDW}}$



Ising and XY Models: Open Systems III

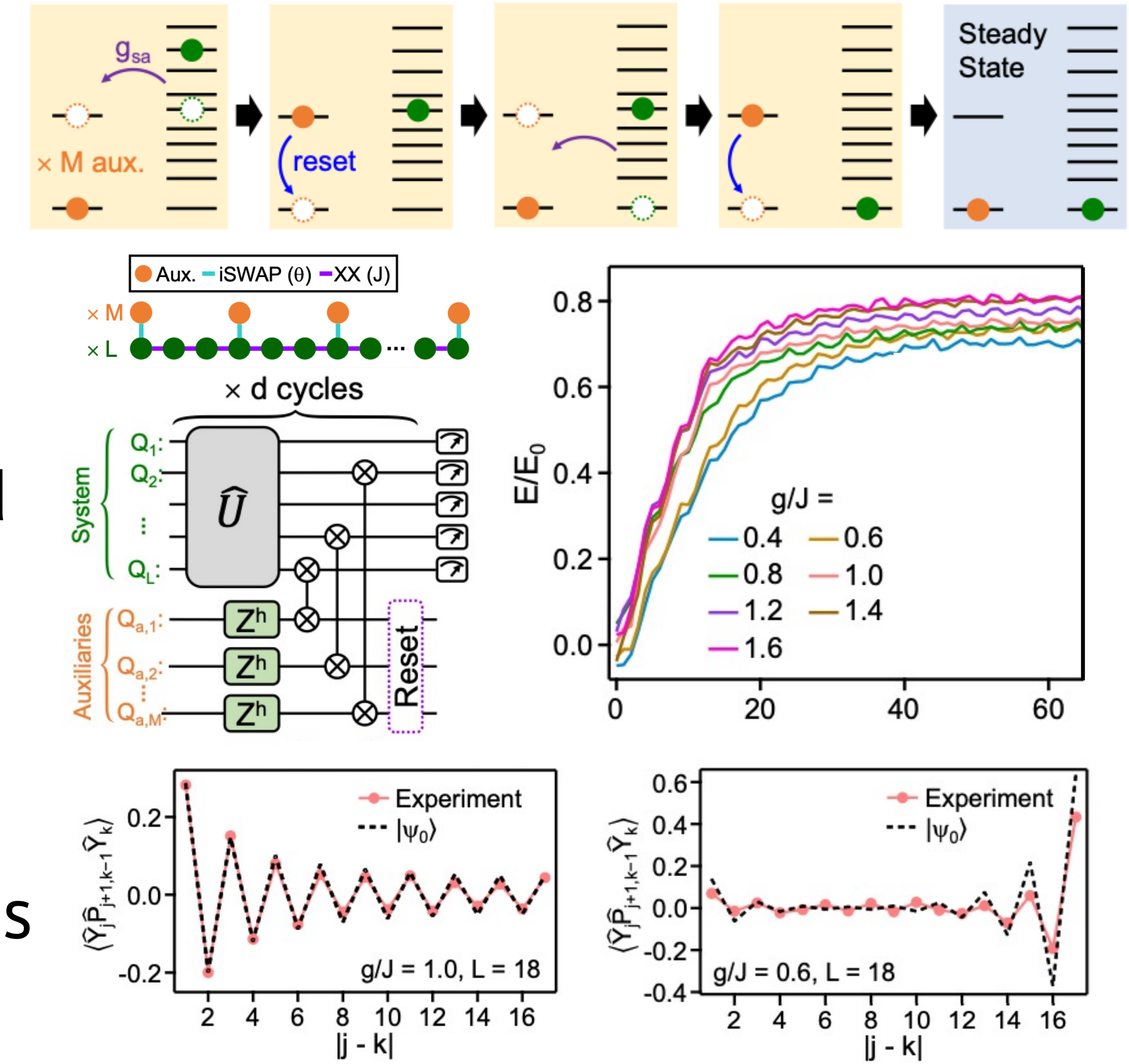
- Strong coupling (Γ~J)
  - Inversion-symmetry broken by dissipation
  - Large amplitude response
- Weak (Γ << J) and ultra-strong (Γ >> J)
  - Restoration of inversion symmetry
    - Bulk is relatively isolated from boundaries
  - Small amplitude response





Dissipation on NISQ Hardware

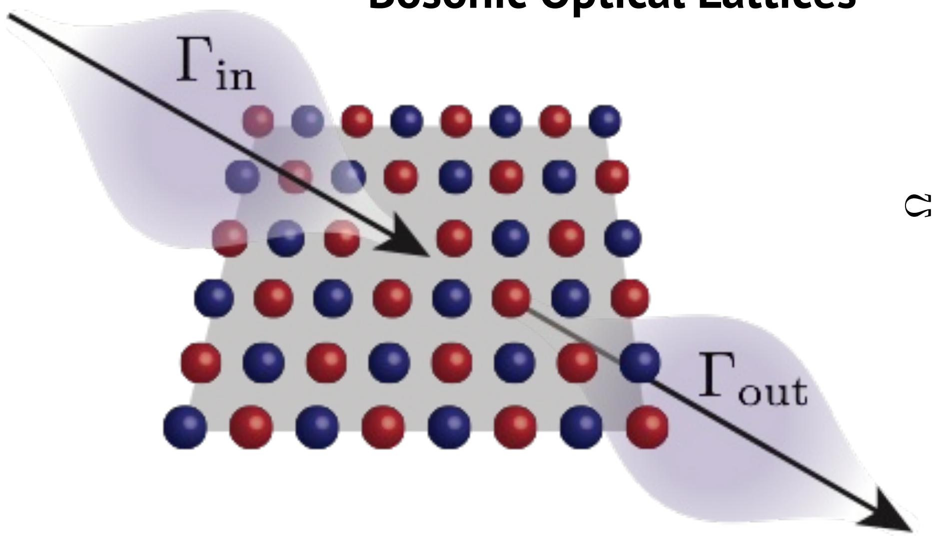
- Google QuantumAI experiment
  - Dissipative Ising model (2D)
  - Use ancillas with resets to simulate dissipative memoryless baths
  - Mi *et al*, 2304.13878
- Cool the system towards its ground state through stroboscopic evolution: unitary evolution followed by dissipative evolution
- Potential to realize other strongly dissipative systems on NISQ devices

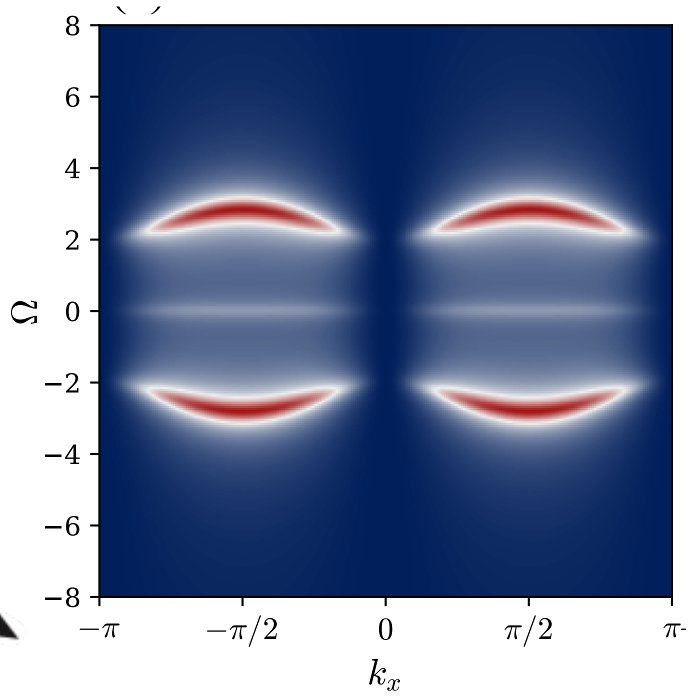


Our Recent Preprint: 2402.06593

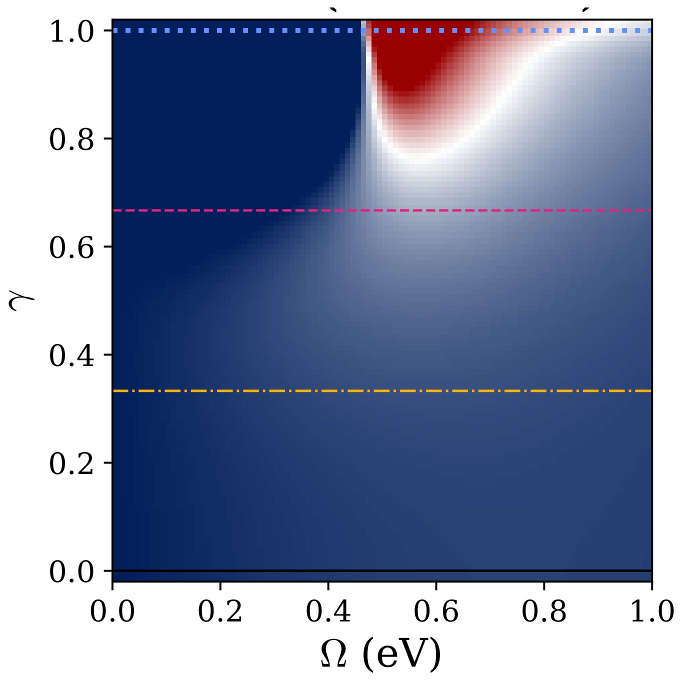
- Accessible introduction to Lindblad-Keldysh Green functions
- Expressions to directly calculate responses

Dynamic Response in Bosonic Optical Lattices



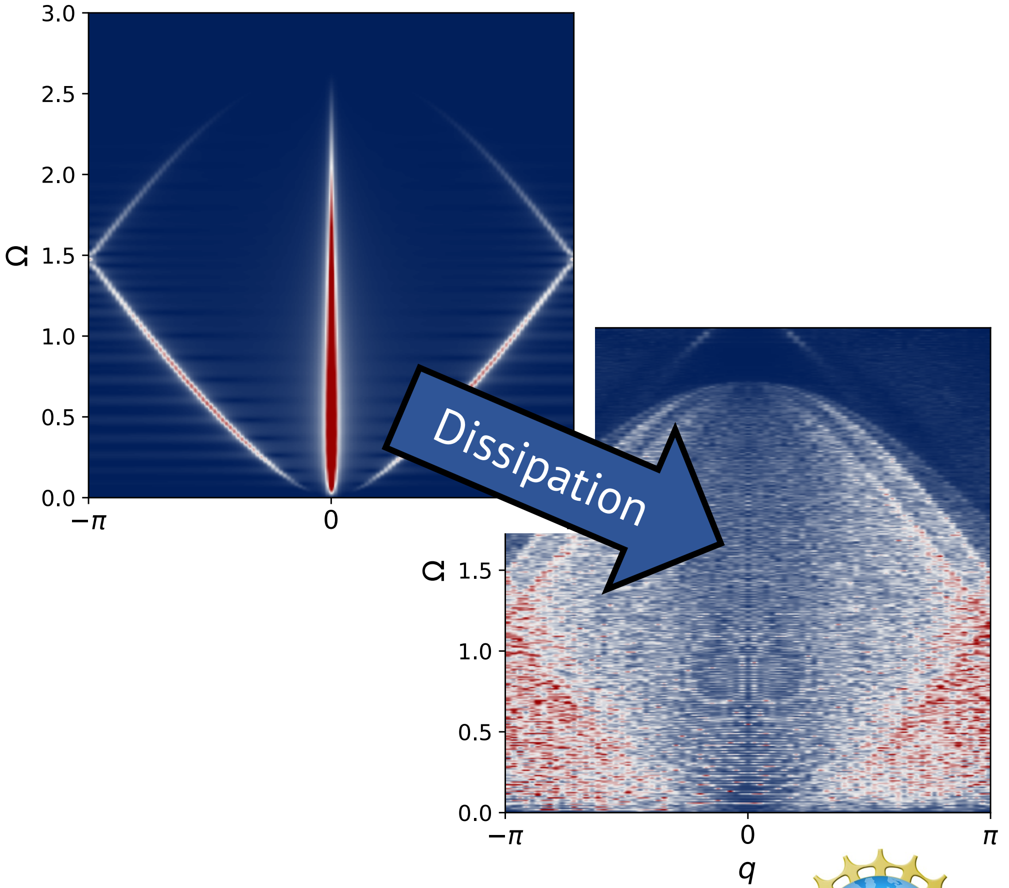


Nonlinear Shift Response in Bernal Bilayer Graphene



Outlook

- Dissipative engineering is a promising direction to control systems and realize out of equilibrium physics
- Finite frequency probes of excitations are readily accessible in our formalism
- Generalizations to interacting systems are the frontier
    - Prethermal plateaus and avoiding thermalization at long-times
    - RPA and collective dissipative phenomena
    - And more!



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