Linear Response Theory for Fermionic Lindbladian Systems

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G66.012 • Open Quantum Systems I

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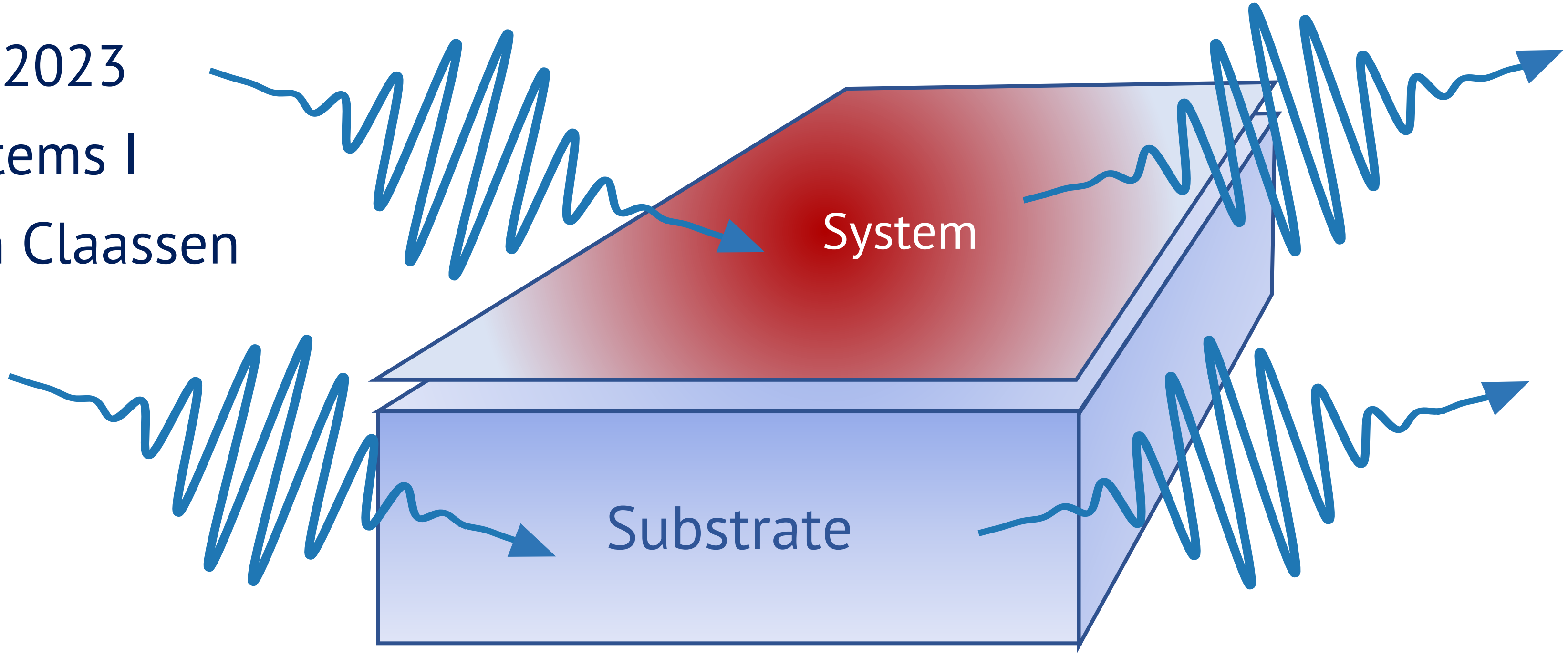
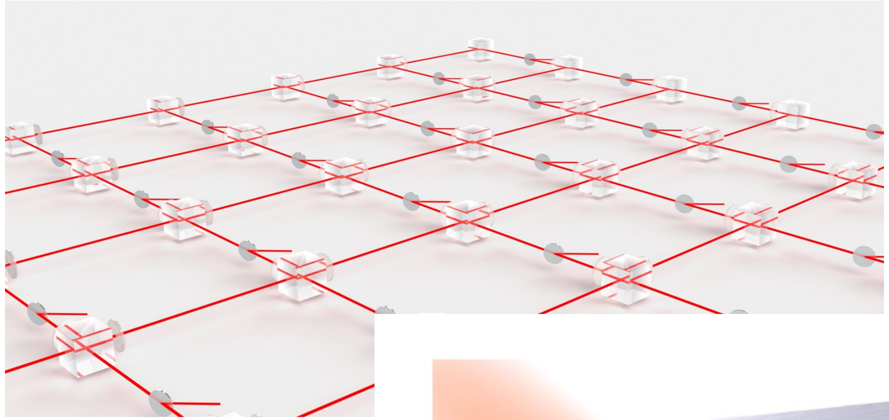
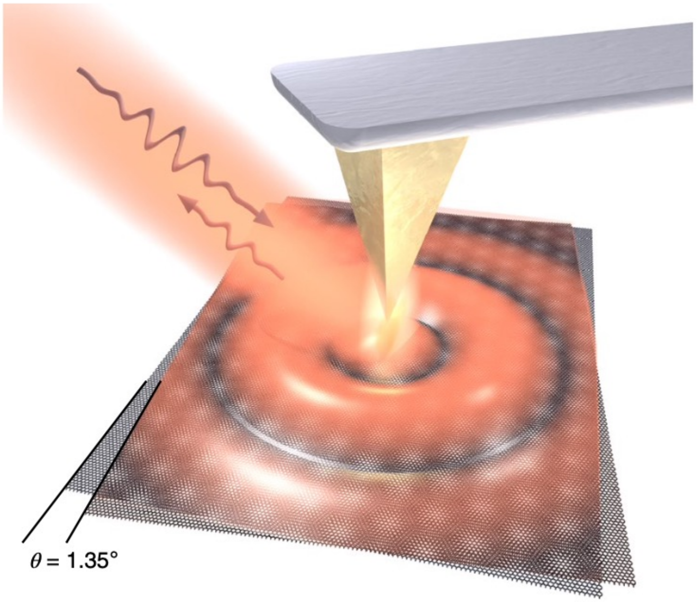


Open Fermionic Systems

- Generic features
  - Dissipation and decoherence
  - Exceptional points
  - Non-equilibrium steady states
- Open Bosonic systems
  - Gain and loss in photonic crystals
  - Non-reciprocal transport in BECs
- Open Fermionic systems
  - Qubits decohering due to an environment
  - Quantum materials hybridized by a substrate





Nat. Phys. **17**, 1161 (2021)

Lindblad Master Equation

- Time evolution in the limit of continuous measurement by a memoryless bath $i\dot{\rho} = \mathcal{L}[\rho]$

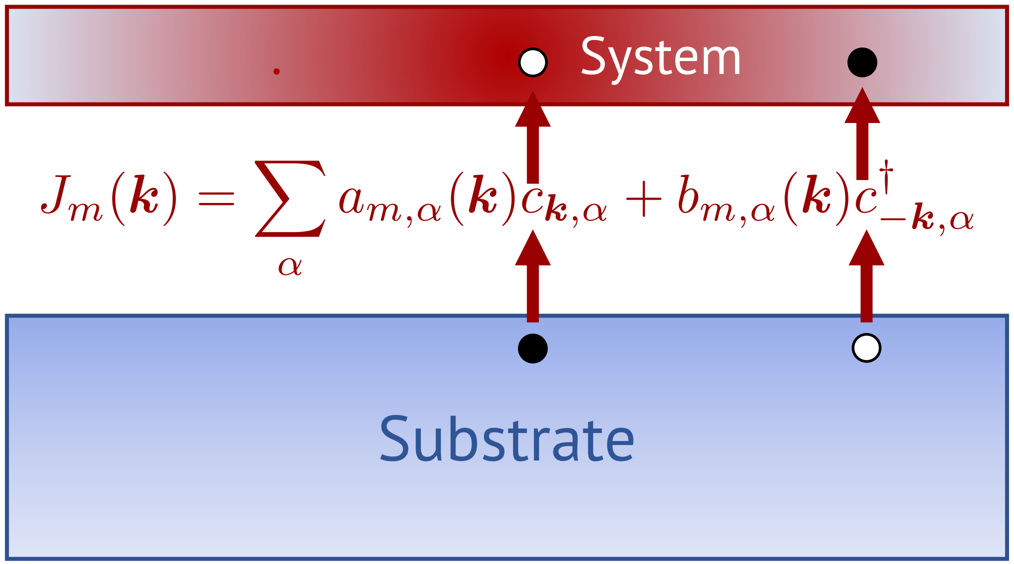
$$i\dot{\rho} = (\mathcal{L}^{\text{coh.}} + \mathcal{L}^{\text{n.h.}} + \mathcal{L}^{\text{jump}})[\rho]$$

- Expressed as

$$\mathcal{L}^{\text{coh.}}[\rho] = [H, \rho]$$

$$\mathcal{L}^{\text{n.h.}}[\rho] = -i\frac{\Gamma}{2}\sum_m \{J_m^\dagger J_m, \rho\}$$

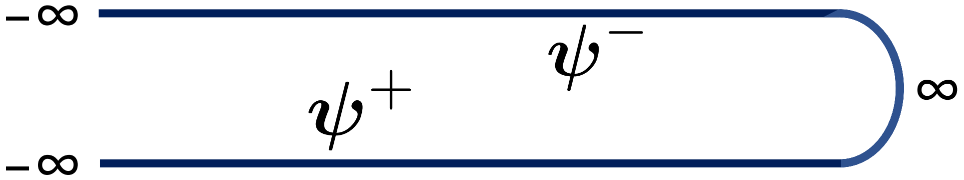
$$\mathcal{L}^{\text{jump}}[\rho] = i\frac{\Gamma}{2}\sum_m 2J_m \rho J_m^\dagger$$



Jump operators linear in fermions ensure the action is quadratic. J_m tunnels in a superposition of particles and holes in bands α.

Single Particle Lindbladian

- We want the normal modes of $\mathcal{L}$ for a system quadratic in fermions
- Express in terms of superfermions ψ^+, ψ^- on the Keldysh contour



- With $\mathcal{L} = \bar{\Psi}[L_{\text{coh}} - iL_{\text{dis}}]\Psi$ for $\Psi = (\psi^+, \bar{\psi}^-, \bar{\psi}^+, \psi^-)$

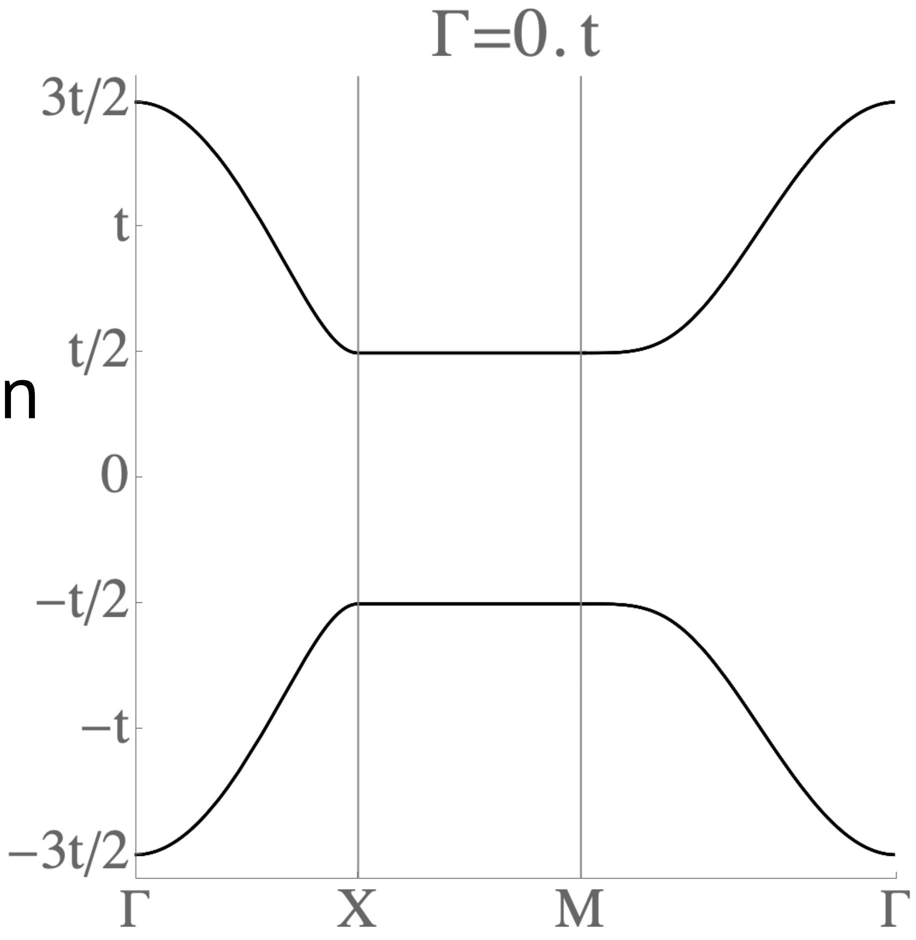
$[A_{\boldsymbol{k}}]_{\alpha,\beta} = a^*_{m,\alpha} a_{m,\beta}$, etc

$$L_{\text{coh}} - iL_{\text{dis}} = \begin{pmatrix} H & 0 & 0 & 0 \\ 0 & H & 0 & 0 \\ 0 & 0 & -H^\top & 0 \\ 0 & 0 & 0 & -H^\top \end{pmatrix} - i\frac{\Gamma}{2} \begin{pmatrix} A - B & -2B & C - C^\top & 2C^\top \\ -2A & B - A & -2C & C - C^\top \\ C^\dagger - C^* & -2C^* & B^\top - A^\top & 2A^\top \\ 2C^\dagger & C^\dagger - C^* & 2B^\top & A^\top - B^\top \end{pmatrix}$$

ST and MC PRB **106**, 161109 (2022): generalization of Prosen's "third quantization" *New J. Phys.* **10**, 043026 (2008)

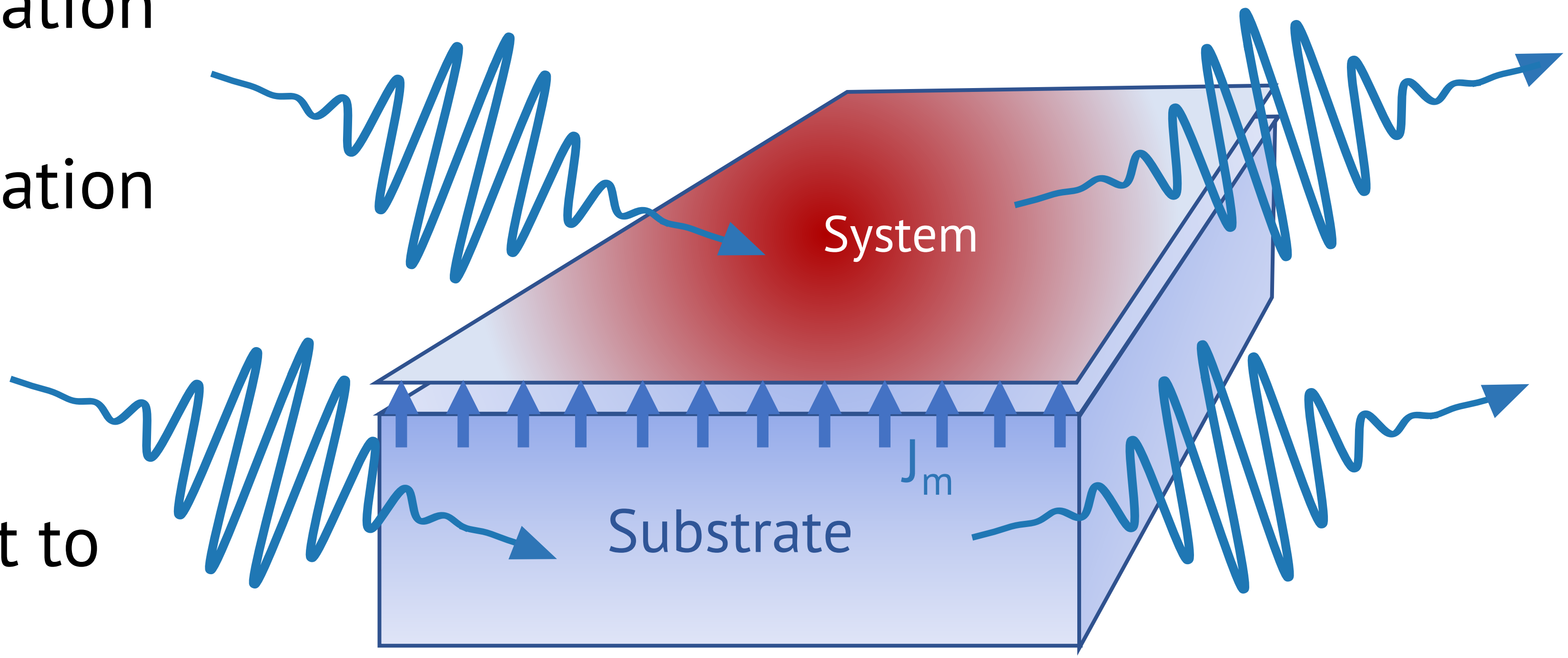
Dissipation Induced Flat Bands

- QWZ model of a 2D Chern insulator
  - Couple to a superconducting substrate
    $$J_n = c_{\boldsymbol{k},n} + c^\dagger_{-\boldsymbol{k},n}$$
  - Find long-lived isolated flat bands
- Our previous work with the SP Lindbladian
  - Long lived flat bands form in the dark space of the dissipation operator
  - Derived symmetry-based conditions for the existence of long-lived flat bands
  - PRB **106**, 161109 (2022)



What's the Response?

- System to perturbation
- System to perturbation and substrate
- System to perturbation and substrate to perturbation



- Response isn't just to H^{NH}, but also the substrate (integrated out to give jumps J_m)

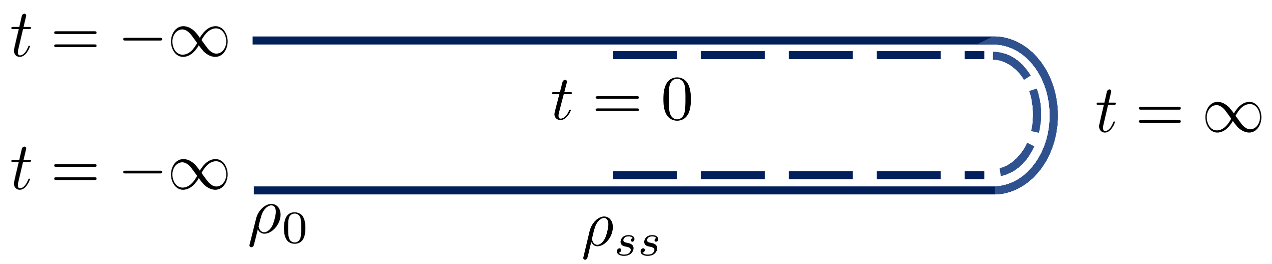
Linear Response Theory

- Kubo formula for equilibrium

$$\langle O(t)\rangle = \langle O\rangle_0 + i\int_0^t d\bar{t}\ \langle [H'(\bar{t}), O(t)]\rangle_0$$

- Kubo formula for the steady state of an open quantum system

$$\langle O(t)\rangle = \langle O\rangle_{ss} + i\int_0^t d\bar{t}\ \langle \mathcal{L}'[O(t)](\bar{t})\rangle_{ss}$$



  - The response of ρ_{ss} is generically very different from that of ρ_0
  - Time evolution changes from $\partial_t O = -i[H', O]$ to $\partial_t O = -i\mathcal{L}'[O]$

What is $\mathcal{L}'$?

- Before diagonalization, the Lindbladian master equation is

system operators

$$\dot{\rho}(t) = -i[H_{\mathrm{sys}}, \rho(t)] + \sum_{i,j} \Gamma_{ij}(s_j \rho(t) s_i^\dagger - s_i^\dagger s_j \rho(t)) + \Gamma_{ji}^*(s_i \rho(t) s_j^\dagger - \rho(t) s_j^\dagger s_i)$$

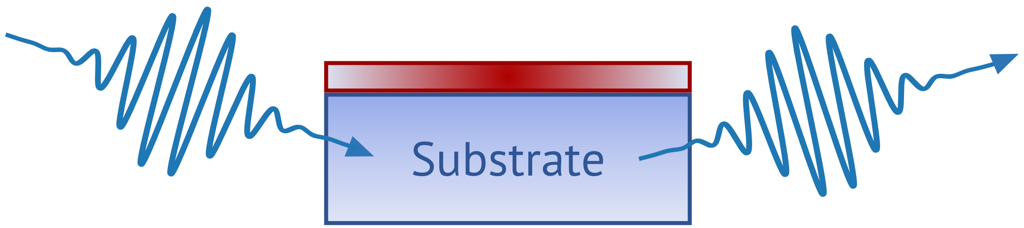
- A generic perturbation changes both ρ and Γ_{ij}, but for simplicity we only show the result of changing Γ_{ij}. We have $\Gamma_{ij} \mapsto \Gamma_{ij} + \Gamma'_{ij}$ with

$$\Gamma'_{ij}(\omega) = \int_0^\infty d\bar{t}\; e^{i\omega\bar{t}}\, \mathrm{Tr}_{\mathrm{sub}}(b^\dagger_{i,I^0}(t) i[\int_0^{t-\bar{t}} d\bar{\bar{t}}\; e^{-iH_{\mathrm{sub}}\bar{\bar{t}}} H'_{\mathrm{sub}}(\bar{\bar{t}}) e^{iH_{\mathrm{sub}}\bar{\bar{t}}}, b_j]_{I^0} \rho_{\mathrm{sub}}(0))$$

$$+ \int_0^\infty d\bar{t}\; e^{i\omega\bar{t}}\, \mathrm{Tr}_{\mathrm{sub}}(i[\int_0^{t} d\bar{\bar{t}}\; e^{-iH_{\mathrm{sub}}\bar{\bar{t}}} H'_{\mathrm{sub}}(\bar{\bar{t}}) e^{iH_{\mathrm{sub}}\bar{\bar{t}}}, b_i^\dagger]_{I^0} b_{j,I^0}(t-\bar{t}) \rho_{\mathrm{sub}}(0))$$

substrate/bath operators

- For light, $H'(t) = j_\mu A^\mu(t)$
- Effects of H' on H_{sys} are as usual

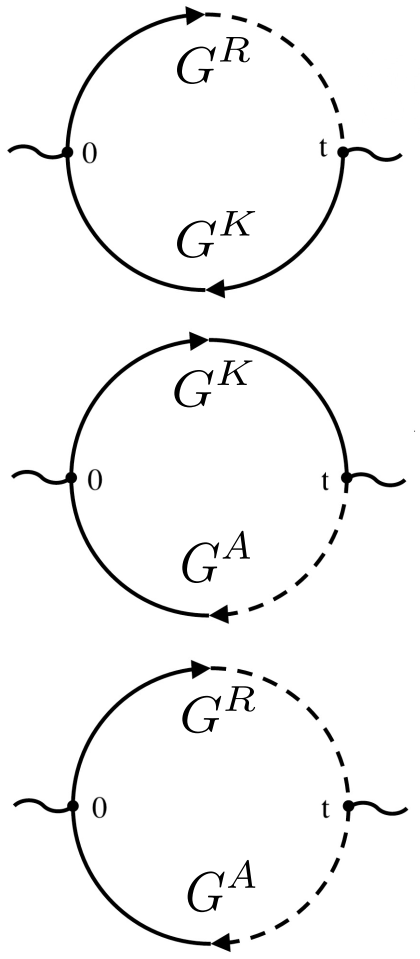


Lindblad-Keldysh Green's Functions

- Now we want the (anti)-commutators of Fermions
- These will become Green's functions
  - In equilibrium: G^R and G^A
  - Out of equilibrium: G^R, G^A and a correlation Green's function G^K
- Obtained from the SP Lindbladian via a (Larkin-Ovchinnikov) rotation and a matrix inversion
- For the particle-number conserving case we have

$$G^{R(A)} = [i\partial_t - H \pm i(\Gamma/2)(A+B)]^{-1}$$
$$G^K = -2G^R i(\Gamma/2)(A-B)G^A$$

- Jump terms lead to complex self energy!



Adapted from Thompson and Kamenev arXiv:2301.02953

Example: Optical Conductivity

- Paramagnetic response $\Pi^{\mu\nu}(t) = -i\langle[\mathsf{J}^\mu(t), \mathsf{J}^\nu(0)]\rangle\theta(t)$ with current operators assumed to be only in the system for simplicity

$$\mathsf{J}^\mu = \sum_{k\alpha\beta} j^\mu_{k\alpha\beta} c^\dagger_{k\alpha} c_{k\beta} \ , \qquad j^\mu_{k\alpha\beta} = \langle u^L_{k\alpha}|\partial_{k_\mu} H^{NH}|u^R_{k\beta}\rangle$$

- Leads to the correlation function

$$\Pi^{\mu\nu}(\Omega) = \pi\Bigg(\sum_n \mathrm{Res}[\mathrm{Tr}[j^\mu G^R(\omega_n) j^\nu G^K(\Omega-\omega_n)], \omega_n] + \sum_m \mathrm{Res}[\mathrm{Tr}[j^\mu G^K(\Omega-\omega_m) j^\nu G^A(\omega_m)], \omega_m]\Bigg)$$

- Note: unlike in Hermitian systems, Re(σ^{dia}) doesn't automatically vanish

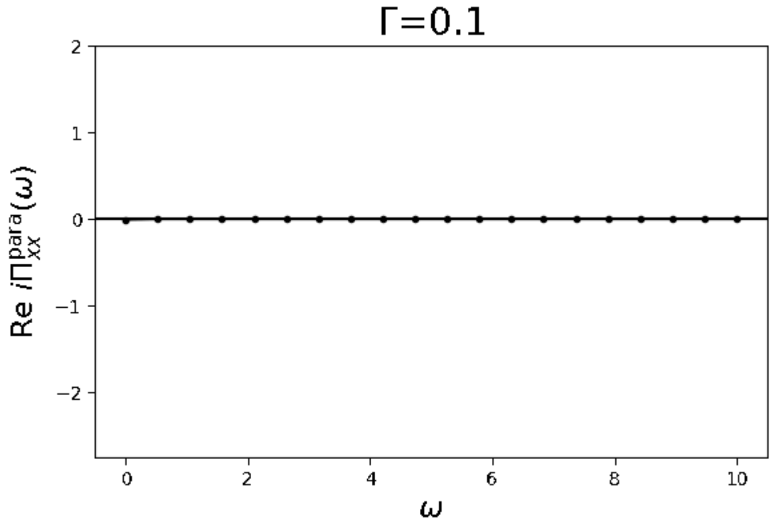
Example: 1D EP Optical Conductivity

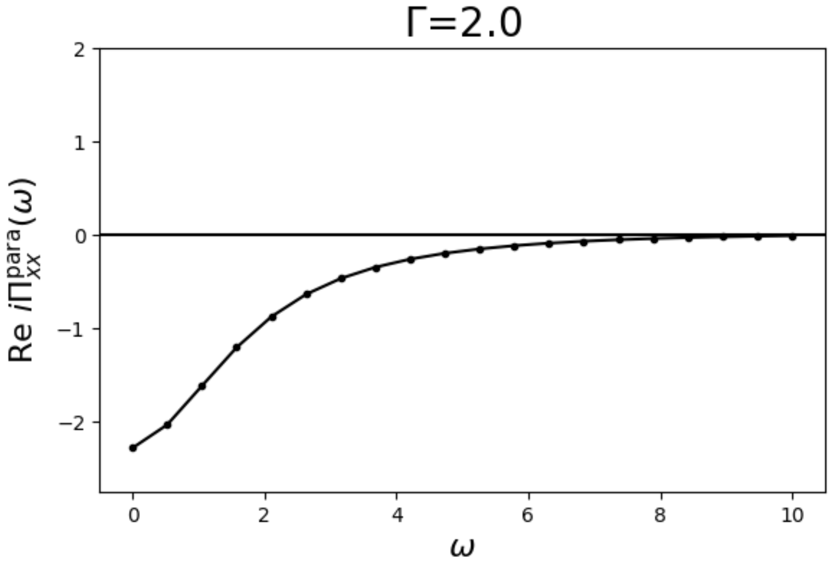
- Model with an exceptional point

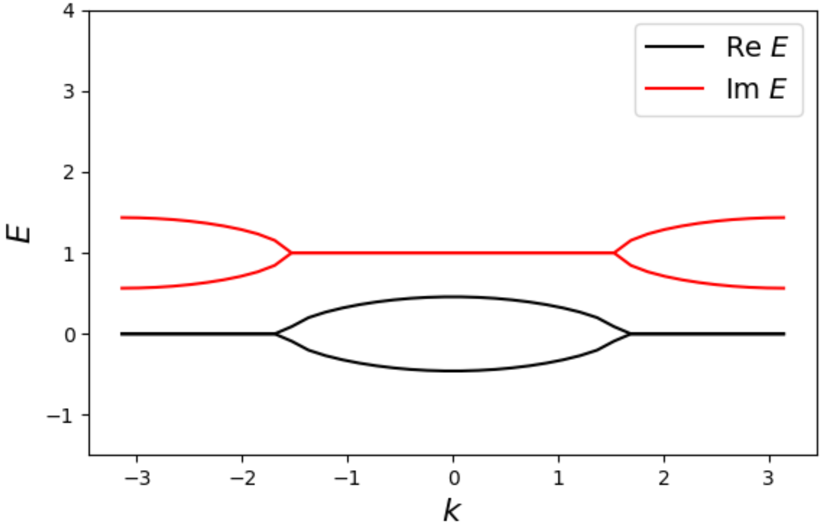
$$H = t(1 + \delta\cos(k_x)\sigma_3) + i(\Gamma/2)(\sigma_0 + \sigma_1),$$

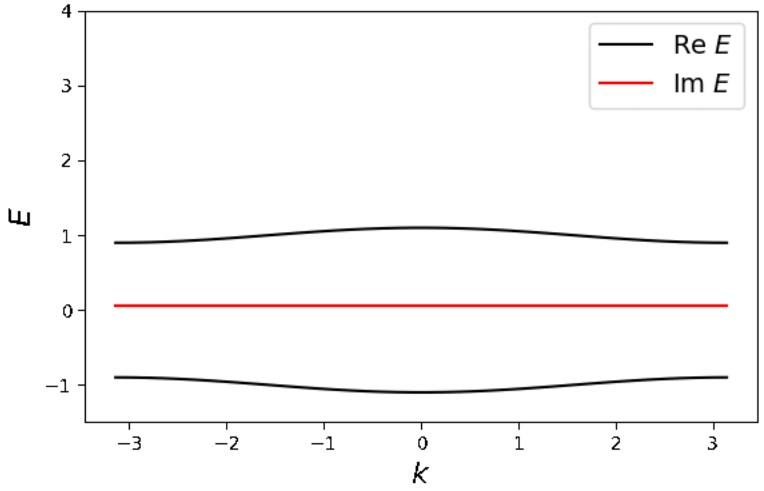
- Lindbladianize

$$H = t(1 + \delta\cos(k_x)), \qquad J = c_1 + c_2$$



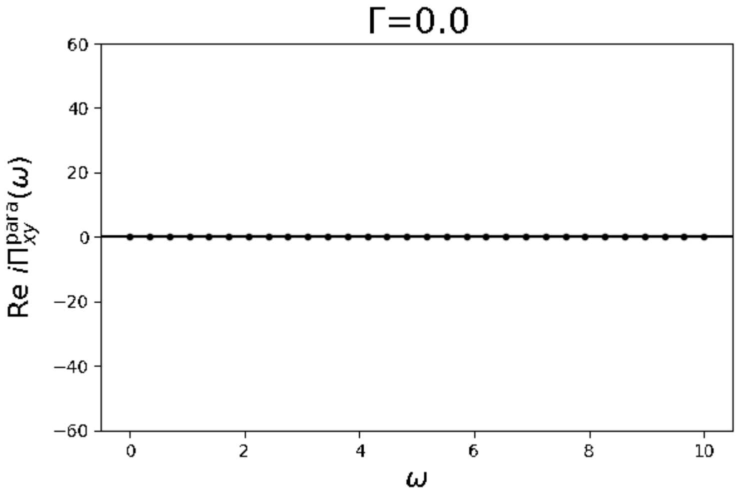


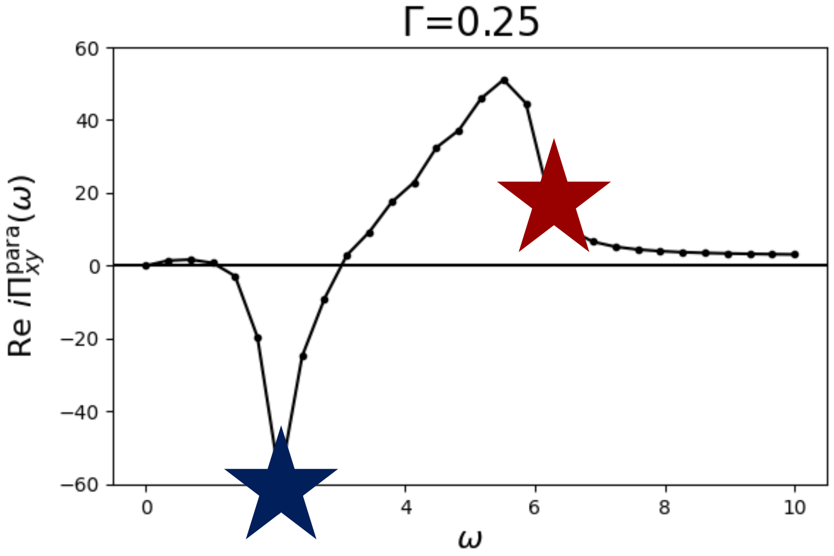


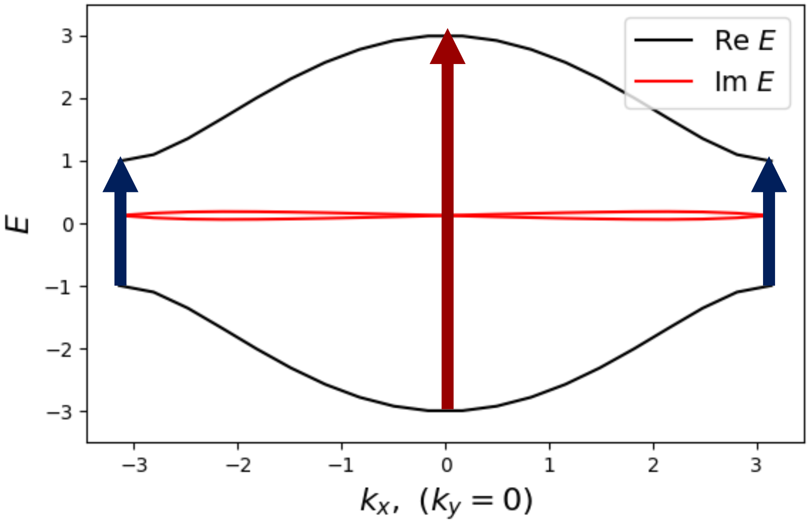


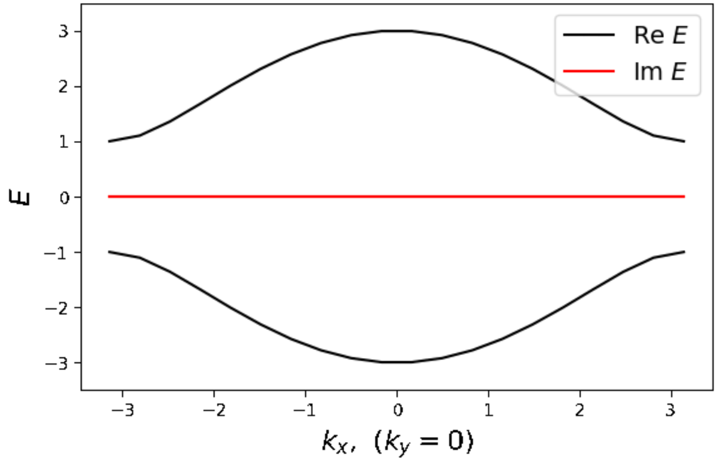
Example: QWZ Optical Conductivity

- Model with non-zero Chern number for the closed system
- Lindbladianize with $J = c_1 + c_2$
- $\sigma_{xy}(0)$ vanishes for open system!
  - Steady state ≠ equilibrium



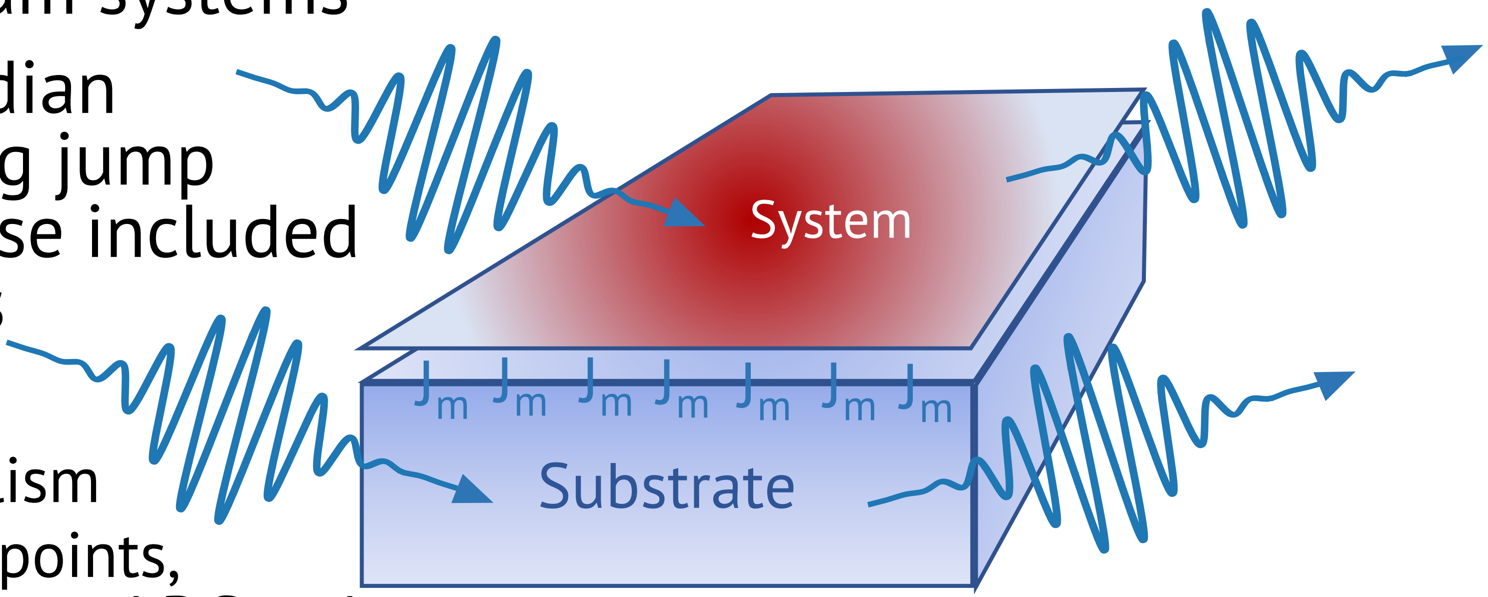






Outlook

- We want to know the response properties of open quantum systems
- The response for Lindbladian systems requires including jump terms that go beyond those included in non-Hermitian systems
- Forthcoming paper
  - Fermionic Lindblad formalism
  - Discussion of exceptional points, parallel transport, topology, and DC and finite frequency conductivity



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