Diffraction grating design for producing stable Laguerre-Gaussian modes

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The discovery of "orbital angular momentum" (OAM) in beams of light has instigated an explosion of new ideas and opportunities across the fields of microscopy, communication, and quantum information. Previous research has shown that light beams, when circularly polarized, can carry spin angular momentum (SAM), and that SAM can take on discrete values of $-\hbar, 0,$ or $\hbar$ per photon. OAM is revolutionary because, while it is still restricted to integer multiples of $\hbar$ per photon, there is no upper or lower bound.

This freedom can be leveraged to improve material imaging capabilities [1] and to create more effective "optical tweezers" for manipulating microscopic particles [2]. OAM can also be used to increase optical data transmission rates by orders of magnitude [3]. Perhaps most notably, OAM is a prime candidate for the creation of a "qudit" for quantum computing [4]. A qudit is a unit of quantum information represented by a multi-level system that can take the place of arrays of qubits. I have developed a novel technique for generating light beams that carry OAM much more stably than previous methods.

Light beams carry OAM when their phase distributions depend on the azimuthal angle around the beam axis. The amount of OAM carried is determined by how many times the phase of the light cycles fully from $-\pi$ to π with one full rotation (Fig. 1). In Fig. 1A, the phase of the light (represented by color) cycles once from $-\pi$ to π (clockwise, by convention) around the beam axis, and thus the beam has a "topological charge" of $m = +1$, and carries $\hbar$ OAM per photon. Similarly, Fig. 1B shows a beam with $m = +2$ topological charge, carrying $2\hbar$ OAM per photon.

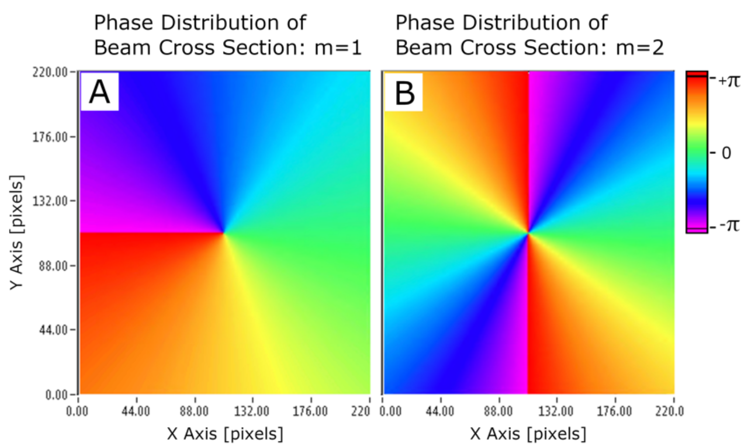

FIG. 1. Phase distributions of two beams of light, with topological charges $m = +1$ (A) and $m = +2$ (B), taken on a cross-section perpendicular to the propagation axis. Phase is denoted by color, measured from $-\pi$ to π.

The points in the centers of these cross-sections, where the phase of the light is ambiguous, are known as "phase singularities." As beams carrying OAM propagate, their phase singularities can move, split, and combine, but the total topological charge is conserved. We refer to "singularity splitting" as the process by which a single phase singularity of order $m > 1$ will split into many smaller singularities of orders ± 1 that add up to a total m. Singularity splitting disrupts the practical applicability of OAM, as it effectively limits the usable modes to $m = -1$, 0, or $+1$. The goal of my research is to use optical models to identify novel ways of improving standard methods of creating OAM beams in order to better produce their ideal analytic forms, which do not undergo singularity splitting.

Beams of light in the paraxial regime (propagation approximately along beam axis) can be expressed as linear combinations of Laguerre-Gaussian (LG) modes, whose complex amplitudes in cylindrical coordinates take the form:

$$u_{m,p}(r,\phi,z) = \frac{Cw_0}{w(z)}\left(\frac{r\sqrt{2}}{w(z)}\right)^{|m|} L_p^{|m|}\left(\frac{2r^2}{w^2(z)}\right) \times \exp\left(\frac{-r^2}{w^2(z)} - im\phi + ikr^2\psi_S(z) + i\psi_G(z)\right)$$

Where $w(z)$ is the beam waist (radial distance at which amplitude decreases by a factor of e), m is the topological charge, $p \geq 0$ is the "radial index," and L_p^m are the generalized Laguerre polynomials. The phases ψ_S and ψ_G are first-order corrections for non-paraxial propagation. Here, we consider the cases for which $p = 0$.

To create such LG modes, we direct a typical Gaussian beam ($m = 0$) at a "pitchfork" spatial light modulator (SLM), which reflects the incident beam and applies a phase retardance equal to

$$R(x,y) = \frac{\pi}{2}\left[1 + \cos\left(\frac{x}{d} - m\tan^{-1}\left(\frac{y}{x}\right)\right)\right]$$

This SLM creates a diffraction pattern where the first order modes have OAM $+m$ and $-m$, and higher order modes have integer multiples of $\pm m$ [5].

To model this situation, I designed and wrote an optical simulation in C++ of a Gaussian beam incident on such a pitchfork grating. The simulation uses the vectorial equivalent of the Huygens-Fresnel principle, treating beams of light as sums of infinitely many spherical point sources. The complex amplitudes of all 6 components of the $\vec{E}$ and $\vec{B}$ fields are computed on the "primary screen," whose center is positioned to capture the first order $+m$ OAM mode diffracted off of the pitchfork SLM. The intensity and phase of the light are then plotted on this screen.

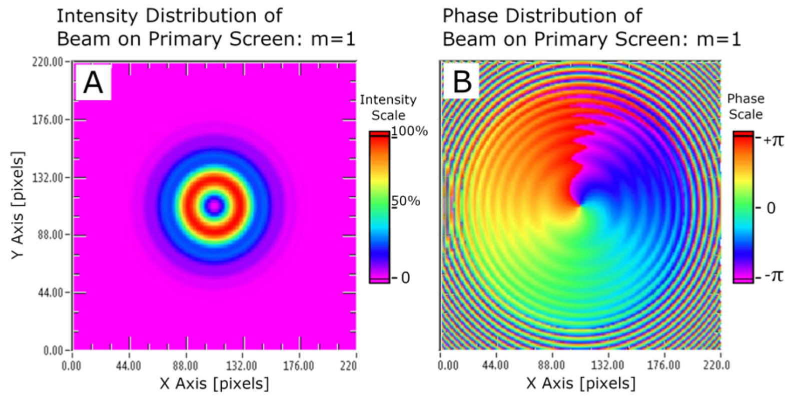

FIG. 2. Intensity (A) and phase (B) distributions of $m = +1$ OAM mode on primary screen.

The intensity and phase of the $m = +1$ mode generated by this simulation on the primary screen are shown in Fig. 2. The phases of the modes created with the pitchfork SLM do not have the clean distributions expected (shown in Fig. 1) given the LG mode equation. The unexpected deviations can be separated into two categories: the "wobble" and the "spiral." The wobble is the azimuthal oscillation of the lines of constant phase, with frequency and amplitude increasing with radial distance. The spiral is the drift of the average value of these oscillations in a gentle constant clockwise curve.

These deviations from the ideal LG mode signify that the actual beam is a superposition of many different lower order LG modes with different p values, which will propagate with different parameters, and thus undergo singularity splitting.

To address these deviations, I studied the equation for an analytic LG mode, and looked for sets of parameters that might create a spiral or wobble. In doing so, I determined that the spiraling is due to the ψ_S phase correction. To first order from the paraxial approximation, we can assume that the beam waist changes hyperbolically with propagation distance.

The changing beam waist causes the wavefronts to be slightly spherically curved, resulting in spiraling of the lines of constant phase. I remedied this spiraling by adding an extra term to the retardance of the SLM, calculated to directly cancel the ψ_S phase for a given expected propagation distance.

While exploring the wobbling, I observed that the SLM I had been using in the model had no way of implementing the radial dependence of the LG mode's amplitude. I thus hypothesized that adding a transmittivity mask to the SLM would yield a correct LG mode and eliminate the wobble. I implemented this transmittivity mask and executed a set of experiments to verify the correction. The resulting beams have no wobble and do not split upon propagation, as shown in Figs. 3B and 3D.

The apparent reduction of wobble in the uncorrected beam (Figs. 3A and 3C) is due to the stronger spiraling drowning out small oscillations, and does not prevent singularity splitting. The residual spiral in the corrected beam is due to higher order adjustments to the paraxial approximation, and can be calculated and removed if needed, although the first order correction is sufficient for most practical applications.

Applying the above retardance and transmittivity masks to a pitchfork SLM significantly improves OAM mode production to avoid singularity splitting. This technique generates OAM modes with a much larger range of stable m values. Enlarging this stable range allows for a multitude of practical innovations, including the creation of stronger optical tweezers, the increase of optical communication efficiency by orders of magnitude, and potentially the development of quantum computers with enough processing power to break modern encryption schemes. My next step will be to create an SLM with my suggested retardance and transmittivity masks in the lab to verify my optical simulation.

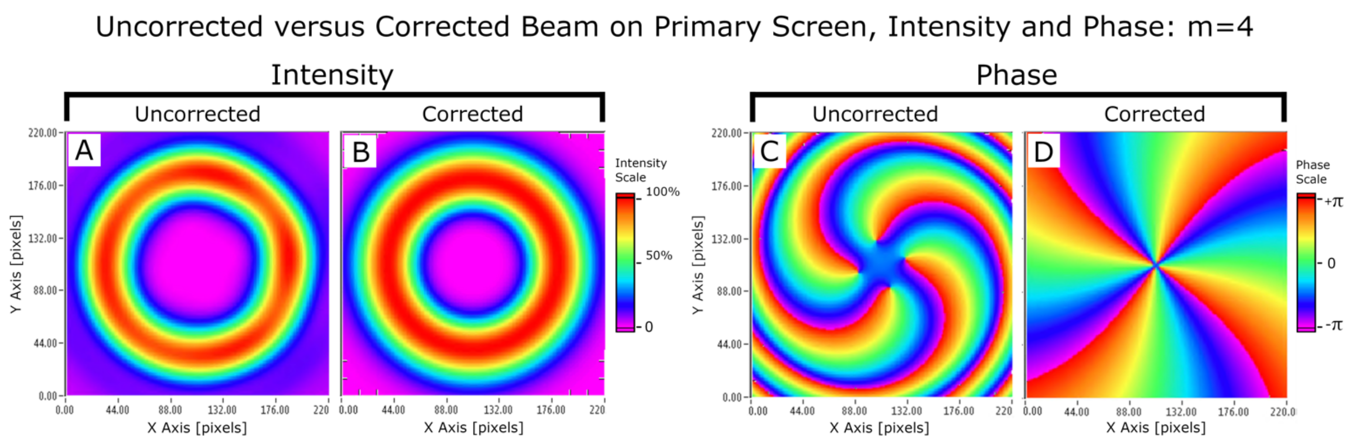

FIG. 3. Intensity (A and B) and phase (C and D) distribution comparison for $m = +4$ OAM modes created with the original pitchfork SLM (A and C) and the SLM with the spiral and wobble corrections (B and D).

References:

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