Secondary Coherent Oscillations in Quantum Dot Systems

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Evolution of Quantum States

Density Matrices

- Solve a differential equation

A density matrix, ρ has $\mathrm{tr}(\rho) = 1$, in particular, for states $|\psi_i\rangle$, and positive real numbers p_i:

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

The von Neumann equation gives time evolution:

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho]$$

There are other useful properties and applications of density matrices, but they are not relevant here.

Unitary Evolution

- Evaluate an integral

When particle number $\langle\psi|\psi\rangle$ is conserved, quantum states evolve with a unitary time-evolution operator:

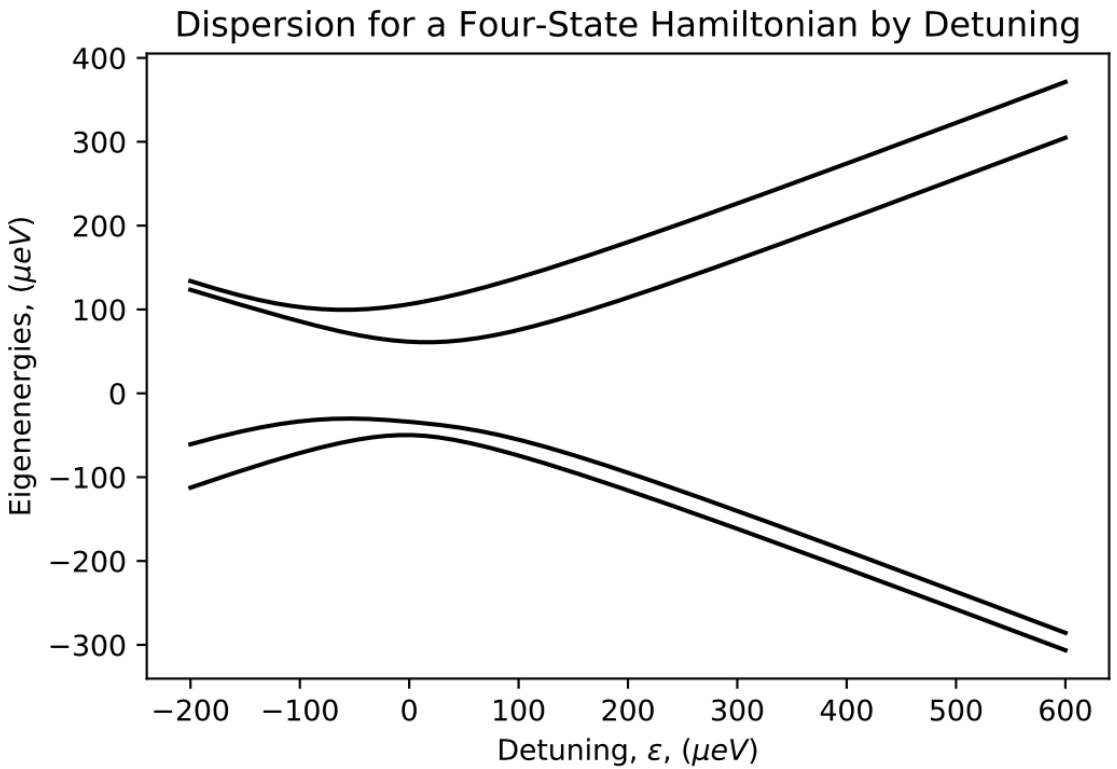
$$|\psi(t)\rangle = \mathscr{U}(t, 0)\,|\psi(0)\rangle$$

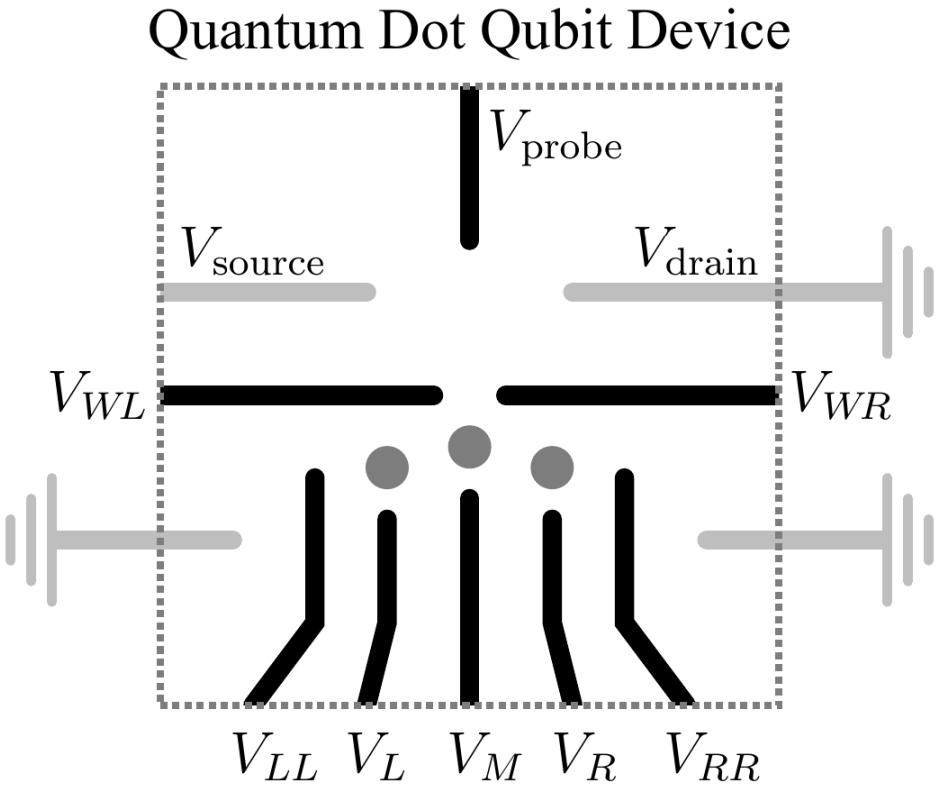
So long as Hamiltonians of different times commute the time-evolution unitary may be found using the formula:

$$\mathscr{U}(t, 0) = \exp\left[-\frac{i}{\hbar}\int_0^t d\bar{t}\; H(\bar{t})\right]$$

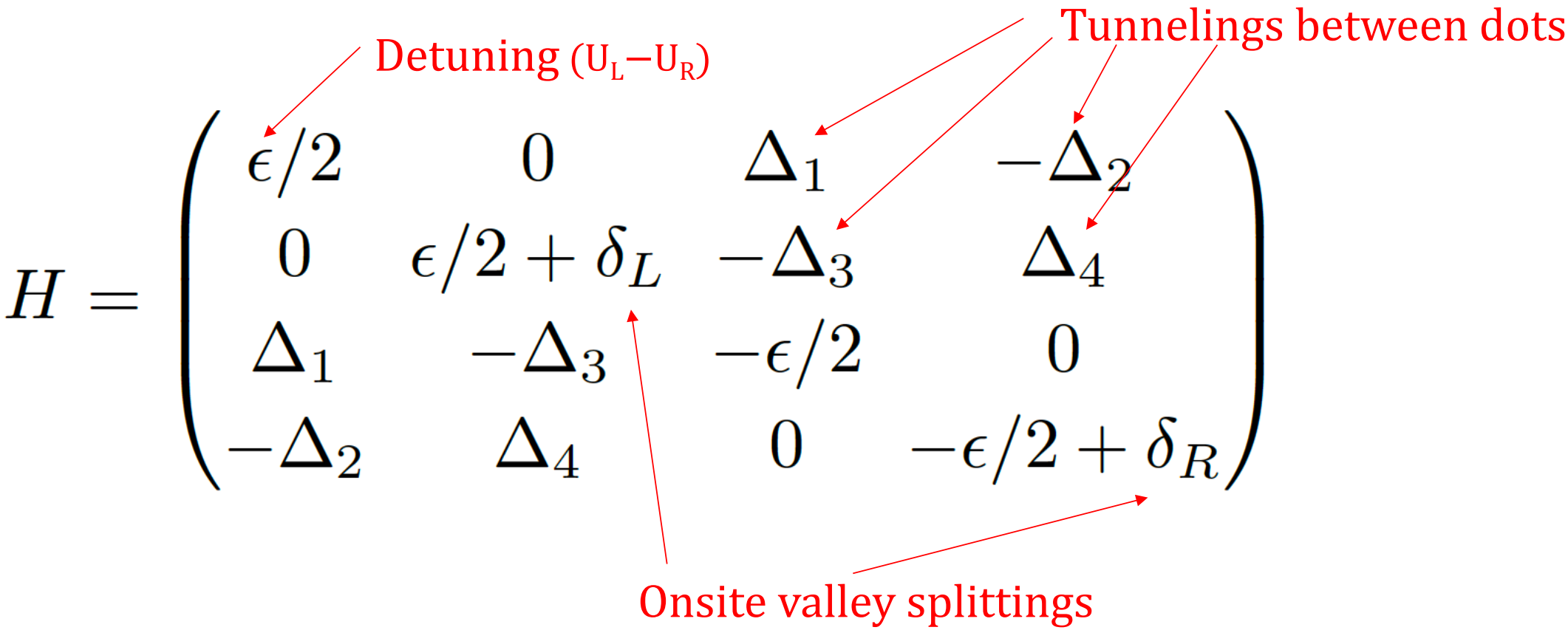
Charge-Based Quantum Dot Qubits

- Discarding all but the charge-tunneling degrees of freedom yields a 4x4 Hamiltonian for time-evolution





The Charge-Tunneling Hamiltonian

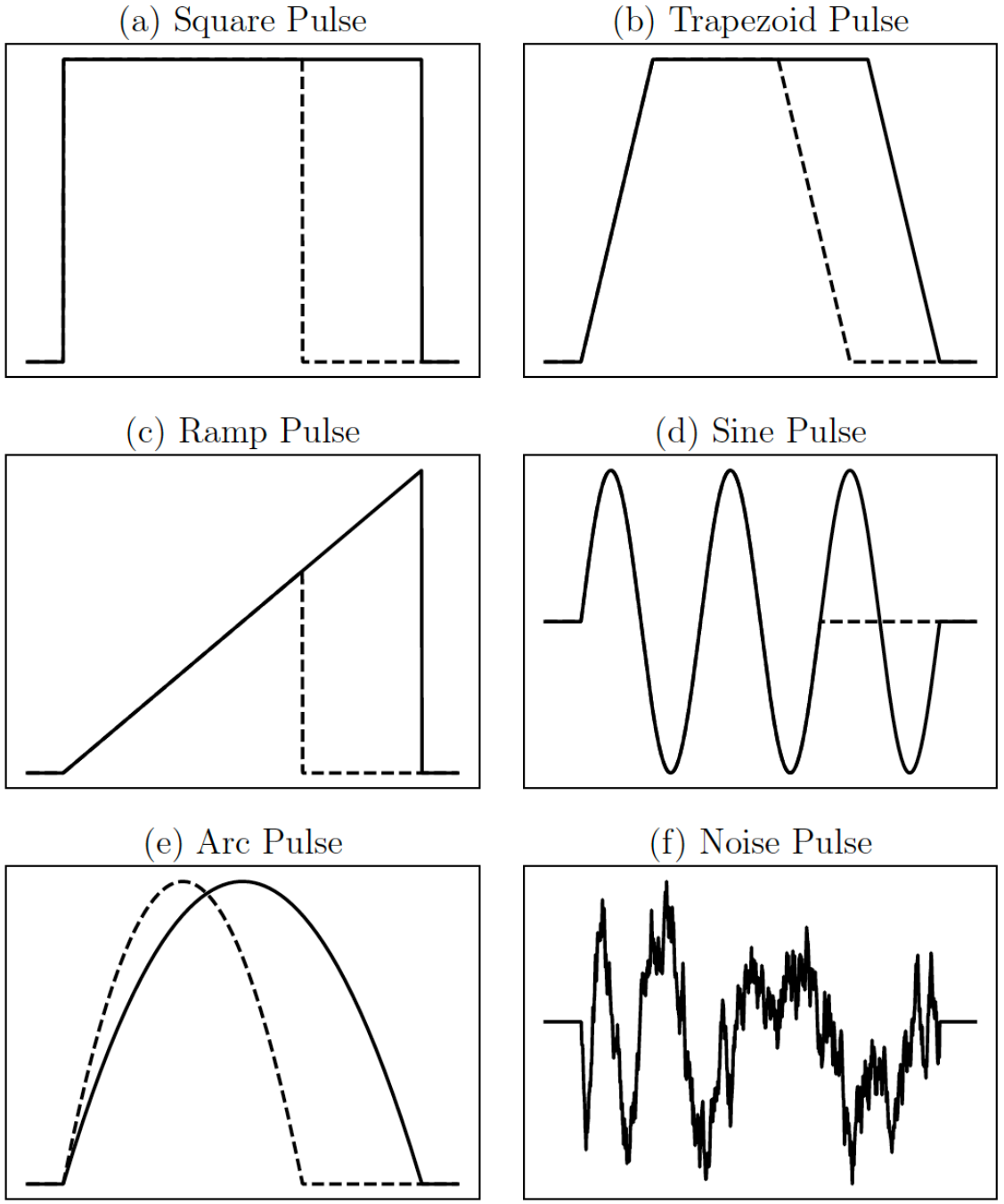


Driving with Pulses

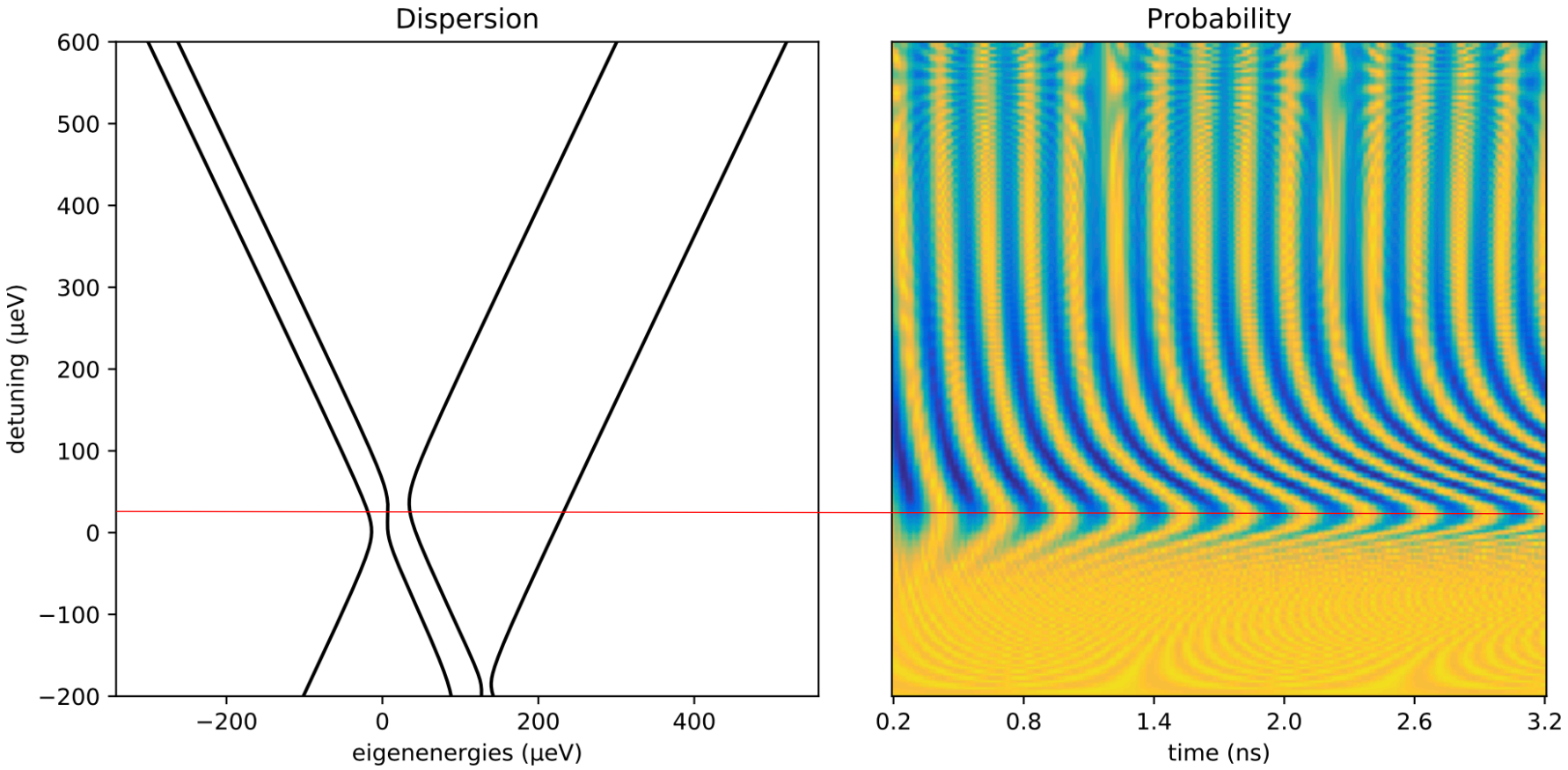
- Systems are driven by electrical pulses

$$U_{site} = qE_{site}$$

- Optimizations are possible in the unitary regime assuming forms of the driving pulse



Coherent Oscillations



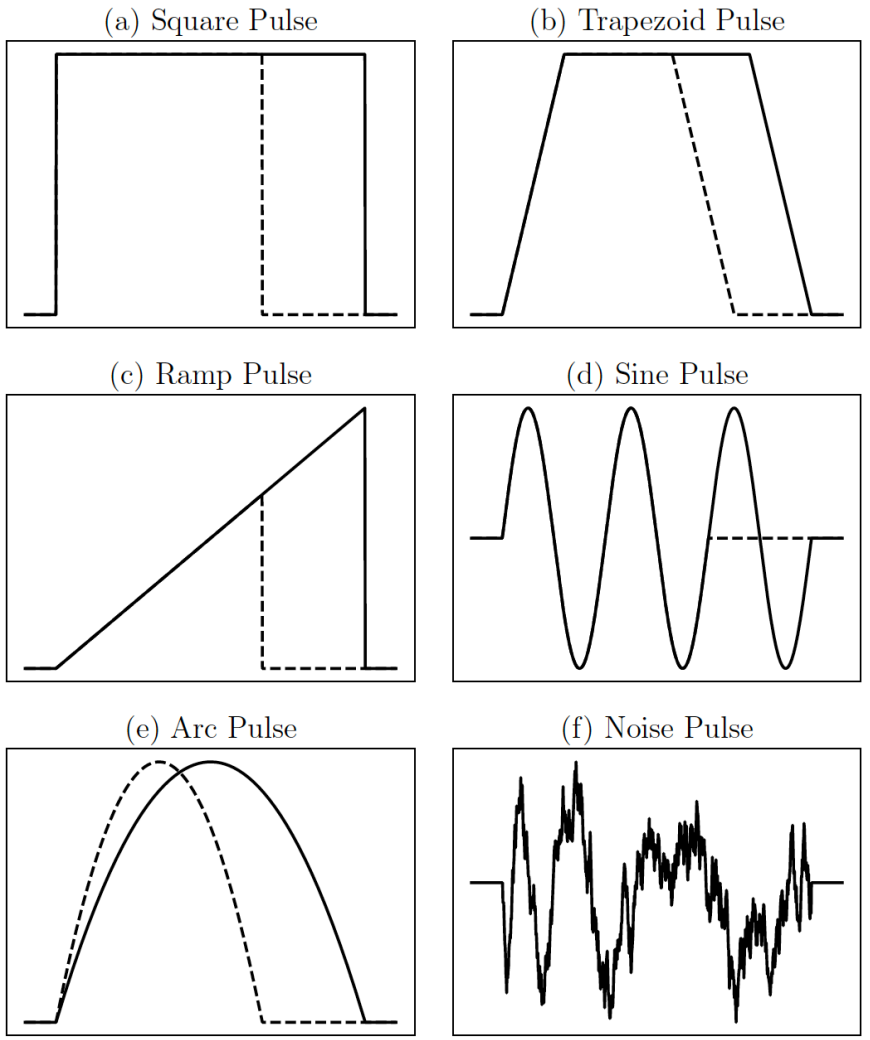
Optimizing Unitary Methods

Now, we may approximate the integral by expressing the unitary evolution as product of h steps of constant H spaced at $\tau = t/h$:

$$\mathscr{U}(t,0) = \mathscr{U}(t,t-\tau)\ldots\mathscr{U}(2\tau,\tau)\,\mathscr{U}(\tau,0) \tag{5}$$

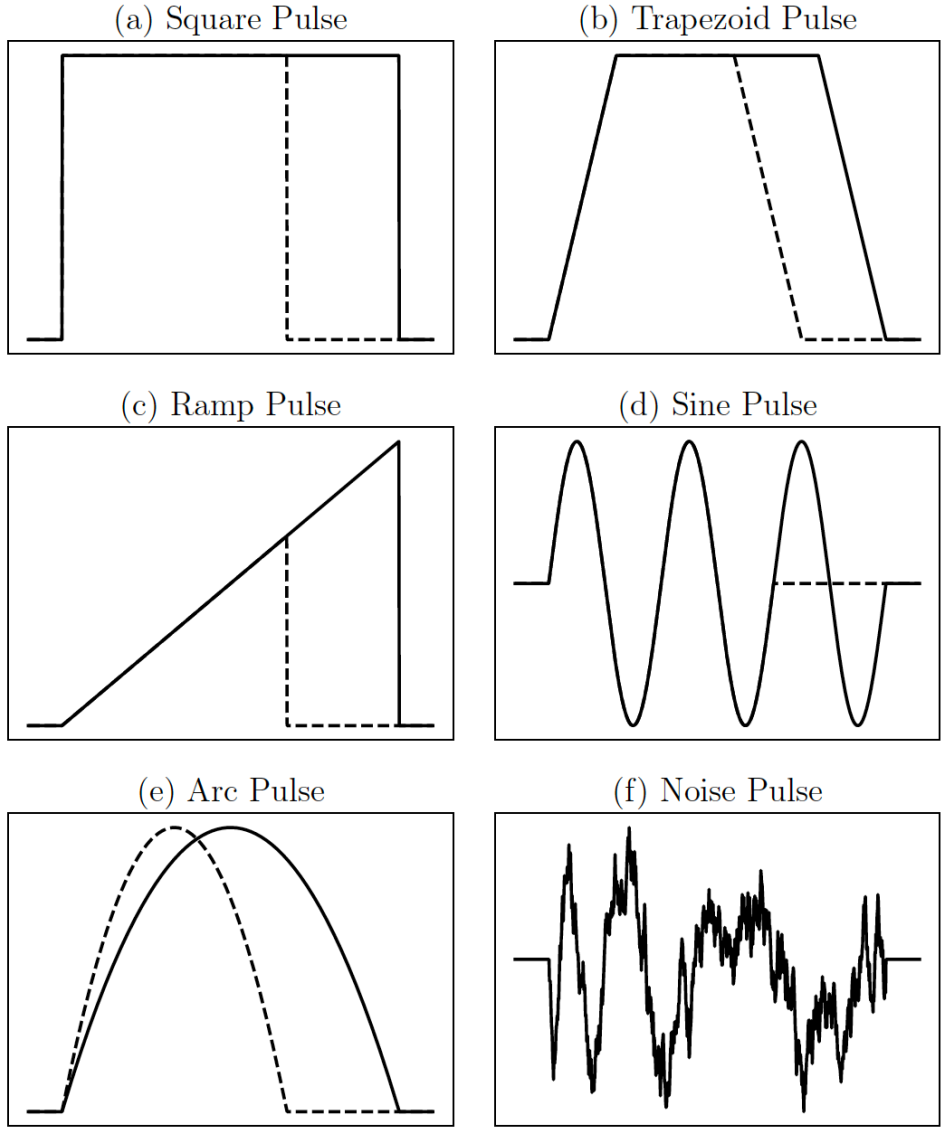
$$\approx \prod_{n=0}^{h} \exp\left[-\frac{i}{\hbar}\tau\, H(n\tau)\right] \tag{6}$$

Carefully selecting the time intervals τ, and how the product is evaluated and stored leads to substantial improvements in computational time.

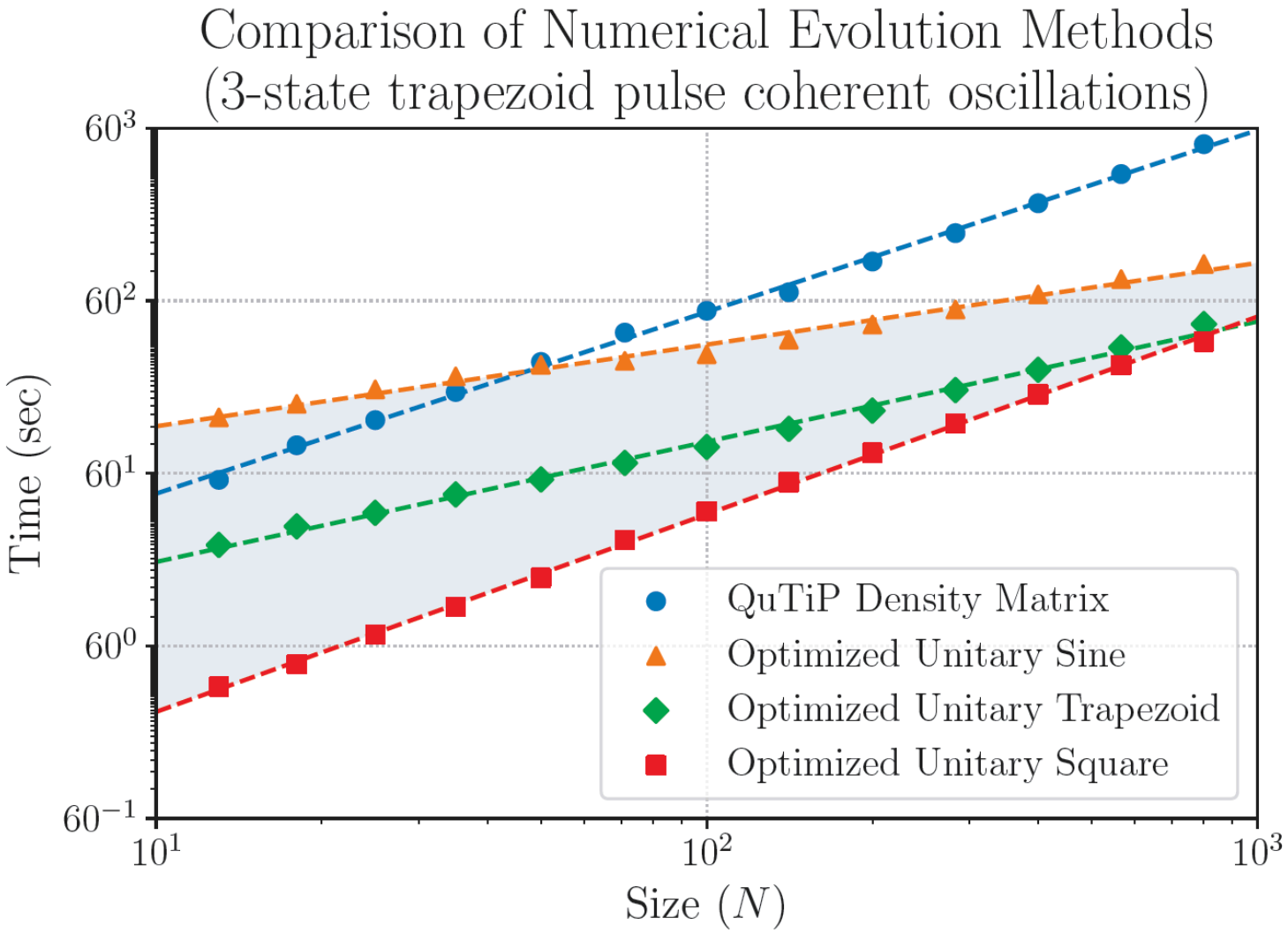


Optimizing Unitary Methods

- Constant Hamiltonian
  - Evaluate in one step
- Repeated features
  - Store in memory
- Extension of previous computations
  - Iteratively multiply



Speeds of Computation



Results of Optimization

- The same results may be found by unitary methods as density matrix methods
- Much faster (120 times)
  - with speed: parameter space

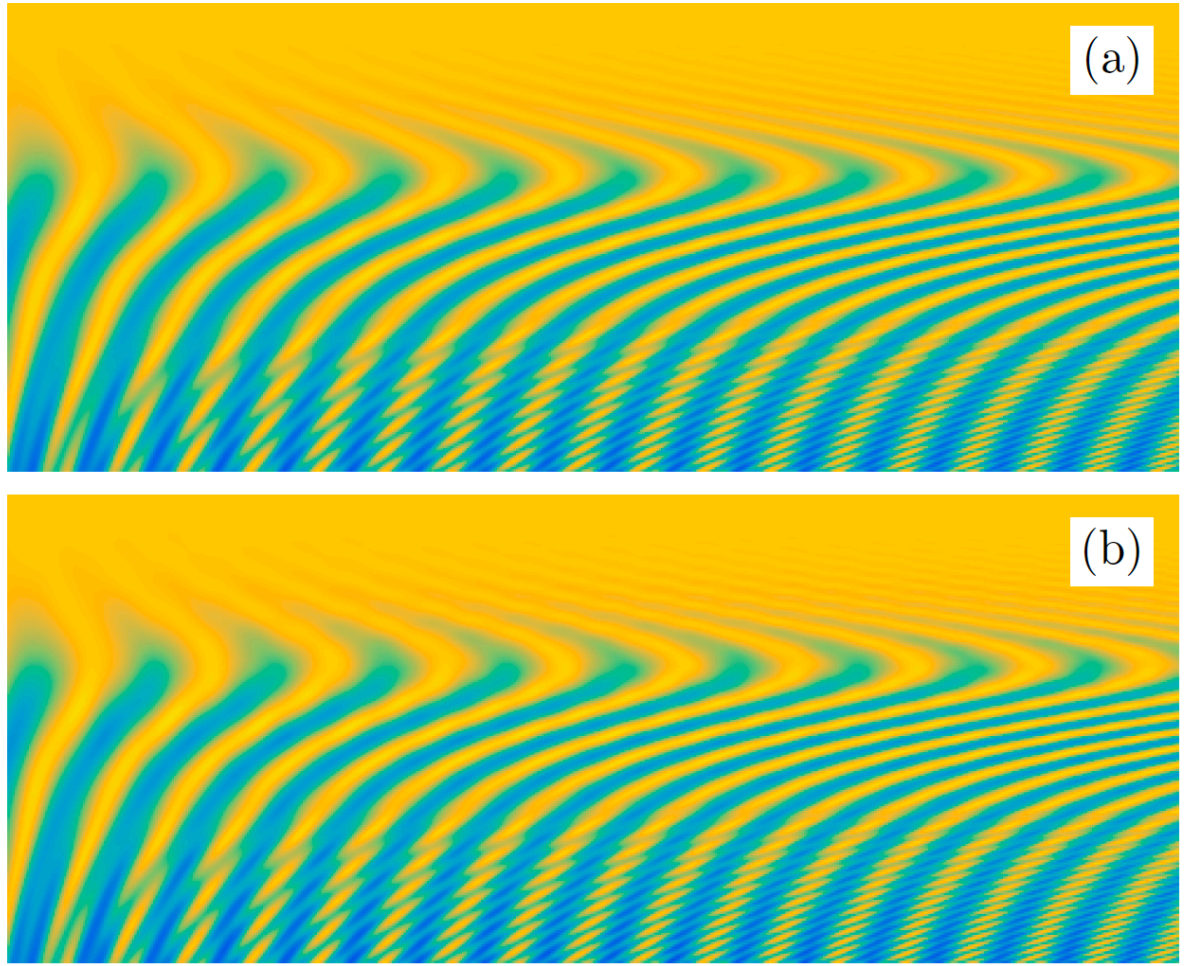
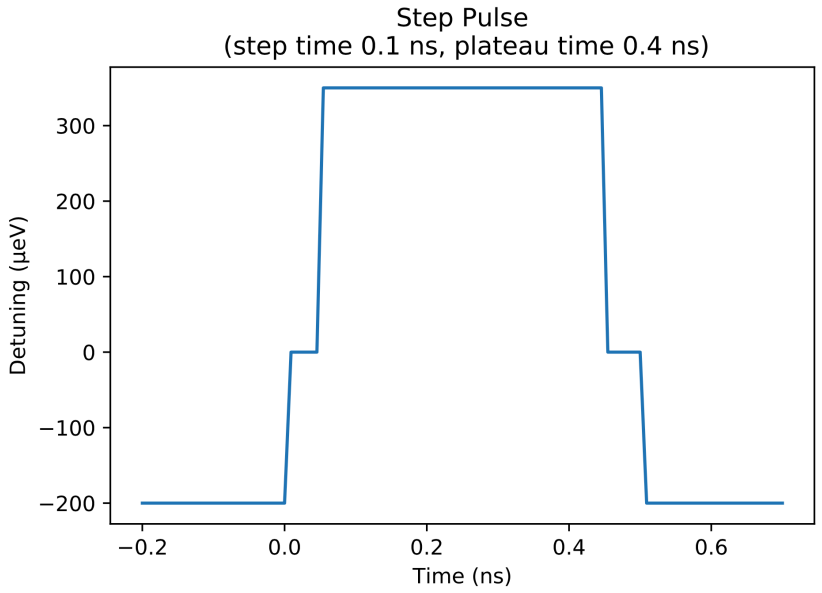
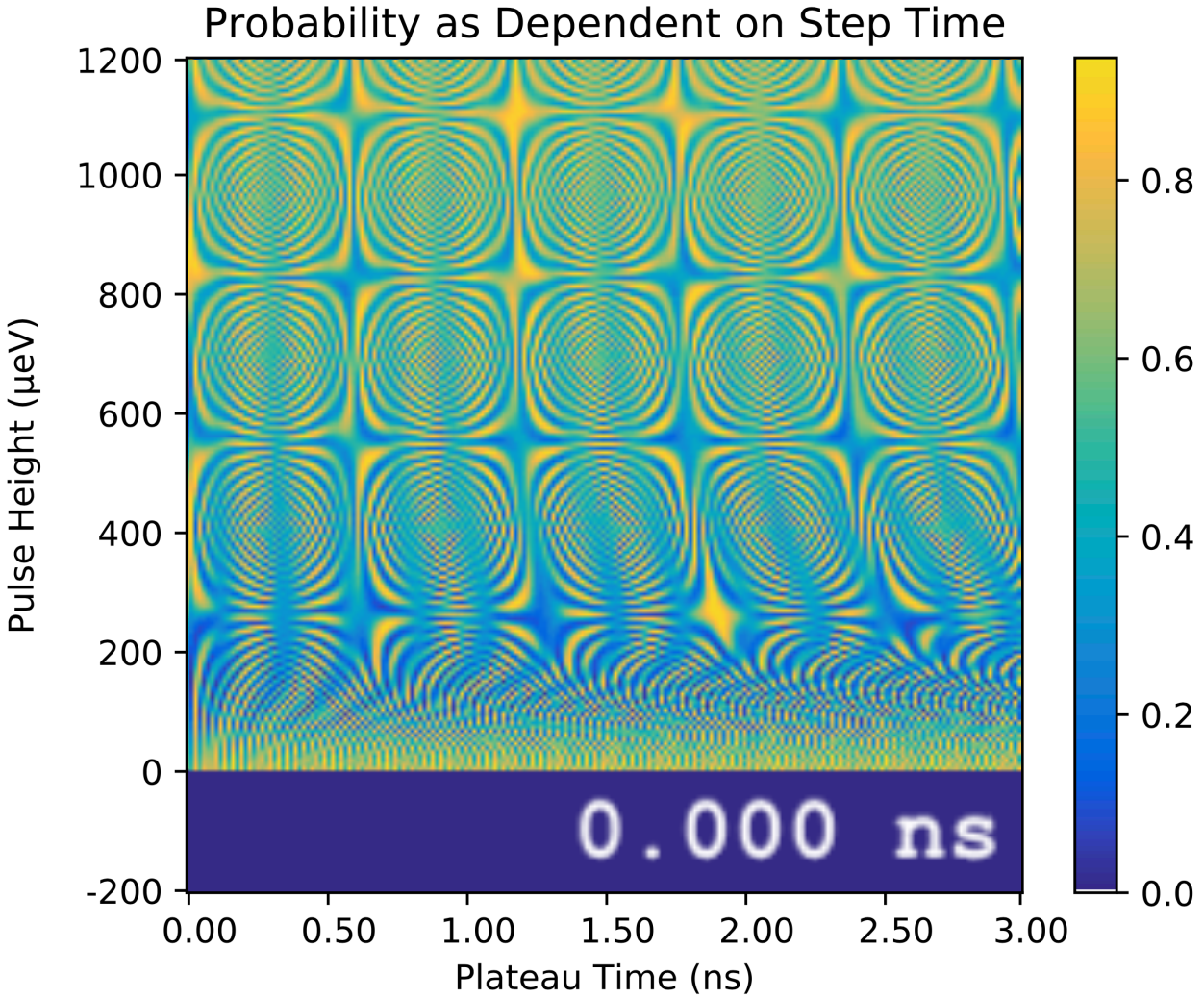


FIG. 4. 500×200 images of coherent oscillations of $\langle L_1 \rangle$ for a four-state system beginning in $|\psi(0)\rangle = |L_1\rangle$, driven by a trapezoidal pulse. The optimized unitary method is 122 times faster than the QuTiP method. Tunnelings are as in Footnote 38. The x-axis is time and ranges from 0.236 to 2.236 ns. The y-axis is detuning and ranges from -50 at the *top* to 100 μeV at the *bottom*. (a) Density matrix methods using QuTiP took 5.92 hr to determine this image. The reporting step size was 2×10^{-11} sec. (b) Optimized unitary method took 2.91 min to determine this image. The step size was 10^{-12} sec.

Parameter Space Exploration is Practical

- Using two-axis step pulse, with a step at 0
- Animation of 200, 200x200 images took 2 hours 57 mins





Questions?