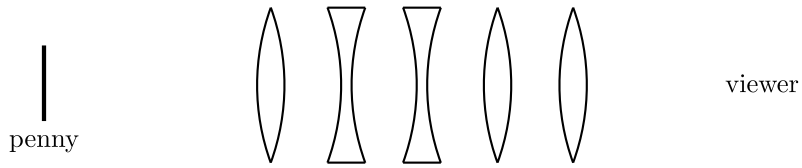
Problem 1. Thin Lens Optics (15 Points)

Consider the following configuration of thin lenses:



If a penny of radius 1 cm is placed 3 cm to the left of the configuration of lenses, at what position **(7 points)**, with what height **(7 points)**, and at what magnification **(1 point)** does an observer on the right see the penny? Assume that all lenses have focal lengths of ± 2 cm.

Problem 2. Thin Film Interference (35 points)

Suppose a metal cylinder is filled with water, refractive index $n > 1$, and the cylinder is spinning so the surface of the water takes on a shape described by $z(r) = h + ar^2$ with respect to the center of the base of the cylinder for constants a and h. If we assume that a is small, the deflection of light due to Snell's law will be small, so assuming normally incident light, light at a radius r will traverse a path of length **(fill in for 5 points)**, which corresponds to a phase shift of **(fill in for 10 points)** for light with vacuum wavelength λ. However, light can reflect off the surface of the water and off the bottom of the cylinder again leading to a cumulative phase shift of **(fill in for 3 points)**, and for b trips through the water the phase shift is **(fill in for 2 points)**. Note: upon reflection from a medium with higher index of refraction, light accumulates a phase of π (assume the index of refraction of the metal is greater than that of water). Upon transmission, light's phase is unchanged.

Assuming that the bottom of the cylinder is perfectly reflective, and every time light is incident on the surface of the water half is reflected, and half is transmitted, for light of vacuum wavelength λ, what is the smallest radius where destructive interference occurs **(5 points)**? What is the intensity of the electric field for destructive interference at this radius if the initial electric field strength is E **(10 points)**?

You may find the geometric series summation formula useful:

$$\sum_{i=1}^{\infty} \frac{1}{p^i} = \frac{1}{p-1} \qquad \text{if } p > 1$$

Problem 3. Atmospheric Muons (20 Points)

When cosmic rays (high energy protons and atomic nuclei) hit molecules in the upper atmosphere, they create muons. Suppose a muon's lifetime is 2.2 μs and it is traveling towards the surface of the Earth at $0.994c$, $\beta \approx 1$, $\gamma \approx 9.1$, note $9.1 \times 2.2 \approx 20$. How long does the muon live in an observer on Earth's frame **(10 points)**? How far does the muon travel in the Earth's frame in this time **(4 points)**? How does this distance compare to the thickness of Earth's atmosphere—100 km **(2 points)**? How does this distance compare to 2.2 μs $\times$ $0.994c$ **(4 points)**? Note: $c \approx 3 \times 10^8$ m/s.

You may find the Lorentz Transform Matrix helpful, $\vec{x}\,' = L\vec{x}$:

$$L = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Problem 4. Time Dilation (30 Points)

Suppose an observer sees a spaceship go past at velocity v and on the spaceship light bounces between two parallel mirrors every 1 ns in the spaceship frame. If the plates are oriented at an angle θ with respect to the direction of motion, how long does it take for the light to bounce between from the first plate to the second plate **(10 points)** and from the second plate to the first plate **(10 points)** in the observer's frame? What is the sum of these two times **(6 points)**? How does the sum depend to θ **(2 points)**? How does the sum depend on γ **(2 points)**?