Exercise 1. Refractive Index I

Suppose that the refractive index of quartz is 1.46 for blue light and 1.45 for red light. If a blue laser and a red laser are shined directly into the 10737 km fiber optic cable from Point Arena California to Maruyama, Japan, which color will make it to Maruyama first? How long will it take each color of light to make it to Maruyama? What is the distance separating the light beams when the first light beam makes it to Maruyama? Note: $c = 2.99792458 \times 10^8$ [m/s].

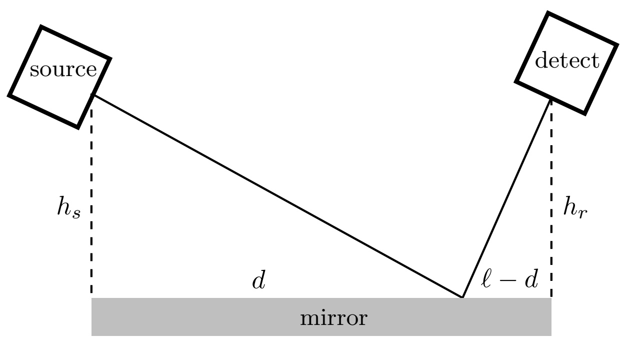
Exercise 2. Refractive Index II

For other types of waves, the speed of wave propagation is described not by an index of refraction, but rather by a dispersion relation $\omega(k)$, and a phase velocity given by $c(\omega) = \omega/k(\omega)$. Considering the case of water waves on the surface of deep water where $\omega(k) = \sqrt{gk}$, explain why it is not possible to to define a sensible refractive index for water waves.

Considering the dispersion relation above, which move faster: large ocean waves with long wavelength, or short ocean waves with small wavelength? In other words, find $c(\lambda)$. Hint: $\lambda = 2\pi c/\omega = 2\pi/k$.

Exercise 3. Reflection I

Consider the setup below. Find all values of d such that the laser beam makes it to the detector. The mirror has length ℓ.



Exercise 4. Reflection II

Now suppose that the mirror is no longer flat, but has a surface normal vector parameterized by $\vec{n} = \hat{y} + \epsilon d\,\hat{x}$ for $0 \le d \le \ell$ and $|\epsilon\ell| < 1$ (so that the vertical distance to the mirror can be approximated as h_s or h_r and so that $\|\vec{n}\| \approx 1$). What are the solutions in this case? Hint: the angle between vectors $\vec{u}$ and $\vec{v}$ is $\theta = \cos^{-1}(\vec{u} \cdot v/\|\vec{u}\|\,\|\vec{v}\|)$.

Exercise 5. Snell's Law I

Diamond has an unusually high refractive index of between 2.44 and 2.40 in the visible range of the spectrum. Assuming that blue light has an index of refraction of 2.44 and red light has an index of refraction of 2.40, what is the angular separation of the red and blue light assuming both are incident at 30 degrees with respect to the surface normal of a diamond? Is the large value of diamond's refractive index the most important characteristic for creating rainbows, or is there some other important characteristic?

Exercise 6. Snell's Law II

Now assume the diamond in question is a large rectangular slab with thickness 1 cm. Draw the paths of the incident, reflected, and refracted light at the top surface and the incident and refracted light at the bottom surface.

Exercise 7. Total Internal Reflection

Suppose that a toroidal fish tank is filled with water of index of refraction 4/3, and the tank's glass has index of refraction, 5/2, if light starts in the glass, what is the critical angle for total internal reflection to occur with respect to the water and air? Supposing the toroid has the shape of a regular polygon; what is the least number of sides the tank must have in order for the light to stay in the glass? Hint: $\theta_{\text{critical}} = \sin^{-1}(n_{\text{water}}/n_{\text{glass}})$ and $\theta_{\text{critical}} = \sin^{-1}(n_{\text{air}}/n_{\text{glass}})$.

Exercise 8. Malus's Law I

Suppose that unpolarized light is incident on a series of three polarizers whose angles are respectively $0°$, $45°$ and $90°$ with respect to the vertical. What is the intensity of light that emerges from the polarizers?

Exercise 9. Malus's Law II

Suppose that unpolarized light is incident on a series of two polarizers whose angles are respectively $0°$ and $90°$ with respect to the vertical. What is the intensity of light that emerges from the polarizers?

Explain your result in the context of the last exercise. How does removing a polarizer result in the change you observe?

Exercise 10. The Complex Refractive Index

The refractive index can be treated as having both real and imaginary parts: $\widetilde{n} = n + i\kappa$. The real part determines the speed of propagation and the imaginary part determines the rate of dissipation. The absorption coefficient is defined as $\alpha = 2\omega\kappa/c$. Interpret each term of the absorption coefficient; what are the units of the absorption coefficient is κ is a dimensionless number?

If $\kappa = 0.4$ in a rock, what is the absorption coefficient for green light $\lambda = 500$ [nm]? Note: $\lambda = 2\pi c/\omega$.

Exercise 11. Beer-Lambert Law

The Beer-Lambert Law says that at a depth d the intensity of light inside of a material with absorption coefficient α is $I_0 \exp(-\alpha d)$ where I_0 is the incident intensity. Continuing the example from the last problem, using the Beer-Lambert Law, what is the intensity of green light 1 [mm] into the rock?

Exercise 12. Reflectance

The reflectance tells us how much of the incident light is reflected. If the reflectance is given by:

$$R = \frac{(n-1)^2 + \kappa^2}{(n+1)^2 + \kappa^2}$$

what is the reflectance of green light from the rock if its refractive index is 1.5?

Given the formula for reflectance, how do you think the values of n and κ are related for a typical shiny metal like silver?