Exercise 1. Speed of Light I

The speed of light in a vacuum is $c = 1/\sqrt{\epsilon_0\mu_0}$, while in a material it is $c_m = 1/\sqrt{\epsilon_m\mu_m}$. Explain why $c_m < c$?

Exercise 2. Speed of Light II

Supposing that nothing can move faster than c, the speed of light's propagation. Argue that light propagating in the $\hat{z}$ direction can only have $\hat{x}$ and $\hat{y}$ polarizations. Equivalently, argue that there can be no longitudinal electromagnetic waves.

Exercise 3. Wave Equation

Show that:

$$E(z,t) = E_0\cos((\omega/c)z - \omega t)$$

Obeys the wave equation:

$$\partial_{zz}E(z,t) = \frac{1}{c^2}\partial_{tt}E(z,t)$$

Exercise 4. Electric versus Magnetic

From $|\vec{B}| = |\vec{E}|/c$, show that a light wave with amplitude 300 [V/m] has magnetic field amplitude 1 [μT]. What does this say about the relative strength of electricity and magnetism?

Exercise 5. Dispersion of Light I

If $E = \hbar\omega$ and $p = \hbar k$ in general and we are told that $E = pc$ for light in a vacuum, what is $\omega(k)$? Is this what you expect?

Exercise 6. Dispersion of Light II

Consider a prism that separates the colors of the rainbow. Given this information, do you expect your finding to be the same or different within the prism? One of the three equations changes. Which is it?

Exercise 7. Poynting Vector I

Draw a picture illustrating the momentum transfer of light encapsultated by the formula below if (a) the material is perfectly absorbent (b) the material is perfectly reflective:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{|\vec{S}|\,|\vec{A}|}{c}(\hat{S}\cdot\hat{A})\hat{S}$$

Exercise 8. Poynting Vector II

Find the acceleration of a perfectly reflective 1 [m^2] sheet that weights 1 [g] due to the pressure of light when uniformly illuminated by normally incident light with electric field magnitude 10 [V/m].

Exercise 9. Solar Sail I (Textbook 32.30)

Consider a disk 9 meters in diameter and 7.5×10^{-6} meters thick. If the intensity of solar energy on the disk is 1400 [W/m^2] (a) What force does the sun's light exert on this sail, assuming it strikes perpendicular to the sail and that the sail was perfectly reflecting? (b) If the sail was made of magnesium, of density 1740 [kg/m^3] what acceleration would the sun's radiation give to the sail?

Exercise 10. Solar Sail II (Textbook 32.54)

Suppose a solar sail-craft uses a large sail and the energy and momentum of sunlight for propulsion. The total power output of the sun is P. How large a sail is necessary to propel a spacecraft of mass m against the gravitational force of the sun? Explain why the answer is independent of the distance from the sun.