Exercise 1. Faraday's Law I

Consider a square wire loop with side-length a and surface normal $\vec{A} = \hat{z}$ inside a magnetic field $\vec{B} = B_0 \sin(\omega t)\hat{z}$. What is the electromotive force induced in the loop?

Exercise 2. Faraday's Law II

Consider a square wire loop with side-length a and surface normal $\vec{A} = \hat{z}$ inside a magnetic field:

$$\vec{B} = B_0 \sin(\pi x/a) \sin(\pi y/a) \cos(\omega t)\hat{z}$$

What is the electromotive force induced in the loop?

Exercise 3. Faraday's Law III

Consider a square wire loop with side-length a and surface normal $\vec{A} = \hat{z}$ moving with velocity $\vec{v} = v\hat{x}$. At $t = 0$, the square current loop is in a region with no magnetic field directly to the left of a region with constant magnetic field $\vec{B} = B_0\hat{z}$. If the current loop has a resistance of R, what is the current induced in the wire loop for $0 \leq t \leq a/v$?

Exercise 4. Faraday's Law IV

Consider a square wire loop with side-length a and surface normal $\vec{A} = \hat{z}$ subject to a magnetic field given by $\vec{B} = B_0 \sin(\omega t)\hat{y} + B_0 \cos(\omega t)\hat{z}$, find the electromotive force induced in the loop as a function of time. What is the power as a function of time if the loop has a resistance R?

Exercise 5. Faraday's Law V

Consider a square wire loop with side-length a and surface normal $\vec{A} = \hat{z}$ moving with velocity $\vec{v} = v\hat{x}$. If the magnetic field is given by $\vec{B} = (\mu_0 I/x)\hat{z}$, what is the electromotive force as a function of time $t \geq 0$ if the initial location is $x(0) = x_0$?