Exercise 1. Lorentz Transform Matrix

Explain the significance of the off-diagonal elements $\beta\gamma$ in the Lorentz Transform matrix:

$$L = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

Exercise 2. Time Dilation I

Suppose an observer sees a spaceship go past at $0.8c$ and on the spaceship light bounces between two parallel plates every 1 ns in the spaceship frame. If the plates are parallel to the direction of motion, how long does it take for the light to bounce between the plates in the observer's frame?

(a) 0.667 [ns]
(b) 1.000 [ns]
(c) 1.333 [ns]
(d) 1.667 [ns]*

Exercise 3. Time Dilation II

Is the answer different if the plates are oriented perpendicular to the direction of motion? Does the light take the same amount of time to go from one plate to the other as it does to come back?

Exercise 4. Space Distortion I

If an astronaut of height h is napping on a spaceship with velocity v relative to an observer in the direction of the astronaut's height, how tall does the astronaut appear to the observer?

(a) h
(b) γh*
(c) h/γ
(d) $\beta\gamma h$

Exercise 5. Space Distortion II

How tall does the astronaut appear is they are standing up (perpendicular to the direction of motion)?

Exercise 6. Velocity Distortion

Suppose that a rebel spacecraft is fleeing from the Empire and sees the Empire's ship receding at $0.2c$. If the Empire shoots a missile at the rebel spacecraft at $0.5c$ in the Empire's frame, how fast is does the missile appear to be moving in the rebel's frame? Is it approaching or receding?

(a) $0.3062c$ receding
(b) $0.3333c$ receding
(c) $0.9186c$ receding
(d) $0.3062c$ approaching
(e) $0.3333c$ approaching*
(f) $0.9186c$ approaching

Exercise 7. Angle Distortion

Suppose that an astronaut on a spaceship passes passes by an observer on an asteroid with velocity v relative to the asteroid. In the astronaut's reference frame they shine a beam of laser light at an angle θ from the horizontal. What is this angle that the laser is shined with respect to the astronaut in the reference frame?

(a) $\theta' = \tan^{-1}(\tan(\theta)/\gamma)$*
(b) $\theta' = \tan^{-1}(\gamma\tan(\theta))$
(c) $\theta' = 1/\tan^{-1}(\tan(\theta)/\gamma)$
(d) $\theta' = 1/\tan^{-1}(\gamma\tan(\theta))$

Exercise 8. Relativistic Momentum

Suppose an electron in Beryllium has a velocity of 2.25×10^6 [m/s] $= 0.0075c$. What is the classical momentum? What is the relativistic momentum?

(a) $p_{\text{classical}} = 0.0075m_e c$, $p_{\text{relativity}} = 0.000056250m_e c$

(b) $p_{\text{classical}} = 0.0075m_e c$, $p_{\text{relativity}} = 0.007499789m_e c$

(c) $p_{\text{classical}} = 0.0075m_e c$, $p_{\text{relativity}} = 0.007500211m_e c$*

(d) $p_{\text{classical}} = 0.0075m_e c$, $p_{\text{relativity}} = 1.000000000m_e c$

Exercise 9. Relativistic Energy I

In the LHC, protons are accelerated to $0.999999991c$. What is the total energy of a proton at this velocity?

(a) $m_p c^2$

(b) $2m_p c^2$

(c) $2357m_p c^2$

(d) $7454m_p c^2$*

Exercise 10. Relativistic Energy II

Compare this to energy given by the ultra-relativistic formula $E = pc = \gamma mvc$.