Problem 1. Solar Sail (10 Points)

At what radius would a perfectly reflective solar sail with area A and mass m that is initially at rest with respect to the sun remain at rest **(10 points)**? Assume that the sun has a mass $m_\odot$ and a luminescent power output of P. Hint: $F_{\text{light}} = \langle S \rangle A/c$.

Problem 2. Ampere's Law on a Limaçon (15 Points)

Suppose that an Amperian loop is described by the limaçon $r(\theta) = 1 + \cos(\theta)$. If the magnetic field on the limaçon is described by $\vec{B}(\theta) = B_0 \sin(\theta/2)\hat{z}$, what is the enclosed current **(15 points)**? It may help to recall the product-to-sum identity: $\cos(u)\sin(v) = \frac{1}{2}\sin(u+v) - \frac{1}{2}\sin(u-v)$.

Problem 3. Elliptical Current Loop (25 Points)

The Biot-Savart Law is:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

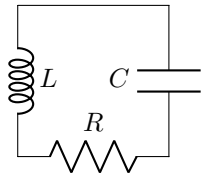
Consider an elliptical current loop with current I and loop radius $\rho(\theta) = ab/\sqrt{(a\sin(\theta))^2 + (b\cos(\theta))^2}$. Assuming cylindrical coordinates (ρ, θ, z), write the integral for the magnetic field at a point at height h on the z axis **(5 points)**. Evaluate the integral in the case $a = b = s$ **(10 points)**.

For **(10 points)**, integrate in the case where $a = s$ and $b = s + \delta s$ for $|\delta s| \ll |s|$. It may help to note that in this case:

$$\frac{1}{(h^2 + \rho^2(\theta))^{3/2}} \approx \frac{1}{(h^2+s^2)^{3/2}} - \frac{3s\,\delta s \sin^2(\theta)}{(h^2+s^2)^{5/2}} \qquad \text{and} \qquad \int_0^{2\pi} d\theta\ \sin^2(\theta) = \pi \tag{1}$$

Problem 4. Damped Circuit (15 Points)

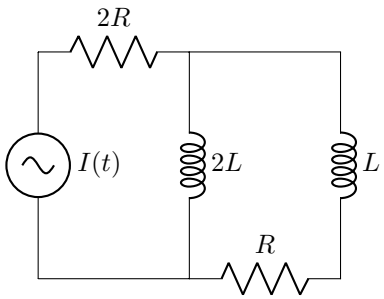
Consider the RLC circuit:



Write a differential equation that models the circuit **(6 points)**. Find the general solution to the differential equation, using for instance the method of characteristic equations **(6 points)**. Apply the boundary conditions $Q(0) = Q_0$ and $\dot{Q}(0) = 0$ to find the solution to the differential equation for these boundary conditions **(3 points)**. Note: $I = \dot{Q}$.

Problem 5. Circuit with Impedance (35 Points)

Consider the driven circuit:



Supposing that $I(t) = I_0\cos(\omega t)$, what is the impedance of the circuit as measured across the current source **(10 points)**? What is the current through the resistor with resistance $2R$ **(5 points)**? Find an expression for the current through the inductor with inductance $2L$ as a function of time **(5 points)**? What is the current through the inductor with inductance L as a function of time **(5 points)**? What is the phase difference between the voltage in the two inductors **(5 points)**? What is the phase difference between the current in the two resistors **(5 points)**?