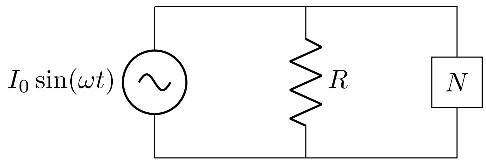
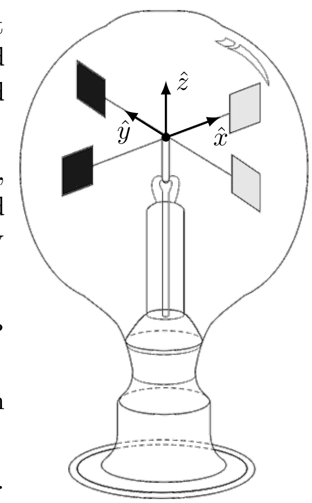
Problem 1. A Nonlinear Circuit (15 Points)

Consider the following circuit:



N is a nonlinear component with impedance $Z_N = i\omega^2 N$. What is the phase difference: $\theta(V_R(t)) - \theta(V_N(t))$ (**15 points**)?

Problem 2. Light Mill (25 Points)

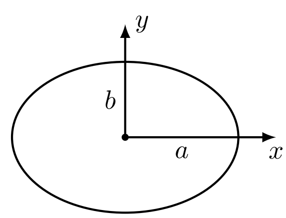


Consider the light mill pictured at right. Assume that the blades of the light mill are initially at rest and at $t = 0$ the apparatus is illuminated by uniform light with magnitude $\vec{E} = E_0 \sin(\omega t)\hat{y}$ and $\vec{B} = B_0 \sin(\omega t)\hat{z}$. Assume that the blades are square and have a sidelength of a and are connected to the spindle by rigid wires of length l, and the system has a moment of inertia I.

(a) Light carries momentum and its transfer is captured by the Poynting vector $\vec{S} = \vec{E} \times \vec{B}/\mu_0$, which is related to the force: $\vec{F} = \vec{S}A/c$. What is the force, $d\vec{F}$, on a sliver with height a and width dr oriented perpendicular to the direction of light propagation if the surface is: perfectly absorbent (**3 points**), perfectly reflective (**2 points**)?
(b) Find the average torques, $\langle d\vec{\tau} \rangle$, on the slivers (**5 points**). Hint: $\sin^2(\theta) = [1 - \cos(2\theta)]/2$.
(c) What is the average torque exerted about the spindle by the pressure of light on the blades? (**10 points**)? Hint: $\langle \vec{\tau} \rangle = \int_{r_0}^{r_1} \langle d\vec{\tau}(r) \rangle$.
(d) Write Newton's law for the rotational motion (**3 points**), which direction does the system start moving as viewed from above (**2 points**). Hint: $\sum \vec{\tau} = I\ddot{\theta}\,\hat{z}$.

Aside: the motion of a light mill is *not* due to the pressure of light, but rather thermal transpiration.

Problem 3. Ampere's Law on an Ellipse (10 Points)



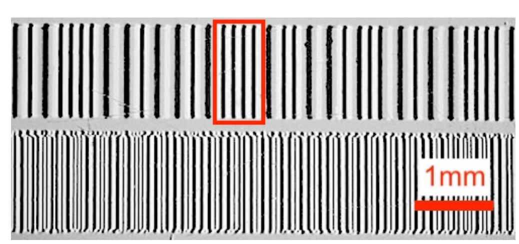
Consider the ellipse parameterized by $\vec{\alpha}(\theta) = (a\cos(\theta), b\sin(\theta))^\top$. The unit tangent to the ellipse is:

$$\hat{T}(\theta) = \frac{(-a\sin(\theta), b\cos(\theta))}{\sqrt{a^2\sin^2(\theta) + b^2\cos^2(\theta)}}$$

and $d\vec{\ell} = r(\theta)d\theta\,\hat{T}(\theta)$, write Ampere's Law (**5 points**), this is referred to as an elliptic integral.

If $B(\theta) =$ constant, what is the value of this integral (**5 points**)? Hint: think physically or think about even and odd functions and note that $\int_0^{2\pi} f(\theta)d\theta$ is the same as $\int_{-\pi}^{\pi} f(\theta)d\theta$.

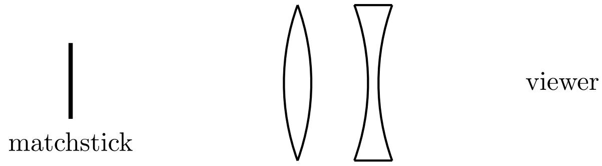
Problem 4. Faraday's Law and Credit Cards (10 Points)



Consider the optical microscope image of a credit card strip pictured at right. The top strip encodes data in binary and the bottom strip provides a uniform distance metric. The magnetic field generated by the dark regions is $B_d = 0.30$ [T] perpendicular to the card, and the magnetic field generated by the light regions is $B_l = 0.05$ [T] perpendicular to the card. If the card is moving right at a uniform velocity v, draw the voltage induced in a thin Faraday's law sensor with area A as a function time as the region in the red box passes under the sensor (**10 points**).

Problem 5. Geometric Optics (10 Points)

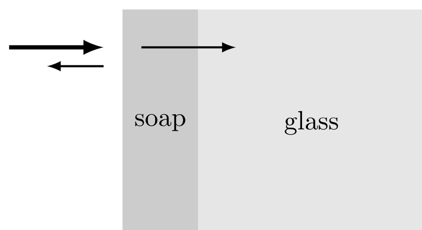
Consider the following configuration of thin lenses:



If a matchstick of length 4 cm is placed 7 cm to the left of the configuration of lenses, at what position **(4 points)**, with what height **(4 points)**, and at what magnification **(2 point)** does an observer on the right see the matchstick? Assume that the convex lens has a focal length of 5 [cm] and the concave lens has a focal length of 3 [cm].

Problem 6. Interference (10 Points)

Consider light with vacuum wavelength λ incident on a soap film of thickness 1 mm and index of refraction $n = 4/3$ covering a glass mirror with glass thickness 3 mm and index of refraction $n = 3/2$. Assuming that there is no phase shift or reflection at interfaces (except for reflection at the surface of the soap-air interface and on the silvered side of the mirror), at what free-space wavelengths will there be constructive interference (**5 points**)? At what free space wavelengths will there be destructive interference (**5 points**)?



Problem 7. Relativistic Laser Angle (10 Points)

Suppose an observer sees a spaceship go past at velocity v and on the spaceship an astronaut shines a laser at an angle θ with respect to the direction of motion in the spaceship frame, what is the angle in the observer's frame? **(10 points)**?

Problem 8. Velocity Distortion (10 Points)

Suppose that a rebel spacecraft is fleeing from the Empire and sees the Empire's ship receding at $0.2c$. If the Empire shoots a missile at the rebel spacecraft at $0.5c$ in the Empire's frame, how fast is does the missile appear to be moving in the rebel's frame (**7 points**)? Is it approaching or receding (**3 points**)?