

1. The amplitude is 5 [cm], so  $x_{\min} = x_0 - A = 15$  [cm]
2. Energy is conserved, so  $1/2kA^2 = 1/2mv^2$ , or  $v = \omega A = 2\pi f A = 0.44$  [m]
3.  $v = \omega r$ , so  $\omega = v/r = 114$  [rad/s]
4.  $v = dx/dt = -A \exp(-\lambda t)[\lambda \cos(\omega t) + \omega \sin(\omega t)] \hat{x}$
5.  $K = mv^2/2 = mA^2 \cos^2(\omega t) \exp(-2\lambda t)/2$
6. If the motion is uniform, then there is no change in velocity after one cycle, so the work is 0:

$$W = \int_{\ell} F \cdot d\ell = \int_{\ell} -\frac{mv^2}{r} \hat{r} \cdot \hat{r}_{\perp} d\ell = -\frac{mv^2}{r} \int_{\ell} \cos(\pi/2) d\ell = -\frac{mv^2}{r} \int_{\ell} 0 d\ell = 0$$

7. Using energy conservation  $kA^2/2 = kx_0^2/2 + mv_0^2/2$ , or  $A = \sqrt{(kx_0^2 + mv_0^2)/k}$
8.  $v = dx/dt = A\omega \cos(\text{stuff that ranges from } -\pi \text{ to } \pi)$ , so  $v_{\max} = A\omega$ , or see (2)
9.  $K = mv^2/2 = m^2v^2/2m = p^2/2m$
10.  $T = 1/f = 2\pi/\omega = 2\pi\sqrt{\ell/g} \approx 2\sqrt{\ell}$ , so  $\ell = 1/4$ , and we demand  $m > 0$  and  $\theta > 0$
11. Forces balance so  $ky - mg = 0$ , or with  $y = \ell - \ell_0$ ,  $\ell - \ell_0 = mg/k$ . Subsequent oscillations are:

$$y(t) = \ell + (\ell_0 - \ell) \cos(\sqrt{k/m} t)$$

12.  $F = ma$  and  $F = -kx - ax^3$ , so  $m\ddot{x} + 0\dot{x} + (k + ax^2)x = 0$ , which is a nonlinear differential equation
13. Use the method of characteristic equations:

$$\begin{aligned} F &= ma = -kx - bv \\ m\lambda^2 + b\lambda + k &= 0 \\ \lambda_{\pm} &= \frac{-b \pm \sqrt{b^2 - 4mk}}{2m} \\ x(t) &= \alpha_+ e^{\lambda_+ t} + \alpha_- e^{\lambda_- t} \end{aligned}$$

With initial conditions:

$$\begin{aligned} \alpha_+ + \alpha_- &= 0 \\ \alpha_+ \lambda_+ + \alpha_- \lambda_- &= 1 \\ \alpha_+ &= \frac{1}{\lambda_+ - \lambda_-}; \quad \alpha_- = \frac{1}{\lambda_- - \lambda_+} \end{aligned}$$

14. As  $\omega_d \rightarrow \omega$ ,  $(k - m\omega_d) \rightarrow 0$ , so if  $b$  is small, the amplitude of oscillations may become (catastrophically) large
15.  $\omega_d = 2\pi f_d = 2\pi 120/60 = 4\pi$ , with  $g \approx 10$ ,  $F_d = mg = 40 \cdot 60 \cdot 10$ , so:

$$A = \frac{2400}{\sqrt{(10^6 - 10^5(4\pi)^2)^2 + (4\pi \cdot 10^4)^2}} = 0.000162 \text{ [m]}$$