Exercise 1. Displacement

A 1 [kg] mass is released from the end of a spring stretched by 5 [cm] from its natural length of 20 [cm]. Assume that there is no dissipation during the spring's oscillations, and the spring constant is 10 [N]. What is the spring's shortest length?

(a) 20 [cm]
(b) 15 [cm]
(c) 10 [cm]
(d) 5 [cm]

Exercise 2. Velocity

A spring and mass combination has a natural frequency of 1 [Hz], and are stretched to an initial displacement of 0.07 [m]. What is the speed of the mass as it passes through the unstretched spring length?

(a) 0.07 [m/s]
(b) 0.14 [m/s]
(c) 0.22 [m/s]
(d) 0.44 [m/s]

Exercise 3. Bullwhip

Bullwhips are known for making a loud noise when they are cracked. The noise originates from the tip of the whip moving faster than the speed of sound, 343 [m/s]. Suppose a bullwhip is 3 [m] long and it moves in a circular arc, what angular frequency must the whip be moving to move faster than sound?

(a) 343 [rad/s]
(b) 114 [rad/s]
(c) 109 [rad/s]
(d) 18.2 [rad/s]
(e) 9.4 [rad/s]
(f) 3 [rad/s]

Exercise 4. Energies I

Suppose we are told that a weight of mass m, moves with $\boldsymbol{x}(t) = A\cos(\omega t)\exp(-\lambda t)\,\hat{\boldsymbol{x}}$ for constants A, ω, λ. Find $\boldsymbol{v}(t)$. Note that energy is not conserved.

(a) $\boldsymbol{v}(t) = A\exp(-\lambda t)[\lambda\cos(\omega t) - \omega\sin(\omega t)]\,\hat{\boldsymbol{x}}$
(b) $\boldsymbol{v}(t) = -A\exp(-\lambda t)[\lambda\cos(\omega t) + \omega\sin(\omega t)]\,\hat{\boldsymbol{x}}$
(c) $\boldsymbol{v}(t) = A\omega\lambda\sin(\omega t)\exp(-\lambda t)\,\hat{\boldsymbol{x}}$
(d) $\boldsymbol{v}(t) = -A\omega\lambda\sin(\omega t)\exp(-\lambda t)\,\hat{\boldsymbol{x}}$

Exercise 5. Energies II

Suppose we are told that a weight of mass m, moves with $\boldsymbol{v}(t) = A\cos(\omega t)\exp(-\lambda t)$ for constants A, ω, λ. Find $K(t)$. Note that energy is not conserved.

(a) $K(t) = mA^2\cos^2(\omega t)\exp(-2\lambda t)/2$
(b) $K(t) = mA^2\cos^2(\omega t)\exp(-2\lambda t)$
(c) $K(t) = A^2\cos^2(\omega t)\exp(-2\lambda t)/2$
(d) $K(t) = A^2\cos^2(\omega t)\exp(-2\lambda t)$

Exercise 6. Uniform Circular Motion

A particle of mass m is moving in uniform circular motion has acceleration, $\boldsymbol{a} = -v^2/\|\boldsymbol{r}\|\,\hat{\boldsymbol{r}}$. Calculate the work done in one cycle. Hint: work is $W = \int_s d\boldsymbol{s}\cdot\boldsymbol{F}$.

(a) $2\pi\|\boldsymbol{v}\|^2$
(b) $-2\pi\|\boldsymbol{v}\|^2$
(c) 0
(d) cannot be determined with the information given

Exercise 7. Amplitude of a Wave

The initial position and velocity of a spring-block system are x_0 and v_0 respectively. What is the amplitude of the oscillations?

Exercise 8. Velocity

Show that the maximum velocity a mass in a spring mass system reaches is ωA. Hint: $\boldsymbol{x}(t) = A\cos(\omega t + \phi)\hat{\boldsymbol{x}}$.

Exercise 9. Kinetic Energy

From $\boldsymbol{p} = m\boldsymbol{v}$ and $K = m\|\boldsymbol{v}\|^2/2$, show $K = \|\boldsymbol{p}\|^2/2m$.

Exercise 10. Making a Clock

You are asked to make a clock, and are given some string of which you cut off a length ℓ, and some weights which you select m, and you release the mass from an angle θ_0 from the vertical direction. Approximate the gravitational constant as $g \approx \pi^2$ [m/s^2]. Which of the following pendulums will have a period T equal to one second? (select all that apply)

(a) $\ell = 0.25$ [m], $m = 1$ [kg], $\theta_0 = 0°$

(b) $\ell = 0.50$ [m], $m = 1$ [kg], $\theta_0 = 0°$

(c) $\ell = 1.00$ [m], $m = 1$ [kg], $\theta_0 = 0°$

(d) $\ell = 0.25$ [m], $m = 1$ [kg], $\theta_0 = 10°$

(e) $\ell = 0.50$ [m], $m = 1$ [kg], $\theta_0 = 10°$

(f) $\ell = 1.00$ [m], $m = 1$ [kg], $\theta_0 = 10°$

(g) $\ell = 0.25$ [m], $m = 1$ [kg], $\theta_0 = 22.5°$

(h) $\ell = 0.50$ [m], $m = 1$ [kg], $\theta_0 = 22.5°$

(i) $\ell = 1.00$ [m], $m = 1$ [kg], $\theta_0 = 22.5°$

(j) $\ell = 0.25$ [m], $m = 1$ [kg], $\theta_0 = 45°$

(k) $\ell = 0.50$ [m], $m = 1$ [kg], $\theta_0 = 45°$

(l) $\ell = 1.00$ [m], $m = 1$ [kg], $\theta_0 = 45°$

Exercise 11. Hanging Oscillator

A spring is screwed into the ceiling, and it hangs down a distance ℓ_0. A mass m is attached to the end of the spring, and the spring hangs down a distance ℓ.

i. Draw a picture/free body diagram.

ii. If the spring has a spring constant k and the weight has mass m, find $\ell - \ell_0$.

iii. The mass is then lifted to ℓ_0 and then released. Describe the subsequent oscillations, $y(t)$.

Exercise 12. Anharmonic Oscillator

Consider an anharmonic spring that exerts a restoring force of $\boldsymbol{F} = -k\boldsymbol{x} - a\boldsymbol{x}^3$ on a weight of mass m. Write the differential equation and explain why the method of characteristic equations is not suited to solving this equation.

Exercise 13. Damped Oscillator

Consider a damped spring that exerts a restoring force of $\boldsymbol{F} = -k\boldsymbol{x} - b\boldsymbol{v}$ on a weight of mass m. Using the characteristic equation, find the position as a function of time, $x(t)$ for an initial displacement $x(0) = 0$, and speed $\dot{x}(0) = 1$.

Exercise 14. Resonant Frequency I

An oscillator may also be driven, for example a bridge by traffic. This additional force may be periodic and expressed as a sinusoid $\boldsymbol{F}_d = \|\boldsymbol{F_d}\| \cos(\omega_d t))\ \hat{\boldsymbol{x}}$.

Through some arduous algebra, it may be found that:

$$A = \frac{\|\boldsymbol{F_d}\|}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

What happens in the limit $\omega_d \to \omega$? What if b is small? Engineers have control over k, b, and specify m for bridges. Why is it important to choose ω carefully?

Exercise 15. Resonant Frequency II

Suppose that a marching band is marching across a bridge with footfalls 120 times per minute.

i. If we assume there are 40 band members each of mass 60 [kg], and their entire weight is pressed down with each step, what is the magnitude of the driving force? You may assume that the gravitational constant is $g = 10$ [m/s^2].

ii. If we assume that the bridge weighs 10^5 [kg] and has a spring constant of 10^6 [N], and a damping constant of $b = 10^4$ [kg/s], what is the maximum amplitude the bridge reaches?