Problem 1. Spring Block System

Suppose that a spring-block system is initially at rest in its equilibrium position. Then a lump of clay, of mass m_c, initially traveling at velocity v_c strikes and sticks to the block at $t = 0$. Describe the subsequent motions, assuming the block has a mass of m_b, and the spring has spring constant k.

This is an inelastic collision, so momentum is conserved, but energy is not.

$$P_i = P_f \Rightarrow m_c v_c = (m_b + m_c) v(0) \Rightarrow v(0) = \frac{m_c v_c}{m_b + m_c}$$

Let $v(0) > 0$, so then with $x(0) = 0$, we have:

$$x(t) = A \sin(\omega t)$$

Where by energy conservation $\frac{1}{2} k A^2 = \frac{1}{2} m v(0)^2$ and the system undergoes SHM, so $\omega = \sqrt{k/m}$

Thus:

$$x(t) = \sqrt{\frac{m_c^2 v_c^2}{k(m_b + m_c)}} \sin\left(\sqrt{\frac{k}{m_b + m_c}}\, t\right)$$

Problem 2. Standing Waves

When you blow air into an open organ pipe, it produces a sound with a fundamental frequency of 440 [Hz]. If you close one end of this pipe, what is the new fundamental frequency?

Open pipes have pressure nodes at both ends, so the resonant frequencies are:

$$f_n = n \frac{v}{2L}, \quad n = 1, 2, 3, 4, \ldots$$

Half closed pipes have one node and one antinode, so:

$$f_n = n \frac{v}{4L}, \quad n = 1, 3, 5, 7, \ldots$$

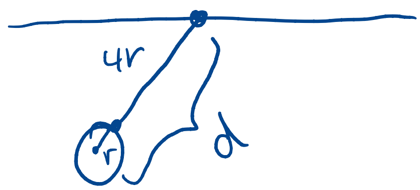
Therefore $f_1^{open} = 1 \cdot \frac{v}{2L} = 440$ [Hz], so we find that:

$$f_1^{closed} = 1 \cdot \frac{v}{4L} = \frac{1}{2} \frac{v}{2L} = 220 \text{ [Hz]}$$

Problem 3. Physical Pendulum

A sphere of mass m and radius r is hung from the ceiling from a string of length $l = 4r$. If the sphere is displaced from equilibrium and released, what is its period of oscillations? Hint: $I_{\text{sphere}} = 2mr^2/5$. Hint: use the parallel axis theorem.

We consider:



Now,

$$I = I_{sphere} + I_{parallel} = \frac{2}{5}mr^2 + md^2 = \frac{127}{5}mr^2$$

For a physical pendulum, we find that:

$$T = 2\pi\sqrt{\frac{I}{mgd}} = \frac{2\pi}{5}\sqrt{127\frac{r}{g}}$$

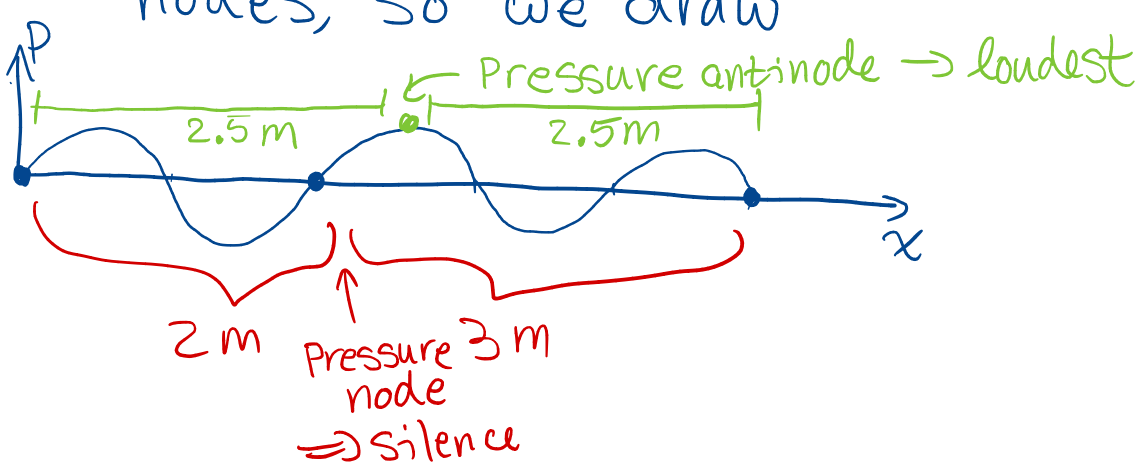
Problem 4. Interference

Suppose that two speakers are separated by 5 [m] and play tones at 170 [Hz] that create standing waves. Suppose you are standing 2 [m] from one speaker and 3 [m] from the other speaker. Describe what you hear if the speakers are displacement antinodes. What would you hear if you were standing directly in between the speakers? The speed of sound is 340 [m/s].

The wavelength of the standing waves is:

$$\lambda = \frac{v}{f} = \frac{340}{170} = 2\ [m]$$

We hear pressure, and the speakers are pressure nodes, so we draw



Problem 5. Diver

A diver is suspended by a cable from a boat at a depth d below the surface of Loch Ness. The diver has a density ρ_d and a volume V; the cable has a radius r and a density ρ_c. The diver thinks she sees something moving in the murky depths and jerks the end of the cable back and forth to send transverse waves up the cable as a signal to her companions in the boat.

(a) What is the tension $T(0)$ in the cable where it is attached to the diver? Hint: if the diver is not accelerating, the forces sum to zero.

(b) Calculate the tension $T(y)$ in the cable a distance y above the diver.

(c) If the speed of transverse waves is $v(y) = \sqrt{T(y)/\mu}$, for linear mass density μ, how long does it take for her signal to reach the surface?

(a) $$\sum_i F_i = 0$$

$$\sum_i F_i = T + \underbrace{F_b}_{gV\rho_w} - \underbrace{F_g}_{gV\rho_d} \Rightarrow T = gV(\rho_d - \rho_w)$$

(b) $$\sum_i F_i = T(y) + \underbrace{F_b^{d}}_{gV\rho_w} + \underbrace{F_b^{c}}_{g\underbrace{\pi r^2 y}_{V_{cable}}\rho_w} - \underbrace{F_g^{d}}_{gV\rho_d} - \underbrace{F_g^{c}}_{g\pi r^2 y \rho_c}$$

$$\Rightarrow T(y) = gV(\rho_d - \rho_w) + g\pi r^2 y(\rho_c - \rho_w)$$
$$\equiv \quad c_1 \quad + \quad c_2 y$$

(c) $$t = \frac{d}{v} = \int_0^d dy \frac{1}{v(y)}$$
$$= \int_0^d dy \sqrt{\frac{\mu}{T(y)}} \qquad \mu = \frac{\pi r^2 y \rho_c}{y} = \pi r^2 \rho_c$$

$u = \frac{c_1 + c_2 y}{\mu}$

$\frac{du}{dy} = \frac{c_2}{\mu}$

$$= \frac{\mu}{c_2} \int_0^{\frac{c_1+c_2 d}{\mu}} du\, u^{-1/2}$$

$$= \frac{2\mu}{c_2} \sqrt{u} \Big|_{u=0}^{u=\frac{c_1+c_2 d}{\mu}} = \frac{2}{c_2}\sqrt{\mu(c_1 + c_2 d)}$$

Problem 6. Doppler

A police car's siren emits a sound wave with $f = 300$ [Hz]. The speed of sound is 340 [m/s] and the air is still. The police car is chasing a speeding car which is moving at 35 [m/s].

(a) What are the wavelengths of the sound in front of and behind the siren if the police car is moving at 40 [m/s]?

(b) At what frequency does the speeding car hear the siren?

(c) At what frequency does the police car hear the siren reflected from the speeding car?

(a) $v = \lambda f \Rightarrow \lambda = \frac{v}{f}$

Behind the siren $v = v_{sound} + v_{police} = 380$, so

$$\lambda = \frac{380}{300} = 1.27 \text{ [m]}$$

In front of the siren $v = v_{sound} - v_{police} = 300$, so

$$\lambda = \frac{300}{300} = 1 \text{ [m]}$$

(b) We recall the doppler shift formula:

$$f^{observed} = \frac{v \pm v^{observer}}{v \pm v^{source}} f^{source} = \frac{340-35}{340-40} 300 = 305 \text{ [Hz]}$$

(c) While the reflected wave has:

$$f^{observed} = \frac{340+40}{340+35} 305 = 309 \text{ [Hz]}$$

Note: the vehicles get closer together with time so we expect the frequency to be "blue shifted" and be greater than 300

Problem 7. Brief Questions

1. What is a transverse wave? What is a longitudinal wave?

A transverse wave is like a snake.

A longitudinal wave is like a worm.

2. Which wave travels left and which travels right? $A\cos(kx+\omega t)$, $A\cos(kx-\omega t)$.

Left traveling: $A\cos(kx+\omega t)$

Right traveling: $A\cos(kx-\omega t)$

3. Which has the fastest, slowest and middle speed of sound among solids, liquids, and gasses?

$v_{solid} > v_{liquid} > v_{gas}$ in general ($v = \lambda f$)

This is because the strength of the interactions and thus the 'frequency' of oscillations increases faster than the 'wavelength' decreases as $g \to l \to s$

4. How many decibels is a 5 [mW] speaker at 2 [m]?

$n_{dB} = 10 \log_{10}\left(\frac{I}{10^{-12}}\right)$, where the intensity is

$$I = \frac{\text{Power}}{\text{Area}} = \frac{5\times10^{-3}}{4\pi \cdot 2^2} \approx 10^{-4} \left[\frac{W}{m^2}\right]$$

So:

$$n_{dB} = 10 \log_{10}(10^8) = 80 \text{ [dB]}$$

5. The wind is blowing in your face at 10 [m/s]. If the speed of sound is 343 [m/s], how long does it take for a friend standing 20 [m] in front of you to hear you?

$$v = \frac{d}{t} \Rightarrow t = \frac{d}{v}$$

where $v = v_{sound} - v_{wind} = 333 \text{ [m/s]}$, so

$$t = \frac{20}{333} = 0.060 \text{ [s]}$$