Physics 1B at UCLA ◊ Formula Sheet (1 of 2)

Perfect Fluid

A perfect fluid is in-compressible and has no viscosity. Their density is

$$\rho = \frac{m}{V}$$

The pressure is then $P = |\boldsymbol{F}|/A$.
In a fluid subject to a gravitational field

$$P(y) = P(0) + \rho\, \boldsymbol{g} \cdot \boldsymbol{y}$$

Archimedes principle: bodies in a fluid feel an upward *buoyant* force equal to the weight of fluid displaced. For flowing fluids the volumetric flow rate is

$$\frac{dV}{dt} = \boldsymbol{A} \cdot \boldsymbol{v}$$

and the continuity equation

$$\boldsymbol{A}_1 \cdot \boldsymbol{v}_1 = \boldsymbol{A}_2 \cdot \boldsymbol{v}_2$$

As a result of energy-conservation we have Bernoulli's equation

$$P_1 + \rho\, \boldsymbol{g} \cdot \boldsymbol{y}_1 + \tfrac{1}{2}\rho|\boldsymbol{v}_1|^2 = P_2 + \rho\, \boldsymbol{g} \cdot \boldsymbol{y}_2 + \tfrac{1}{2}\rho|\boldsymbol{v}_2|^2$$

Periodic Motion

Periodic motion repeats in time with a period T. Reexpressing in frequency f, angular frequency ω, velocity c, wavelength λ and wavenumber k, we have

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{\lambda}{c} = \frac{2\pi}{ck}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{c}{\lambda} = \frac{ck}{2\pi}$$

$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi c}{\lambda} = ck$$

$$c = \frac{\lambda}{T} = f\lambda = \frac{2\pi f}{k} = \frac{\omega}{k} = \frac{2\pi}{kT} = \frac{\omega\lambda}{2\pi}$$

$$\lambda = cT = \frac{c}{f} = \frac{2\pi}{k} = \frac{2\pi c}{\omega}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{2\pi}{cT} = \frac{\omega}{c}$$

Pendulums

Pendula oscillate with

$$\omega = \sqrt{mgd/I}$$

where d is the distance from the center of rotation to the center of mass. The moment of inertia is

$$I = \sum\nolimits_i m_i d_i^2$$

For a simple pendulum (a small mass at the end of a string), $I = md^2$ so

$$\omega = \sqrt{g/d}$$

Simple Harmonic Motion

The simple harmonic motion ODE is

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

whose solution is

$$x(t) = A\cos(\omega t - \phi)$$

In spring-block motion this arises as

$$\boldsymbol{F} = m\boldsymbol{a} = -k_s \boldsymbol{x}$$

so that

$$\frac{d^2\boldsymbol{x}}{dt^2} = -\left(\sqrt{\frac{k_s}{m}}\right)^2 \boldsymbol{x}$$

Driven-Damped Oscillation

We may consider a more nuanced ODE

$$\frac{d^2x}{dt^2} - b\frac{dx}{dt} + \omega_0^2 x = F_0\cos(\Omega t)$$

Whose solution is

$$x = Ae^{-bt/2}\cos(\omega t - \phi)$$

with

$$\omega = \sqrt{\omega_0^2 - b^2/4t}$$

and

$$A = \frac{F_0}{\sqrt{(\omega^2 - \Omega^2)^2 + b^2\Omega^2}}$$

So we see that in the absence of driving, the amplitude goes to zero (at long times), although there may be some transient motion. This is classified as

- $b < 2\omega$: underdamped
- $b = 2\omega$: critically damped
- $b > 2\omega$: overdamped

Traveling Waves

Waves can travel ($-$ right, or $+$ left)

$$y(x,t) = A\cos(kx \mp \omega t)$$

which solve the wave equation

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$

Waves can be superimposed so that

$$y_{\text{tot}}(x,t) = \sum\nolimits_i y_i(x,t)$$

Mechanical Waves

The wave velocity on a string with mass density μ and tension F is

$$c = \sqrt{F/\mu}$$

The average power transmitted is

$$\langle P\rangle = \tfrac{1}{2}\frac{F}{c}\omega^2 A^2 = \tfrac{1}{2}\omega k A^2$$

Sound Waves

Sound is longitudinal waves with

$$c = \sqrt{B/\rho}$$

where B is the bulk modulus which have maximum power (displacement d)

$$P_0 = Bkd$$

The intensity is then

$$I = \frac{1}{2}\frac{P_0^2}{\rho c}$$

Which is often quoted in decibels

$$\beta = 10\log_{10}\left(\frac{I}{10^{-12}\ \text{W/m}^2}\right)$$

Beats form from two different tones

$$f_{\text{beats}} = |f_1 - f_2|$$

The Doppler effect is

$$f_{\text{listen}} = \frac{c + v_{\text{listen}}}{c + v_{\text{source}}} f_{\text{source}}$$

Standing Waves

Standing waves are when nodes ($y = 0$ positions) are t-independent. For a string of length L with fixed ends the standing-wave frequency is

$$f_n = n\frac{c}{2L}$$

In an open pipe standing waves form with frequencies (where $n = 1, 2, 3, \ldots$)

$$f_n = n\frac{c}{2L}$$

and in a stopped pipe the frequencies

$$f_n = \frac{2n-1}{2}\frac{c}{2L}$$

Charge Elements

Differential charge elements are

line	$\lambda(x)\ dx$
strip	$\sigma(x) w\ dx$
disc	$\sigma(r) 2\pi r\ dr$
cylinder	$\rho(r) 2\pi r h\ dr$
sphere	$\rho(r) 4\pi r^2\ dr$

where $dq \sim \lambda dx \sim \sigma dA \sim \rho dV$.

Helpful Integrals

The following may come in useful

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$$

$$\int \frac{x\ dx}{(x^2+a^2)^{3/2}} = -\frac{1}{(x^2+a^2)^{1/2}}$$

$$\int \frac{dx}{(x^2+a^2)^{1/2}} = \ln|x + (x^2+a^2)^{1/2}|$$

$$\int \frac{x\ dx}{(x^2+a^2)^{1/2}} = (x^2+a^2)^{1/2}$$

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Electric Fields

Electric fields induce a force on charges

$$\boldsymbol{F} = q\boldsymbol{E}$$

Charges create electric fields

$$\boldsymbol{E}_{\text{point charge}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\boldsymbol{r}}$$

For a more general charge distribution

$$\boldsymbol{E} = \frac{1}{4\pi\epsilon_0}\int_q \frac{dq}{r^2}\hat{\boldsymbol{r}}$$

Electric fields add as vectors

$$\boldsymbol{E}_{\text{tot}} = \sum\nolimits_i \boldsymbol{E}_i$$

Electric Potentials

The potential energy is $U = qV$ where $\boldsymbol{F} = -\boldsymbol{\nabla} U$ so from $\boldsymbol{F} = q\boldsymbol{E}$ we have

$$\boldsymbol{E} = -\boldsymbol{\nabla} V$$

and the electric field is conservative so

$$V = \int_r^\infty \boldsymbol{E}\cdot d\boldsymbol{r}$$

The potential of a point-charge is

$$V_{\text{point charge}} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

For a more general charge distribution

$$V = \frac{1}{4\pi\epsilon_0}\int_q \frac{dq}{r}$$

Electric potentials add as scalars

$$V_{\text{tot}} = \sum\nolimits_i V_i$$

Gauss's Law

Gauss's law is

$$\Phi = \frac{q_{\text{enc}}}{\epsilon_0}$$

Where the electric flux is

$$\Phi = \int_A d\boldsymbol{A}\cdot\boldsymbol{E}$$

and the enclosed charge is

$$q_{\text{enc}} = \int_V dq = \int_V \rho\, dV$$

If $\boldsymbol{E}$ is parallel to $\boldsymbol{A}$ everywhere and the charge is symmetric then $d\boldsymbol{A}\cdot\boldsymbol{E} = 1$, so

$$|\boldsymbol{E}| = \frac{q_{\text{enc}}}{\epsilon_0 A}$$

Conductors & Insulators

There is no net charge inside a conductor: all charges are on the surface or in cavities, hence $\boldsymbol{E}$ is zero inside a conductor (if there is a charge in a cavity, the opposite charge is on the bounding surface); the potential on any one surface of a conductor is the same (so field lines are perpendicular to the surface). Insulators have fixed charges.

Capacitors

The capacitance is

$$C = q/\mathcal{E} \iff \mathcal{E} = q/C$$

in a parallel-plate configuration this is

$$C = \epsilon A/d$$

The energy stored is then

$$U = \frac{q^2}{2C} = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}Q\mathcal{E}$$

For capacitors in series

$$C_{\text{tot}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots\right)^{-1}$$

For capacitors in parallel

$$C_{\text{tot}} = C_1 + C_2 + C_3 + \dots$$

Resistors

Resistance in terms of the resistivity

$$R = \rho L/A$$

For resistors in series

$$R_{\text{tot}} = R_1 + R_2 + R_3 + \dots$$

For resistors in parallel

$$R_{\text{tot}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\right)^{-1}$$

The voltage across a resistor is

$$\mathcal{E} = jR$$

Where the current is defined as

$$j = \frac{dq}{dt}$$

The power dissipated in a resistor is

$$P = j\mathcal{E} = j^2\mathcal{E} = \mathcal{E}^2/R$$

Kirchhoff's Rules

Voltage is single-valued, so the sum of voltages around any loop is zero:

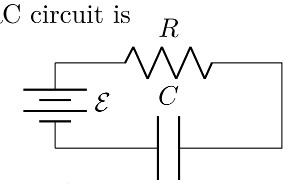
$$\sum\nolimits_{\circlearrowleft} \mathcal{E} = 0$$

Charge is conserved (no accumulation), so at any junction currents sum to zero:

$$\sum\nolimits_{\perp} j = 0$$

RC Circuits

The RC circuit is



which can charge as

$$q(t) = C\mathcal{E}(1 - e^{-t/RC})$$

or discharge as

$$q(t) = q(0)e^{-t/RC}$$

where the current is $j = dq/dt$.

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