Worksheet 2 Solns.

(1) $3\times10^8 = \lambda\cdot 6.50\times10^5 \Rightarrow \lambda = 462\,[m]$

explaining why radio towers are tall

(2) $f = \frac{v}{\lambda} = \frac{343}{1.72} = 200\,[Hz]$

(3) First find wavelength

$\lambda = \frac{v}{f} = \frac{343}{440} = 0.78\,[m]$

Then find length

$L = n\frac{\lambda}{2} = \frac{\lambda}{2} = 0.39\,[m]$

(4) $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{8.5\times10^{-6}/0.01}} = 343$

$\Rightarrow F = 343^2 \cdot \frac{8.5\times10^{-6}}{0.01} = 100\,[N]$

The eardrum doesn't break under such force because distance is small

(5) Begin by finding the wavevector $k = \frac{\omega}{v} = 0.1\left[\frac{1}{m}\right]$

Now, find the wavelength $\lambda = \frac{2\pi}{k} = 62.8\,[m]$

(6) $P = F v_y = F\, \underbrace{\partial_x y}\, \partial_t y$

Pulls out kx, so scales by k, which here is 2

(7) Transverse: a sine wave moving along looks like a snake, while a longitudinal wave is more linear.

(8) $P = F\, v_y(x,t) = F\, \partial_x y\ \partial_t y$

$$\partial_x y = kA \cos(kx - \omega t)$$

$$\partial_t y = -\omega A \cos(kx - \omega t)$$

$$P = k\omega A^2 F \cos^2(kx - \omega t)$$ ← Power is positive by convention

$$= 2 \cdot 4.71 \cdot 0.1^2 \cdot 50 \cos^2\left(2x - \tfrac{3\pi}{2} t\right)$$

$$= 4.71 \cos^2\left(2x - \tfrac{3\pi}{2} t\right)$$

(9) $P = 4.71 \underbrace{\cos^2\left(2x - \frac{3\pi t}{2}\right)}$

maximum value is +1

at $2x - \frac{3\pi}{2} t = n\pi \underset{x=0}{\Rightarrow}$ at times $\frac{2n}{3}$ in seconds

$$P_{max} = 4.71\ [W]$$

(10) $P_{ave} = \dfrac{\int_0^{n\pi} d\theta \cos^2(\theta)}{n\pi} \cdot 4.71 = \frac{1}{2} \cdot 4.71 = 2.35\ [W]$

(11) $v_y = \partial_t y$

$$= (0.15 \sin(5x - 0.075t))$$

$$= 0.15 \sin(5x - 0.075t)$$

$$\Rightarrow v_{max} = \pm 0.15 \left[\tfrac{m}{s}\right]$$

(12) $I = \dfrac{\text{Power}}{\text{Area}} = \dfrac{P}{4\pi \cdot 4.3^2} = 0.026$

$$\Rightarrow P = 6.04\ [W]$$

$$I(3.1) = \frac{P}{A} = \frac{6.04}{4\pi \cdot 3.1^2} = 0.0500 \left[\frac{W}{m^2}\right]$$

(13) $\text{Cost} = \frac{\$\,\text{per}}{\text{each}} \cdot \text{number} = \frac{\$0.11}{kWh} \cdot 6.04\,W \cdot 1h \cdot \frac{1kW}{1000W}$

$$= \$0.000664$$

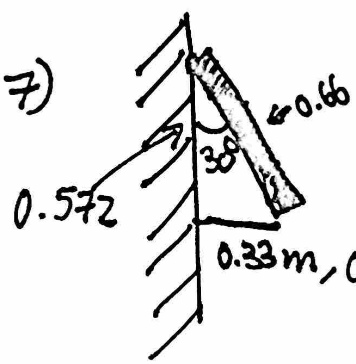
(14) $f = \frac{v}{\lambda}$, $\lambda = 2L$ for fundamental, $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{500}{\frac{0.003}{0.4}}} = 327 \left[\frac{m}{s}\right]$

$$f = \frac{327}{2 \cdot 0.4} = 409\,[Hz]$$

(15) $f = \frac{327}{2 \cdot 0.4} \cdot n \leq 10000 \rightarrow n = 24$

(16) [See 14] $\rightarrow 327 \left(\frac{m}{s}\right) \rightarrow$ very close to the speed of sound in air

(17)



$F_g = mg = 45 \cdot 9.8 = 441$

$F(0.572) = 441 \cdot 0.1666 \Rightarrow F = 127.3\,[N]$

$0.1666 = 0.33 \sin\left(\frac{\pi}{6}\right)$

0.33 m, 0.092 kg $\rightarrow \mu = \frac{0.092}{0.33} = 0.278 \rightarrow v = \sqrt{\frac{F}{\mu}}$, $f = \frac{v}{\lambda}$

$$f = 32.4\,[Hz]$$

(18) $v = 343 + 13 = 356$

$$t = \frac{l}{v} = \frac{33}{356} = 0.0927\,[s]$$

(19) I'm not sadistic, so left is on the left, right is on the right

(think of ωt as ϕ in Ch 14)

(20) Extrema lie at $\partial_x y(x) = 0$

$\hookrightarrow -\sin(x) + \cos(x) = 0 \rightarrow \sin(x) = \cos(x)$

which occurs at $x = \pi/4$

(21) Begin by noting

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2}, \quad \sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$$

So,

$$\sin(x + \pi/2) = \frac{e^{ix} e^{i\pi/2} - e^{-ix} e^{i\pi/2}}{2i}$$

$$= \frac{e^{i\pi/2}}{2i} \left(e^{ix} - e^{-ix} e^{i\pi} \right)$$

$$\text{identity} \rightarrow = \frac{1}{2} \left(e^{ix} + e^{-ix} \right)$$

(22) $\sin(-x) = -\sin(x)$, so $\sin(x)$ is odd

$\cos(-x) = \cos(x)$, so $\cos(x)$ is even

(23) Begin by evaluating the partials:

$$\partial_{xx} y = -k^2 A \cos(kx - wt)$$

$$\partial_{tt} y = -w^2 A \cos(kx - wt)$$

Equate:

$$A \cos(kx - wt) = \underbrace{\frac{w^2}{k^2}}_{\frac{1}{v^2}} \cdot A \cos(kx - wt)$$

(24) $v = \frac{w}{k} = \frac{1.5}{50} = 0.03 \left[\frac{m}{s}\right]$

(25) i. Shaking a rope takes energy — dissipation exists

ii. look at a buoy, or notice that strings don't jump off a violin

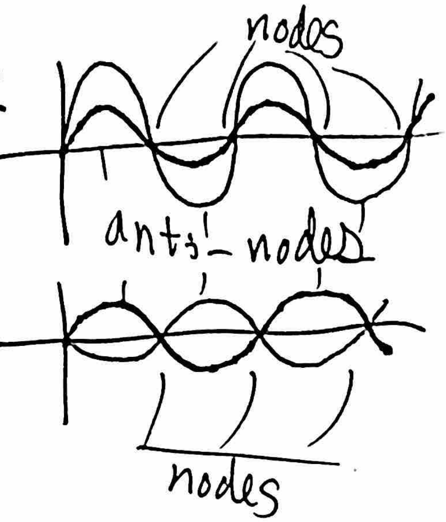
iii. (1) waves as a tide is coming in

(2) talking in a moving car

(26)

constructive

destructive



(27) constructive: phase difference = $0, 2\pi, 4\pi$

destructive: phase difference = $\pi, 3\pi, 5\pi$

(28) The medium for sound is air, while the medium for light is the electromagnetic field. However, stimulating (acoustically) solids and liquids with acoustics does produce sonoluminescence at high frequencies.

(29) Longitudinal waves would have to travel faster than c, which cannot be.

(30) $p = \hbar k$

units

p: mv → $kg \; \frac{m}{s}$

$\hbar k$: $\underbrace{J\cdot s \quad \frac{1}{m}}_{N\cdot s}$ → $\underbrace{kg\frac{m}{s^2}\cdot s}_{kg\frac{m}{s}}$

agreement