Exercise 1. Displacement

A 1 [kg] mass is released from the end of a spring stretched by 5 [cm] from its natural length of 20 [cm]. Assume that there is no dissipation during the spring's oscillations, and the spring constant is 10 [N]. What is the spring's shortest length?

(a) 25 [cm]
(b) 20 [cm]
(c) 15 [cm] ***
(d) 10 [cm]
(e) 5 [cm]
(f) 0 [cm]

Exercise 2. Velocity

A spring and mass combination has a natural frequency of 1 [Hz], and are stretched to an initial displacement of 0.07 [m]. What is the speed of the mass as it passes through the unstretched spring length?

(a) 0 [m/s]
(b) 0.07 [m/s]
(c) 0.14 [m/s]
(d) 0.22 [m/s]
(e) 0.44 [m/s] ***
(f) 0.49 [m/s]

Exercise 3. Bullwhip

Bullwhips are known for making a loud noise when they are cracked. The noise originates from the tip of the whip moving faster than the speed of sound, 343 [m/s]. Suppose a bullwhip is 3 [m] long and it moves in a circular arc, what angular frequency must the whip be moving to move faster than sound?

(a) 343 [rad/s]
(b) 114 [rad/s] ***
(c) 109 [rad/s]
(d) 18.2 [rad/s]
(e) 9.4 [rad/s]
(f) 3 [rad/s]

Exercise 4. Energies I

Suppose we are told that a weight of mass m, moves with $\boldsymbol{x}(t) = A\cos(\omega t)\exp(-\lambda t)\,\hat{\boldsymbol{x}}$ for constants A, ω, λ. Find $\boldsymbol{v}(t)$. Note that energy is not conserved.

(a) $\boldsymbol{v}(t) = A\exp(-\lambda t)[\lambda\cos(\omega t) - \omega\sin(\omega t)]\,\hat{\boldsymbol{x}}$
(b) $\boldsymbol{v}(t) = -A\exp(-\lambda t)[\lambda\cos(\omega t) + \omega\sin(\omega t)]\,\hat{\boldsymbol{x}}$ ***
(c) $\boldsymbol{v}(t) = A\omega\lambda\sin(\omega t)\exp(-\lambda t)\,\hat{\boldsymbol{x}}$
(d) $\boldsymbol{v}(t) = -A\omega\lambda\sin(\omega t)\exp(-\lambda t)\,\hat{\boldsymbol{x}}$

Exercise 5. Energies II

Suppose we are told that a weight of mass m, moves with $\boldsymbol{v}(t) = A\cos(\omega t)\exp(-\lambda t)$ for constants A, ω, λ. Find $K(t)$. Note that energy is not conserved.

(a) $K(t) = mA^2\cos^2(\omega t)\exp(-2\lambda t)/2$ ***
(b) $K(t) = mA^2\cos^2(\omega t)\exp(-2\lambda t)$
(c) $K(t) = A^2\cos^2(\omega t)\exp(-2\lambda t)/2$
(d) $K(t) = A^2\cos^2(\omega t)\exp(-2\lambda t)$

Exercise 6. Uniform Circular Motion

A particle of mass m is moving in uniform circular motion has acceleration, $\boldsymbol{a} = -v^2/\|\boldsymbol{r}\|\,\hat{\boldsymbol{r}}$. Calculate the work done in one cycle. Hint: work is $W = \int_s d\boldsymbol{s}\cdot\boldsymbol{F}$.

(a) $2\pi\|\boldsymbol{v}\|^2$
(b) $-2\pi\|\boldsymbol{v}\|^2$
(c) 0 ***
(d) cannot be determined with the information given

Exercise 7. Runner

A runner may be approximated as a system with two physical pendulums. We consider one of them. If a leg takes 90 steps per minute, weighs 12 [kg], is 0.77 [m] long, and has a moment of inertia of 2.07 [kg m^2], how far along the leg is the center of mass? Hint: For a physical pendulum, $\omega = \sqrt{mgd/I}$.

(a) 3.088 [m]
(b) 0.309 [m] ***
(c) 0.154 [m]
(d) 0.074 [m]

Exercise 8. Making a Clock I

You are asked to make a clock, and are given some string of which you cut off a length ℓ, and some weights which you select m, and you release the mass from an angle θ_0 from the vertical direction. Approximate the gravitational constant as $g \approx \pi^2$ [m/s^2]. Which of the following pendulums will have a period T equal to one second? (select all that apply)

(a) $\ell = 0.25$ [m], $m = 1$ [kg], $\theta_0 = 0°$
(b) $\ell = 0.50$ [m], $m = 1$ [kg], $\theta_0 = 0°$
(c) $\ell = 1.00$ [m], $m = 1$ [kg], $\theta_0 = 0°$
(d) $\ell = 0.25$ [m], $m = 1$ [kg], $\theta_0 = 10°$ ***
(e) $\ell = 0.50$ [m], $m = 1$ [kg], $\theta_0 = 10°$
(f) $\ell = 1.00$ [m], $m = 1$ [kg], $\theta_0 = 10°$
(g) $\ell = 0.25$ [m], $m = 1$ [kg], $\theta_0 = 22.5°$ ***
(h) $\ell = 0.50$ [m], $m = 1$ [kg], $\theta_0 = 22.5°$
(i) $\ell = 1.00$ [m], $m = 1$ [kg], $\theta_0 = 22.5°$
(j) $\ell = 0.25$ [m], $m = 1$ [kg], $\theta_0 = 45°$ ***
(k) $\ell = 0.50$ [m], $m = 1$ [kg], $\theta_0 = 45°$
(l) $\ell = 1.00$ [m], $m = 1$ [kg], $\theta_0 = 45°$

Exercise 9. Making a Clock II

You are asked to make a clock, and are given some string of which you cut off a length ℓ, and some weights which you select m, and you release the mass from an angle θ_0 from the vertical direction. Approximate the gravitational constant as $g \approx \pi^2$ [m/s^2]. Which of the following pendulums will have a period T equal to one second? (select all that apply)

(a) $\ell = 0.25$ [m], $m = 1$ [kg], $\theta_0 = 45°$ ***
(b) $\ell = 0.50$ [m], $m = 1$ [kg], $\theta_0 = 45°$
(c) $\ell = 1.00$ [m], $m = 1$ [kg], $\theta_0 = 45°$
(d) $\ell = 0.25$ [m], $m = 2$ [kg], $\theta_0 = 45°$ ***
(e) $\ell = 0.50$ [m], $m = 2$ [kg], $\theta_0 = 45°$
(f) $\ell = 1.00$ [m], $m = 2$ [kg], $\theta_0 = 45°$
(g) $\ell = 0.25$ [m], $m = 4$ [kg], $\theta_0 = 45°$ ***
(h) $\ell = 0.50$ [m], $m = 4$ [kg], $\theta_0 = 45°$
(i) $\ell = 1.00$ [m], $m = 4$ [kg], $\theta_0 = 45°$
(j) $\ell = 0.25$ [m], $m = 8$ [kg], $\theta_0 = 45°$ ***
(k) $\ell = 0.50$ [m], $m = 8$ [kg], $\theta_0 = 45°$
(l) $\ell = 1.00$ [m], $m = 8$ [kg], $\theta_0 = 45°$

Exercise 10. Amplitude of a Wave

We are told a motion is $\boldsymbol{x}(t) = [a\cos(\omega t) + b\sin(\omega t)]\hat{\boldsymbol{x}}$. We want to express this as a single cosine wave of amplitude A with a phase shift ϕ: $\boldsymbol{x}(t) = A\cos(\omega t + \phi)$. What is A?

(a) $A = a + b$
(b) $A = |a| + |b|$
(c) $A = \sqrt{a^2 + b^2}$ ***
(d) $A = \sqrt{a^2 + b^2 - 2ab\cos(\omega t)}$

Exercise 11. Velocity

Show that the maximum velocity a mass in a spring mass system reaches is ωA. Hint: $\boldsymbol{x}(t) = A\cos(\omega t + \phi)\hat{\boldsymbol{x}}$.

Exercise 12. Kinetic Energy

From $\boldsymbol{p} = m\boldsymbol{v}$ and $K = m\|\boldsymbol{v}\|^2/2$, show $K = \|\boldsymbol{p}\|^2/2m$.

Exercise 13. Double Angle

Show that $2\sin(ax)\cos(ax) = \sin(2ax)$. Hint: Euler's formula is $\exp(i\theta) = \cos(\theta) + i\sin(\theta)$.

Exercise 14. Another Runner

A runner's right leg weighs 15.4 [kg], and is approximated by a cylinder of length 0.83 [m]. Find the rotational kinetic energy for a runner moving at 85 steps per leg per minute.

Hint: the formula for moment of inertia of a thin cylinder oscillating about its end is $I = ml^2/3$, $\boldsymbol{L} = \boldsymbol{I} \times \boldsymbol{\omega}$, and $K = \boldsymbol{\omega} \cdot \boldsymbol{L}/2$.

Exercise 15. Hanging Oscillator

A spring is screwed into the ceiling, and it hangs down a distance l_0. A mass m is attached to the end of the spring, and the spring hangs down a distance l.

i. Draw a picture/free body diagram.

ii. If the spring has a spring constant of $k = 5$ [N] and an unstretched length of 10 [cm], and the mass has a weight of 200 [g], find $l - l_0$. Let $g = 9.8$ [m/s^2].

iii. The mass is then lifted to l_0 and then released. Describe the subsequent oscillations, $\boldsymbol{x}(t)$. Hint: work symbolically and then substitute numbers at the end.

Exercise 16. Anharmonic Oscillator

Consider an anharmonic spring that exerts a restoring force of $\boldsymbol{F} = -k\boldsymbol{x} - a\boldsymbol{x}^3$ on a weight of mass m. Using the characteristic equation, find the position as a function of time, $x(t)$ for an initial displacement $x(0) = 1$, and speed $\dot{x}(0) = 0$.

Exercise 17. Damped Oscillator

Consider a damped spring that exerts a restoring force of $\boldsymbol{F} = -k\boldsymbol{x} - b\boldsymbol{v}$ on a weight of mass m. Using the characteristic equation, find the position as a function of time, $x(t)$ for an initial displacement $x(0) = 0$, and speed $\dot{x}(0) = 1$.

Exercise 18. Resonant Frequency I

An oscillator may also be driven, for example a bridge by traffic. This additional force may be periodic and expressed as a sinusoid $\boldsymbol{F}_d = \|\boldsymbol{F_d}\| \cos(\omega_d t))\ \hat{\boldsymbol{x}}$.

Through some arduous algebra, it may be found that:

$$A = \frac{\|\boldsymbol{F_d}\|}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

What happens in the limit $\omega_d \to \omega$? What if b is small? Engineers have control over k, b, and specify m for bridges. Why is it important to choose ω carefully?

Exercise 19. Resonant Frequency II

Suppose that a marching band is marching across a bridge with footfalls 120 times per minute.

i. If we assume there are 40 band members each of mass 60 [kg], and their entire weight is pressed down with each step, what is the magnitude of the driving force? You may assume that the gravitational constant is $g = 10$ [m/s^2].

ii. If we assume that the bridge weighs 10^5 [kg] and has a spring constant of 10^6 [N], and a damping constant of $b = 10^4$ [kg/s], what is the maximum amplitude the bridge reaches?

Exercise 20. Feedback

My goal is that the PLF sessions will be as useful as possible. I believe that this means shifting from lectures, discussion sections, and homework to focus on tripping points and problem solving techniques. To do this, my plan is to focus PLF sessions around worksheets, with plenty of room for questions and for all of us to learn. *In doing this, the PLF sessions do not cover all of the material from lecture.* While I do my best to be complete and accurate, please keep in mind that I am not the professor or a TA. In this worksheet, I have included a variety of exercise types: numerical exercise, algebraic exercises, and derivations. These are broken into multiple choice, short answer, long answer, and challenge problems. Let me know how this is working for you! What do you want to see more of? Less of? Things we haven't covered but think we should?

(a) this format is good!

(b) tell me, or message me!