Worksheet 1 Solutions

(1)

$$x(t) = \cancel{A}\, l + A\cos(\omega t)$$

$$x(t) = 20 + 5\cos(\omega t)$$

$$\min(x(t)) = 20 + 5\cos(\pi)$$

$$= 20 - 5$$

$$= 15 \text{ [cm]}$$

(2)

$$x(t) = A\cos(\omega t)$$

$\omega = 2\pi f$

$$= 0.07\cos(2\pi t)$$

$$v(t) = -2\pi \cdot 0.07 \sin(2\pi t)$$

$t = 1/4$ when passes through

$$= -2\pi \cdot 0.07 \sin(\pi/2)$$

$$= -2\pi \cdot 0.07$$

$$\|v(t)\| = 0.44 \text{ [m]}$$

(3)

$$v = \frac{\text{circumference}}{\text{Period}} = \frac{2\pi r}{T} = 2\pi r \cdot f = r\omega$$

$$\omega = \frac{v}{r} = \frac{343}{3} = 114.3 \left[\frac{\text{rad}}{\text{s}}\right]$$

(4)

$$x(t) = A\cos(\omega t)e^{-\lambda t}\,\hat{x}$$

$$v(t) = \left[-A\omega\sin(\omega t)e^{-\lambda t} - A\lambda\cos(\omega t)e^{-\lambda t}\right]\hat{x}$$

$$= -Ae^{-\lambda t}\left[\lambda\cos(\omega t) + \omega\sin(\omega t)\right]\hat{x}$$

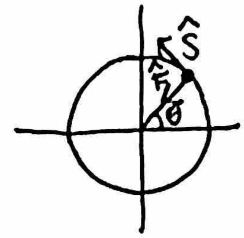
(5)

$$v(t) = A\cos(\omega t)e^{-\lambda t}\,\hat{x}$$

$$K = \frac{mv^2}{2}$$

$$K(t) = \frac{mA^2\cos^2(\omega t)e^{-2\lambda t}}{2}$$

(6) $\vec{a} = -\dfrac{v^2}{\|r\|}\hat{r}$, $F = ma$



$$W = \int_S ds \cdot F = \int_0^{2\pi} d\theta \; F\cos\left(\tfrac{\pi}{2}\right)$$

angle between $\vec{F}$ and $\hat{s}$

$A \cdot B = AB\cos(\varphi)$

$$= \int_0^{2\pi} d\theta \; 0$$

$$= 0$$

hence motion is "uniform" circular

(7) $f = \dfrac{90}{60} = 1.5 \rightarrow \omega = 2\pi f = 3\pi$

$$\omega = \sqrt{\frac{mgd}{I}} \Rightarrow d = \frac{I\omega^2}{mg} = \frac{2.07 \cdot (3\pi)^2}{12 \cdot 9.8} = 1.56\,[m]$$

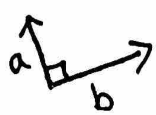
this shows the leg is <u>not</u> a S.H.O.

(8)(9) $T = 2\pi\sqrt{\dfrac{l}{g}} \longrightarrow m, \theta$ are irrelevant (unless $\theta = 0$)!

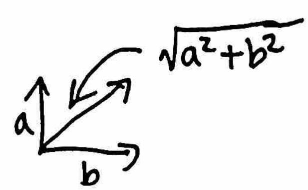
if $g = \pi^2 \longrightarrow T = 2\sqrt{l} \longrightarrow$ one second period

$\longrightarrow l = \frac{1}{4} = 0.25\,[m]$

(10) visualize as phasors: (rotate the axis at ω angular frequency, while keeping the angle fixed)



combining by vector addition



(11) $x(t) = A\cos(\omega t + \phi)$

$v(t) = -A\omega \underbrace{\sin(\omega t + \phi)}_{\text{bounded by } [-1,1]} \longrightarrow$ max is $v(t) = A\omega$

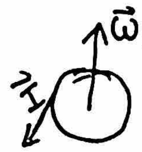
(12) $$k = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$$

(13) $$e^{i\theta} = \cos(\theta) + i\sin(\theta) \longrightarrow \cos(\theta) = \frac{e^{i\theta}+e^{-i\theta}}{2}$$
$$\sin(\theta) = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

$$2\sin(ax)\cos(ax) = 2\left[\frac{e^{iax}-e^{-iax}}{2i}\right]\left[\frac{e^{iax}+e^{-iax}}{2}\right]$$
$$= \frac{1}{2i}\left[e^{2iax}+1-1-e^{-2iax}\right]$$
$$= \sin(2ax)$$

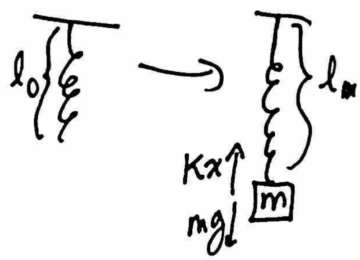
(14) This is not a very good question because $\vec{\omega}$ is a non-linear function of time

What I was going for was at the <u>mean</u> ω, what would this be, which works out as $\omega = \frac{85}{60}\cdot 2\pi = 8.90$, $I = \frac{ml^2}{3} = 3.54$



$$k = \frac{\omega \cdot I \times \omega}{2} = \frac{I\omega^2\cos(0)\sin\frac{\pi}{2}}{2}$$
$$= \frac{I\omega^2}{2} = 140.9\,[J]$$

(15)



$\Delta x \equiv l - l_0 \leftarrow$ defn.

$mg = kx \leftarrow$ at rest

$$\Delta x = \frac{mg}{k} = \frac{0.2\cdot 9.8}{5} = 0.392\,[m]$$

$$x(t) = l - \Delta x\cos(\overset{\sqrt{k/m}}{\omega} t + \phi)$$ ϕ here

$$= 0.1 + 0.392 - 0.392\cos(5t)$$

$$= 0.492 - 0.392\cos(5t)$$

(16) This problem is not solvable using analytic techiques. Alas.

(17) See message in the groupme.

(18) If $\omega_d \to \omega$, then $\omega_d = \sqrt{\frac{k}{m}}$, so $A = \dfrac{\|F_a\|}{\sqrt{b^2 k/m}}$

If $b \to 0$ as well, then $A \to \infty$

Bad things happen if $A \to \infty$. We want to avoid this.

(19) $\omega_d = \dfrac{120}{60} \cdot 2\pi = 4\pi$

$F_d = mg = 40 \cdot 60 \cdot 10 = 2400$ [N]

$$A = \frac{2400}{\sqrt{\left(10^6 - 10^5 \cdot (4\pi)^2\right)^2 + (4\pi \cdot 10^4)^2}} = 0.000162 \text{ [m]}$$

So this bridge is well designed to handle this load.