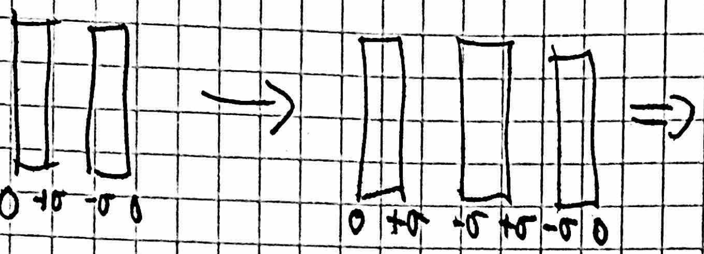
MT 2B Answers

(1) I (a) $U = qV = q\frac{kQ}{r}$

(b) $U = qV = q\frac{(k(Q-Q))}{r} = 0$

but electric field is not 0

II It stays constant if the slab is thin



$\Rightarrow$ Electric field in vaccum is σ / ε_0

$E = -\nabla V = \frac{\sigma}{\varepsilon_0} \Rightarrow \nabla V$ is linear in distance for both cases, or any number of (thin) slabs

III Typo! the insulator has a surface charge of $3Q$, not $9Q$

Metal surfaces will have equal potential, so:

$V_{c_1} = V_{c_2} = V_I$

$V_I = \frac{3kq}{3a} = kq \Rightarrow Q_{c_1} = 1Q$

$Q_{c_2} = 2Q$

$Q_I = 3Q$

Problem 2

A long coaxial cable carries a uniform volume charge density ρ on the inner cylinder of radius a, and a uniform surface charge density σ on the outer cylinder of radius b. This surface charge is negative, and is of just the right magnitude that the cable as a whole is electrically neutral.

- Find the electric field for $s < a$
- Find the electric field for $a < s < b$
- Find the electric field for $b < s$
- Plot $|\boldsymbol{E}|$ as a function of s

For $s < a$, begin with Gauss's Law:

$$\oint_S \boldsymbol{E} \cdot \hat{\boldsymbol{n}}\, da = \frac{q}{\varepsilon_0} = \frac{\rho V}{\varepsilon_0} = \frac{\rho \cdot \pi r^2 l}{\varepsilon_0}$$

$$\oint_S \boldsymbol{E} \cdot \hat{\boldsymbol{n}}\, da = E \cdot 2\pi r l$$

$$\boldsymbol{E} = \frac{\rho \cdot \pi r^2 l}{2\varepsilon_0 \pi r l}\,\hat{\boldsymbol{r}} = \frac{\rho \cdot r}{2\varepsilon_0}\,\hat{\boldsymbol{r}}$$

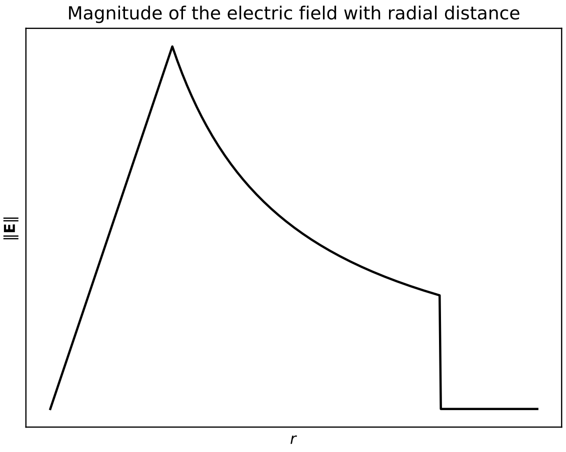
For $a < s < b$, begin with Gauss's Law:

$$\oint_S \boldsymbol{E} \cdot \hat{\boldsymbol{n}}\, da = \frac{q}{\varepsilon_0} = \frac{\rho V}{\varepsilon_0} = \frac{\rho \cdot \pi a^2 l}{\varepsilon_0}$$

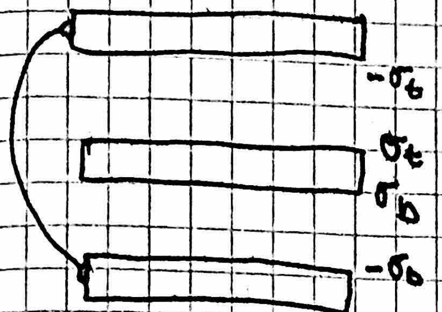
$$\oint_S \boldsymbol{E} \cdot \hat{\boldsymbol{n}}\, da = E \cdot 2\pi r l$$

$$\boldsymbol{E} = \frac{\rho \cdot \pi a^2 l}{\varepsilon_0 2\pi r l}\,\hat{\boldsymbol{r}} = \frac{\rho a^2}{2\varepsilon_0 r}\,\hat{\boldsymbol{r}}$$

For $b < s$, note that there is no enclosed charge, so $\boldsymbol{E} = 0\,\hat{\boldsymbol{n}}$.



(3)



(a) $V(B) - V(A) = \frac{\sigma_t}{\varepsilon_0} \cdot 3$ ← distance

$V(C) - V(B) = \frac{\sigma_b}{\varepsilon_0} \cdot 7$

(b) $V(C) = V(A)$ because of wire, so

$V(C) - V(B) = -(V(B) - V(A))$

(c)
$$\left.\begin{array}{l} \frac{3\sigma_t}{\cancel{\varepsilon_0}} = -\frac{7\sigma_b}{\cancel{\varepsilon_0}} \\ \sigma_t + \sigma_b = 1 \end{array}\right\} \Rightarrow \begin{array}{l} \sigma_t = +\frac{7}{4} \left[\frac{\mu C}{cm^2}\right] \\ \sigma_b = -\frac{3}{4} \left[\frac{\mu C}{cm^2}\right] \end{array}$$