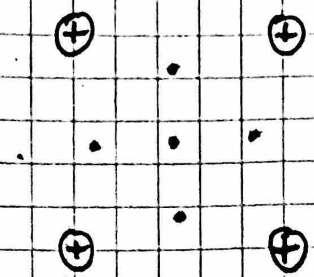
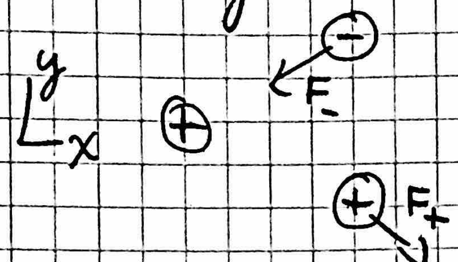
MT2A Answers

(1) I → 5



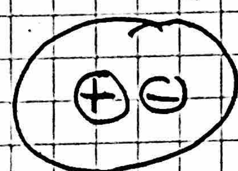
II → $-\hat{y}$



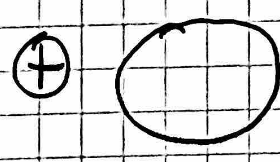
$F_{total} = F_- + F_+$ is in $-\hat{y}$

Note: E-field is non-uniform

III (a) False



(b) False

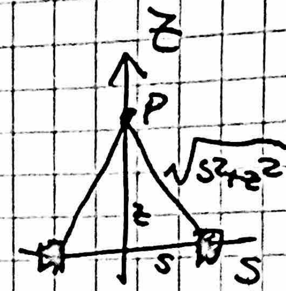


(c) Kinda vague, but going for $\underbrace{\nabla \cdot E = \frac{\rho}{\varepsilon_0}}_{\Phi = \frac{Q}{\varepsilon_0}}$

if so, then True

(can be false <u>on</u> a surface)

(2) (A) The potential is:



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho}{r}\, dV$$

$$= \frac{\sigma}{4\pi\varepsilon_0}\int \frac{dV}{\sqrt{s^2+z^2}} = \frac{\sigma}{4\pi\varepsilon_0}\int_a^b \frac{2\pi s}{\sqrt{s^2+z^2}}\, ds$$

= (u-substitution with $u = s^2+z^2$, or given the integral)

$$= \frac{\sigma}{4\varepsilon_0}\, 2\sqrt{u}\,\Big|_{a^2+z^2}^{b^2+z^2} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{b^2+z^2} - \sqrt{a^2+z^2}\right)$$

(B) $E = -\nabla V = -\partial_z V\,\hat{z}$

$= \cdots$ calculus $\cdots$

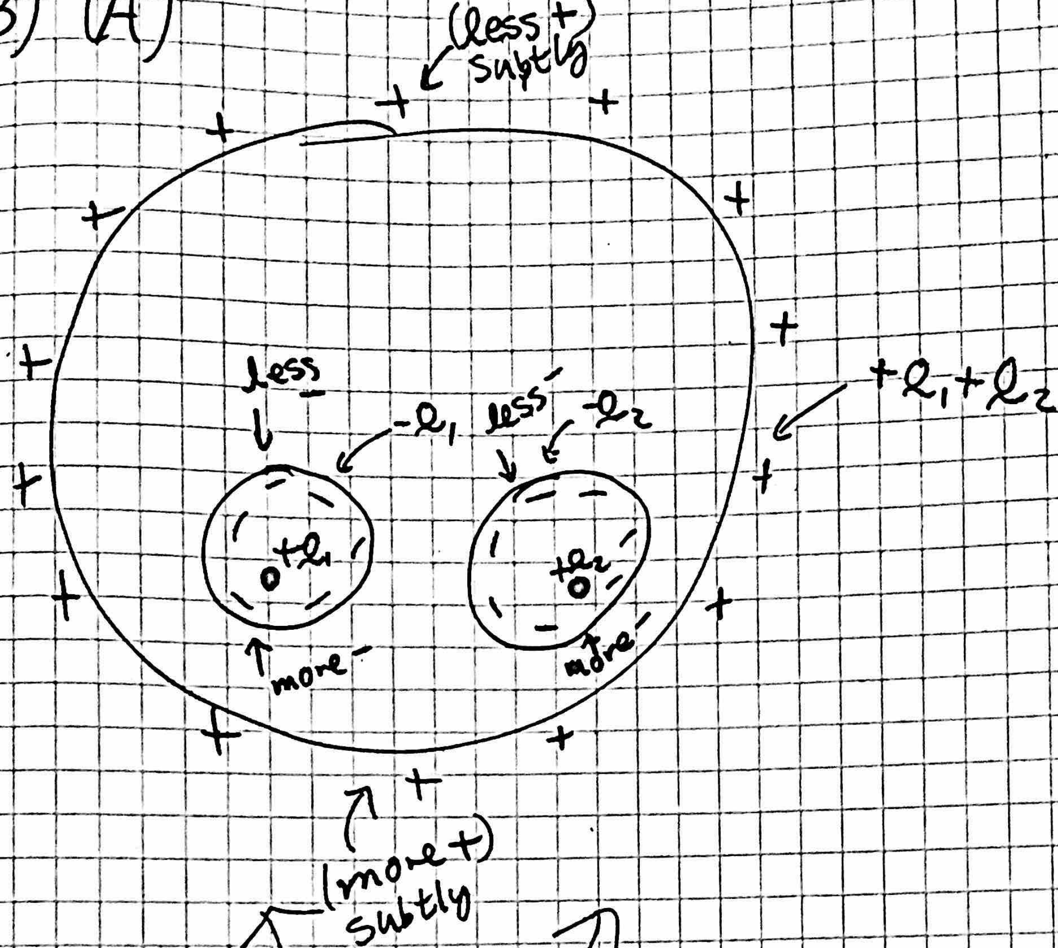
$$= -\frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{z^2+b^2}} - \frac{z}{\sqrt{z^2+a^2}}\right)\hat{z}$$

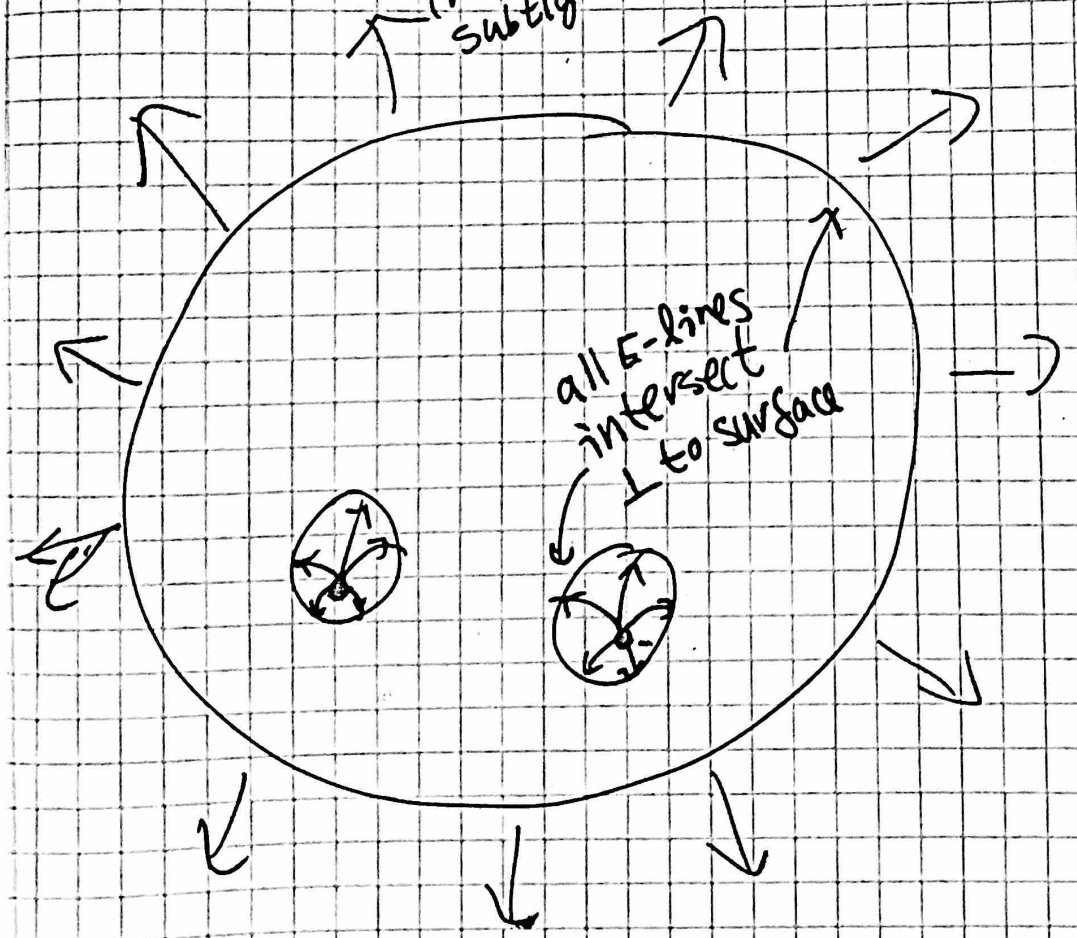
as expected by symetry

(C) $a \to 0,\ b \to R$

$$E = -\frac{\sigma}{2\varepsilon_0} z\left(\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{\sqrt{z^2}}\right)\hat{z}$$

(3) (A)


(less + subtly)
less -
$-e_1$
less -
$-e_2$
$+e_1$
$+e_2$
$+e_1 + e_2$
more -
more -
(more + subtly)


all E-lines intersect ⊥ to surface

(3)(B) Since $r \gg r_{conductor}$, to good approximation, we may assume that the charge dist. looks like a point charge collection. As such two answers (that are simple) are ok:

First: $F = qE = \frac{kq(Q_1+Q_2)}{r^2}$

Second: $F = qE = q(E_1+E_2) = kq\left(\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2}\right)$

These two are asymptotically equal (to the true value) as r becomes large as you could verify from the law of cosines.

(4) (a) $q = \int dq = \int_{R_0}^{R_1} \overbrace{\rho_0 \left(1 - \frac{r^3}{R_1^3}\right)}^{\rho} \overbrace{4\pi r^2 dr}^{dV}$

$= 4\pi\rho_0 \int_{R_0}^{R_1} dr \left(r^2 - \frac{r^5}{R_1^3}\right)$

$= 4\pi\rho_0 \left(\frac{R_1^3 - R_0^3}{3} - \frac{R_1^6 - R_0^6}{6R_1^3}\right)$

$= \frac{4\pi\rho_0}{3}\left(\frac{R_1^3}{2} + \frac{R_0^6}{2R_1^3} - R_0^3\right)$

(b) inner $\leftarrow$ 0 net charge inside conductor

$\hookrightarrow -\left(Q + \frac{4\pi\rho_0}{3}\left(R_1^3/2 + R_0^6/2R_1^3 - R_0^3\right)\right)$

outer $\rightarrow$ 0 (since grounded, so $V = 0 = \frac{kQ_{enc}}{r}$ here)

(d) is not grounded

a $\rightarrow$ the same

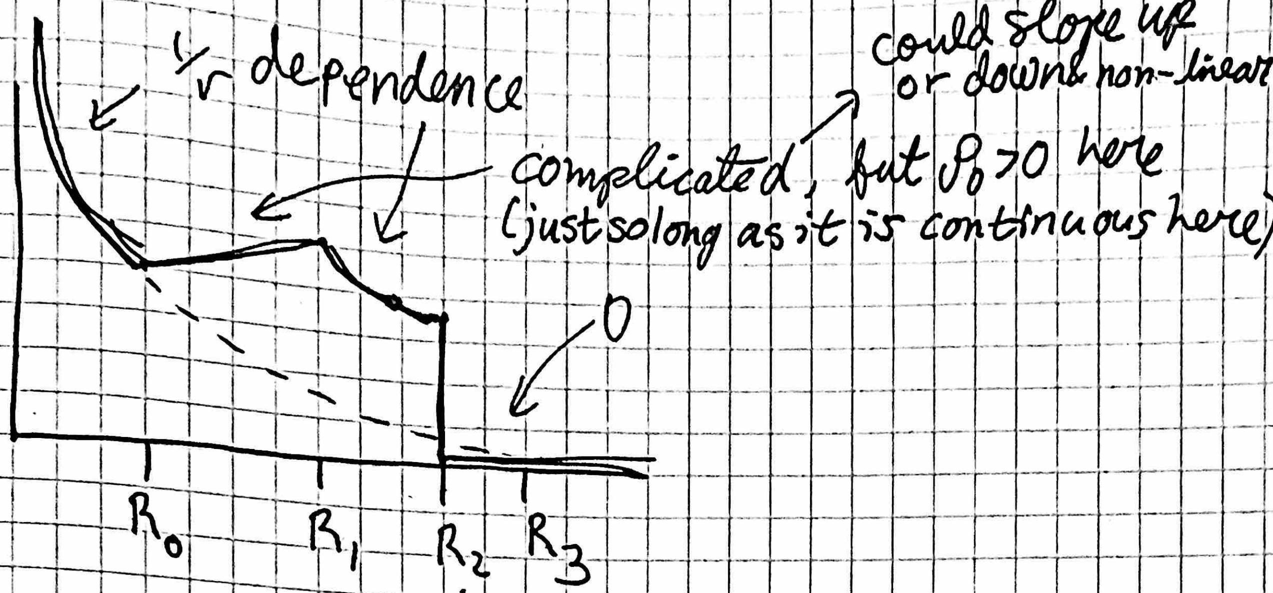
b $\rightarrow$ inner $\rightarrow$ the same

outer $\rightarrow$ $-$(inner) so charge conserved

(c) Find $q(r)$, then use Gauss's Law

$0 < r < R_0$	$+Q$
$R_0 \leq r \leq R_1$	$Q + \int_{R_0}^{r}$ "integral from a"
	$Q + \frac{4\pi\rho_0}{3}\left(r^3 - R_0^3 - \frac{r^6 - R_0^6}{2R_1^3}\right)$
$R_1 < r < R_2$	$Q + \frac{4\pi\rho_0}{3}\left(R_1^3 - R_0^3 - \frac{R_1^6 - R_0^6}{2R_1^3}\right)$
$R_2 < r$	0

(c)



Electric field is, by symmetry

$$\vec{E} = \frac{q_{enc}}{\varepsilon_0 A}\hat{r} = \frac{q(r)}{\varepsilon_0 4\pi r^2}\hat{r}$$

Where $q(r)$ is as on the last page