Electric Charge$^{(21)}$

The force experienced by a charge q in an electric field E is:

$$\boldsymbol{F} = q\boldsymbol{E} \tag{1}$$

Charges create electric fields. For a point charge[2]:

$$\boldsymbol{E}_{\text{point charge}} = \frac{kq}{r^2}\hat{\boldsymbol{r}} \tag{2}$$

For a more general distribution of charge:

$$\boldsymbol{E} = \int_{\text{charge}} \frac{k\ dq}{r^2}\hat{\boldsymbol{r}} \tag{3}$$

Electric fields add with *vector* addition:

$$\boldsymbol{E}_{\text{total}} = \sum_i \boldsymbol{E}_i \tag{4}$$

The component of the electric field in one direction $\hat{\boldsymbol{z}}$ is[3]:

$$\boldsymbol{E}_z = (\boldsymbol{E} \cdot \hat{\boldsymbol{z}})\hat{\boldsymbol{z}} = (\|\boldsymbol{E}\|\hat{\boldsymbol{E}} \cdot \hat{\boldsymbol{z}})\hat{\boldsymbol{z}} = \|\boldsymbol{E}\|\cos(\alpha)\hat{\boldsymbol{z}} \tag{5}$$

Potentials$^{(23)}$

The potential energy U is the charge q times the potential V:

$$U = qV \tag{6}$$

The electric force is conservative so $\boldsymbol{F} = -\nabla U$, and from Eq. 1, we find:

$$\boldsymbol{F} = -\nabla U \implies \boldsymbol{E} = -\nabla V \iff V = \int_r^\infty \boldsymbol{E} \cdot d\boldsymbol{r} \tag{7}$$

Integrating, we find the potential of a point charge is:

$$V_{\text{point charge}} = \frac{kq}{r} \tag{8}$$

Similarly, for a more general distribution of charge:

$$V = \int_{\text{charge}} \frac{k\ dq}{r} \tag{9}$$

Electric potentials add with *scalar* addition:

$$V_{\text{total}} = \sum_i V_i \tag{10}$$

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[2] $k = 1/4\pi\epsilon_0$ is a constant equal to 8.988×10^9 [N m^2/C^2]

[3] Example 21.9 in the 13$^{\text{th}}$ edition is a good demonstration of where this is useful, e.g. if you have equal amounts of charge on both sides of the point of consideration

Gauss's Law$^{(22)}$

Gauss's Law is a key topic on the midterm[4]:

$$\Phi = \frac{Q_{\text{enclosed}}}{\epsilon_0} \tag{11}$$

The flux is:

$$\Phi \equiv \int_A d\boldsymbol{A} \cdot \boldsymbol{E} \tag{12}$$

The enclosed charge is:

$$\frac{Q_{\text{enclosed}}}{\epsilon_0} = \frac{1}{\epsilon_0}\int_V dq = \frac{1}{\epsilon_0}\int_V \rho dV \tag{13}$$

If $\boldsymbol{E}$ is parallel to $\boldsymbol{A}$ everywhere on the Gaussian surface ($d\boldsymbol{A} \cdot \boldsymbol{E} = 1$), and the charge distribution is symmetric ($\|\boldsymbol{E}\|$ is uniform on the surface), *then* we have the very nice relation:

$$\|\boldsymbol{E}\| = \frac{Q_{\text{enclosed}}}{\epsilon_0 A} \tag{14}$$

Calculus Review

Some differential charge elements, $dq \sim \lambda dx \sim \sigma dA \sim \rho dV$ (these might be good to put on your index card):

$$\text{line} \quad \lambda(x)dx \tag{15}$$

$$\text{strip} \quad \sigma(x)wdx \tag{16}$$

$$\text{disc} \quad \sigma(r)2\pi r dr \tag{17}$$

$$\text{cylinder} \quad \rho(r)2\pi r h dr \tag{18}$$

$$\text{sphere} \quad \rho(r)4\pi r^2 dr \tag{19}$$

~~Integrals to know (in order of importance):~~ If you need an integral, it will be provided.

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}} \tag{20}$$

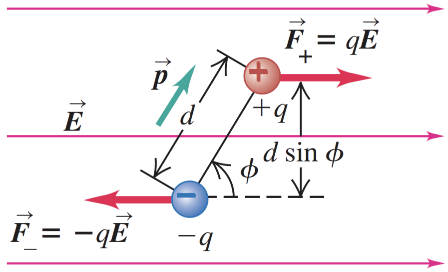
$$\int \frac{x\ dx}{(x^2+a^2)^{3/2}} = -\frac{1}{(x^2+a^2)^{1/2}} \tag{21}$$

$$\int \frac{dx}{(x^2+a^2)^{1/2}} = \ln|x + (x^2+a^2)^{1/2}| \tag{22}$$

$$\int \frac{x^2\ dx}{(x^2+a^2)^{3/2}} = -\frac{x}{(x^2+a^2)^{1/2}} + \ln|x + (x^2+a^2)^{1/2}| \tag{23}$$

[4] Gauss's Law is an application of Stokes Theorem from Math 32B

~~Special Topics: Dipoles~~[21] Dipoles will not be covered on the midterm.



The dipole moment is for charges $+q$ and $-q$ separated by d:

$$\boldsymbol{p} = q\boldsymbol{d} \iff p = qd \tag{24}$$

The electric field works to align the dipole with the field:

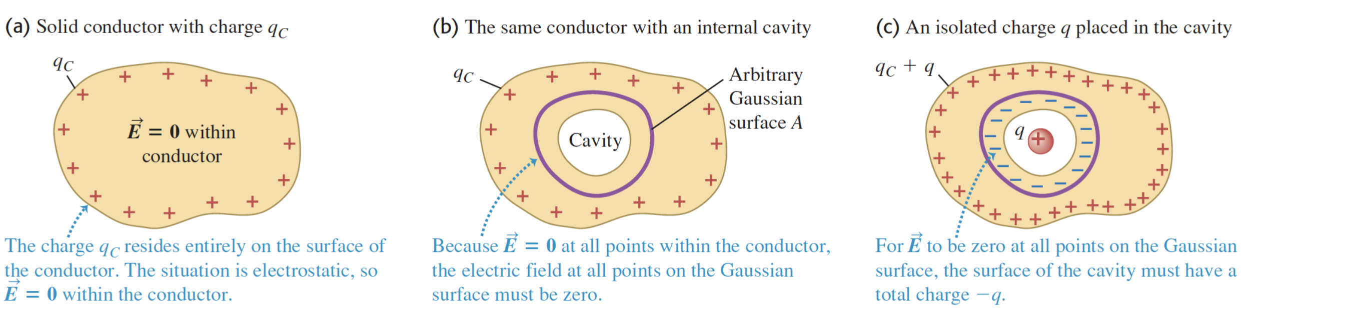
$$\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{E} \iff \tau = pE\sin(\phi) \tag{25}$$

The potential energy of a dipole in a field is:

$$U = -\boldsymbol{p} \cdot \boldsymbol{E} = -pE\cos(\phi) \tag{26}$$

Special Topics: Conductors and Insulators[22]

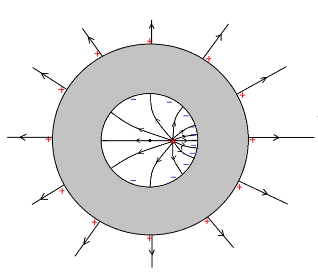
22.23 Finding the electric field within a charged conductor.



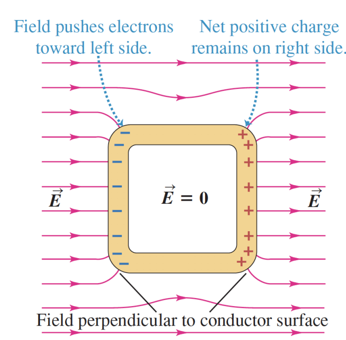
Conductors have some properties:

- There is no net charge inside a conductor: all charges reside on the surface
- The potential on any one surface of a conductor is the same (so field lines are $\perp$ to the surface)

(left) Electric field lines of a point charge in a cavity of a conductor.



(right) There is no electric field in a cavity within a conductor subjected to an electric field.



Insulators have fixed charged distributions, do not share the above properties with conductors.