Problem 1. (25 Points)

A cylindrical bucket, open at the top to the air, is filled with 30.0 [cm] of water and is 10.0 [cm] in diameter. Note that $\rho_{\text{water}} = 1000$ [kg/m^3], and 1 [atm] $= 1.01 \times 10^5$ [Pa].

(a) How much is the pressure at the bottom of the bucket? (5 points)

(b) If we place a lead ball with mass 11 [kg] inside the water and then measure the weight of the ball using a balance under the water, how much is the reading? Note $\rho_{\text{lead}} = 1100$ [kg/m^3]. (10 points)

(c) We take out the lead ball and secure the lower half of this bucket in an atmosphere of 1×10^5 [Pa] and then drill a cricular hole with a cross section of 1.25 [cm^2] in the left bottom of the bucket. Water flows into the bucket at a rate of $2.5 \times 10^{-2.5}$ [m^3/s].

(1) What flow speed out of the hole keeps the water level constant? (5 points)

(2) How high will the water in the bucket rise? (5 points)

Problem 2. (20 Points)

A spring with spring constant k has a displacement x under a force F.

(a) How much is the displacement of 1/3 of the spring experience? (3 points)

(b) If this spring is cut into three equal width pieces, what is the spring constant of each piece? (10 points)

(c) If we hang three springs with spring constant k in parallel and attach a mass m to the end of them, what is the oscillation frequency?

Problem 3. (15 Points)

An object is attached to a spring with one end fixed to the wall and moves frictionlessly on the ground. When it is displaced by 0.6 [m] to the right of its equilibrium position, it has a velocity of 2.2 [m/s] to the right and an acceleration of 8.4 [m/s^2] to the left.

(a) How much is the spring constant? (5 points)

(b) How much is the amplitude? (5 points)

(c) How much further from this point will the object move before it stops and starts to move back to the left? (5 points)

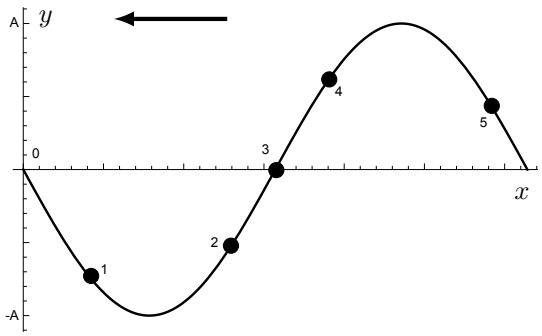
Problem 4. (20 Points)

A transverse wave is propagating along the $-x$ direction on a string with a speed of 2 [m/s], and a frequency of 10 [Hz]. At $t = 0$, the displacement at $x = 0$ is at its maximum negative value $A = 0.2$ [m].

(a) Write down the wave function. (10 points)

(b) Plot $v_y(x, t)$ at $t = 0$. (5 points)

(c) At a certain time, the wave looks like the one in the figure. Please draw the direction of the velocity and acceleration on points $1, 2, 3, 4$ and 5. (5 points)



Problem 5. (20 Points)

You are designing a two-string instrument with metal strings 35 [cm] long. Both strings are under the same tension. S_1 has a mass of 7 [g] and produces middle C in its fundamental mode.

(a) Plot the first three normal modes of S_1 and label the nodal point(s) as N and antinodal points as A. (5 points)

(b) What is the tension on S_1? (5 points)

(c) What should be the mass of S_2 so that it will produce $A^\sharp$ as its second harmonic? (5 points)

(d) To extend the range of your instrument, you include a fret located just under the strings, but not normally touching them. What is the x so that when you press S_1 tightly against it, this string will produce $C^\sharp$ in its second overtone?

Note: C = 262 [Hz], $C^\sharp = 277$ [Hz], and $A^\sharp = 466$ [Hz]. Note: converting to angular frequency gives the grade cutoffs.