Physics 180C: Critical Field of Superconducting Indium and Lead

Spenser Talkington,* Madeline Gullen, Joshua Lee, Teresa Le, and Stuart Brown
Department of Physics and Astronomy, University of California at Los Angeles,
405 Hilgard Avenue, Los Angeles, California 90095, USA
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Superconductors are perfect diamagnets up to a critical field where they become normal metals. This means that if a superconductor is the core of an inductor, then as the temperature and field are varied, the critical field as a function of temperature can be determined by the change in resonant frequency of an LC circuit. We studied samples of indium and lead at a variety of temperatures and field strengths and observed that both indium and lead samples experienced jumps in frequency consistent with our expectations for superconductors. We then used the BCS model of weak-coupling superconductors to determine the critical temperatures and fields of indium and lead as 3.261±0.043 K, 0.01989±0.00051 T and 7.14±0.12 K, 0.07531±0.0014 T respectively. Additionally, we show that within the BCS theory, indium is a weak-coupling superconductor and lead is a strong-coupling superconductor.

INTRODUCTION

Many metals undergo a sharp transition between a finite resistance and zero resistance at a low temperature known as the critical temperature. This was first observed in 1911 by Kamerlingh-Onnes, where the resistance of a mercury sample dropped from 0.12 Ω just above 4.2 K to an immeasurably small resistance just below 4.2 K [1]. The discovery of superconductivity was followed in 1933 by Meissner and Ochsenfeld's discovery that these superconductors acted as perfect diamagnets and expel all magnetic fields up to a critical field where the superconductivity breaks down [2].

A first phenomenological theory was constructed by Ginzburg and Landau in 1950 to describe the critical behavior at the superconducting transition [3]. In 1957, Bardeen, Cooper, and Schrieffer presented their microscopic theory of electron-phonon pairing into a bosonic condensate at low temperatures [4]. While the BCS Model described all superconductors known at the time, the Type I superconductors, it predicted that no superconducting condensate could exist above ~ 20 K. In the 1980s, Type II superconductors with up critical temperatures of over 100 K were discovered [5, 6].

Here we study two samples of the Type I superconductors: Indium and Lead, and relate the measured behavior to BCS Theory. In particular, BCS theory predicts a sharp transition at the critical field H_c from superconducting to normal. In weak-coupling BCS theory, H_c is expected to be 0 at the critical temperature T_c, and reach a maximal value at $T = 0$ in accordance with:

$$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right] \tag{1}$$

Using the jump in resonant frequency of an LC circuit at the superconducting transtion, we determined the critical temperatures and fields of Indium and Lead as 3.261 ± 0.043 K, 0.01989 ± 0.00051 T and 7.14 ± 0.12 K, 0.07531 ± 0.0014 T respectively.

BCS Theory

The thermodynamic ground state of most normal metals is superconductivity. This is because "the interaction between electrons resulting from virtual exchange of phonons (may) dominate the repulsive screened values of the repulsive screened Coulomb interaction," leading to a lower energy state [4]. This state is the condensation of bosonic Cooper pairs into a collective ground state at energy Δ below the Fermi energy. Cooper pairs are spin-0 (opposed spins), but a strong magnetic field may make it favorable for the individual electronic spins to align with the field with energy $-\mu H$ rather than form a condensate. This competition will determine whether the actual state is superconducting or normal at a given field and temperature. Now, we consider the first law of thermodynamics for this magnetic system:

$$dE = TdS - MdH \tag{2}$$

The difference in entropies between phases at T_c is:

$$S_N - S_S = -H_c(T)\,\frac{\partial H_c(T)}{\partial T} \tag{3}$$

Correspondingly the difference in specific heats are:

$$C_N - C_S = -T_c\left(\frac{\partial H_c}{\partial T}\right)^2_{T_c} \tag{4}$$

This indicates that at $H_c = 0$, the transition is second order, involving the divergence of a second derivative, while with $H_c \neq 0$, the transition is first order, involving the discontinuity of a first derivative.

In weak-coupling BCS theory, where $kT_c \ll \hbar\omega_{\text{phonon}}$, the superconducting energy gap $2\Delta_0$ at $T = 0$ is given in terms of the Euler-Mascheroni constant γ:

$$\Delta_0 = \frac{\pi}{e^\gamma}\,kT_c \approx 1.76388\,kT_c \tag{5}$$

The difference in free energy between the normal and superconducting states is given by the density of states at the Fermi Energy:

$$E_N - E_S = \frac{1}{2} D(\epsilon_F) \Delta_0^2 \tag{6}$$

Which is also:

$$E_N - E_S = \frac{H_c(0)^2}{2} \tag{7}$$

Therefore:

$$H_c(0) = \sqrt{D(\epsilon_F)}\, \Delta_0 \tag{8}$$

Now, in free electron theory, the density of states is:

$$D(\epsilon_F) = \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \epsilon_F^{1/2} \tag{9}$$

Combining equations we have:

$$H_c(0) = \left[\frac{2^{1/4}}{e^\gamma} V^{1/2} \left(\frac{m}{\hbar^2}\right)^{3/4}\right] \epsilon_F^{1/4}\, kT_c \tag{10}$$

Where with T_c in Kelvin and ϵ_F in electron volts, a unit volume of 1 cm^3, and the critical field in Tesla at $T = 0$ for a weak-coupling BCS superconductor is:

$$H_c(0) = 0.00502157\, \epsilon_F^{1/4}\, T_c \tag{11}$$

Using this expression, and that for indium $T_c = 3.408$ K and $\epsilon_F = 8.63$ eV, and that for lead $T_c = 7.195$ K and $\epsilon_F = 9.47$ eV [7–9], we find the BCS theory predicts:

$$H_c^{\text{indium}}(0) = 0.02933 \text{ T} \tag{12}$$

$$H_c^{\text{lead}}(0) = 0.06338 \text{ T} \tag{13}$$

Where the accepted experimental values are [8–10]:

$$H_c^{\text{indium}}(0) = 0.02857 \text{ T} \tag{14}$$

$$H_c^{\text{lead}}(0) = 0.08034 \text{ T} \tag{15}$$

Therefore, we see that indium is in the weak coupling limit while lead is not. This is to be expected since the weak coupling limit is strictly, $T_c \to 0$ K, and 3.4 K is closer to 0 K than 7.2 K.

EXPERIMENTAL METHODS

Superconductors are perfect diamagnets, while normal metals are not. This means that if a superconductor is the core of an inductor, then as the temperature and field are varied, $H_c(T)$ can be determined by the change in resonant frequency of an LC circuit. In particular, a solenoid has inductance:

$$L = \mu \frac{N^2 A}{l} \tag{16}$$

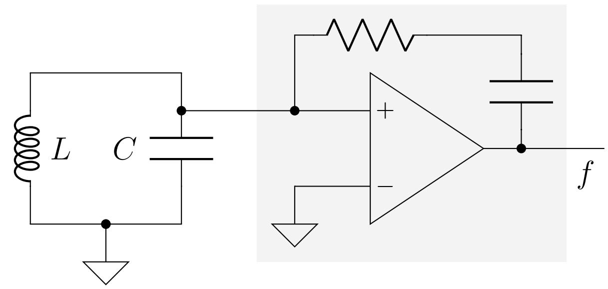

FIG. 1. LC resonant circuit with frequency selector shaded in gray. The resonant frequency of the LC circuit is $f = 1/2\pi\sqrt{LC}$. The circuit works by utilizing the fact that the impedance of the LC oscillator are only large at the resonant frequency, and so the opamp correspondingly amplifies this frequency f. There is a DC-blocking capacitor on the feedback of the opamp.

When the sample becomes superconducting the permittivity μ decreases, the inductance decreases, and the resonant frequency increases. When this happens is compared with the physical conditions to determine $H_c(T)$. The resonant frequency is measured using a circuit as shown in Fig. 1.

Temperature

Type I superconductors need to be cooled to less than 10 K to become superconducting. Since the ambient temperature is roughly 300 K, this requires substantial cooling and insulation from the environment. This was achieved using the setup illustrated in Fig. 2.

First the sample was inserted, then the system was isolated by the outer vacuum and vacuums with gas were created, then the system was cooled to 77K with liquid nitrogen, and finally liquid helium was introduced and cooled to 1.7 K by pumping on it. The temperature was then controlled by warming the sample to a temperature above 1.7 K with an inductive heater coil. Temperature was measured through the resistance of a thermometer as shown in Fig. 3.

Applied Field

We varied the applied field on the superconducting samples by varying the current in a superconducting solenoid immersed in the liquid helium as shown in Fig. 4. The current was supplied using a constant current source that was controlled by a voltage supplied by a lock-in amplifier controlled by a computer. We were given that the solenoid generated 0.1699 T/A and determined in Fig. 5 that the constant current source produced 0.2833 A/V supplied, so the applied field in terms of the voltage was:

$$H = 0.04813V \tag{17}$$

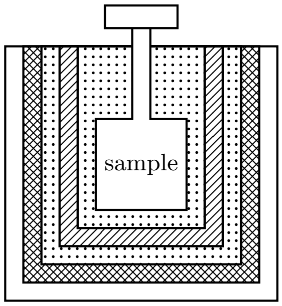

FIG. 2. Cross section of cooling apparatus used to cool sample as low as 1.7 K. From the outside in: pure vacuum, liquid nitrogen at 77 K, vacuum with a little helium gas, liquid helium pumped on by a vacuum at 4.2 to 1.7 K, vacuum with a little helium gas, and sample.

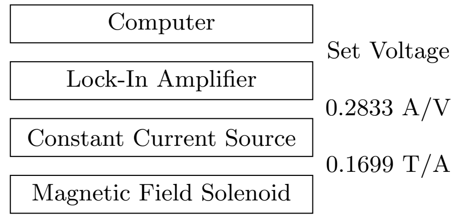

FIG. 4. The magnetic field experienced by the sample comes from a superconducting solenoid in the liquid helium that encompasses the sample. The field of this solenoid is given as 0.1699 T/A, where the current is produced by a constant current source. The current produced by the constant current source was determined to be 0.2833 A/V for the currents we used, see Fig. 5 below. While the voltage is produced by a lock-in amplifier in accordance to values set with a computer.

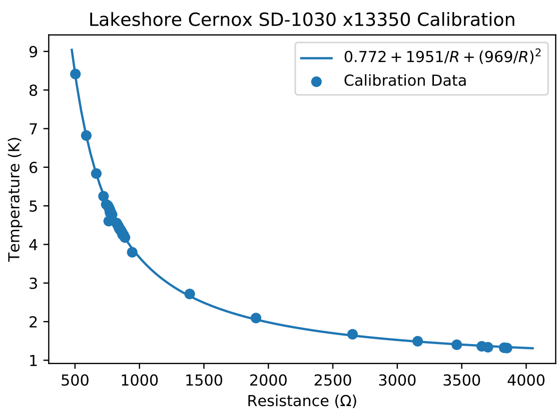

FIG. 3. Determination of temperature as a function of resistance from given temperature-resistance data for the thermometer/temperature controller.

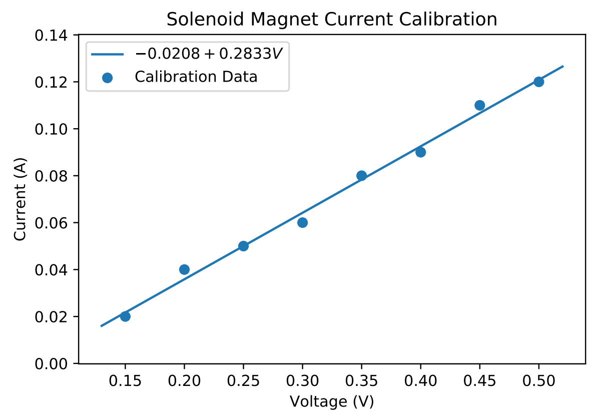

FIG. 5. Determination of the current produced by the constant current source for voltage provided. Note the small values and resulting quantization error in the measured current.

Measurement

We collected data using the following procedure:

1. Set temperature using resistance
2. Vary the field
    - Perform voltage sweep
    - Measure resonant frequency
3. Save data

We performed this procedure for eight temperatures from 1.7 K to the critical temperature of the indium and lead samples.

We then determined the critical field at each of these temperatures by performing a Gaussian fit to the derivative of the resonant frequency with respect to the applied field as shown in Fig. 6.

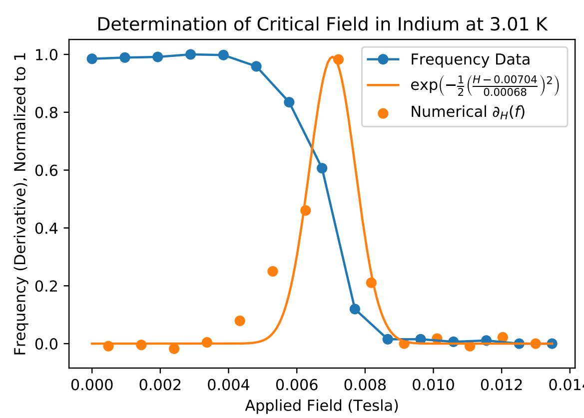

FIG. 6. The critical field was determined by numerically taking the derivative of the frequency with respect to the applied field and then performing a Gaussian fit. This method works for both sharp and noisy transitions, where we report $H_c = \text{mean} \pm \text{std}$ of the Gaussian.

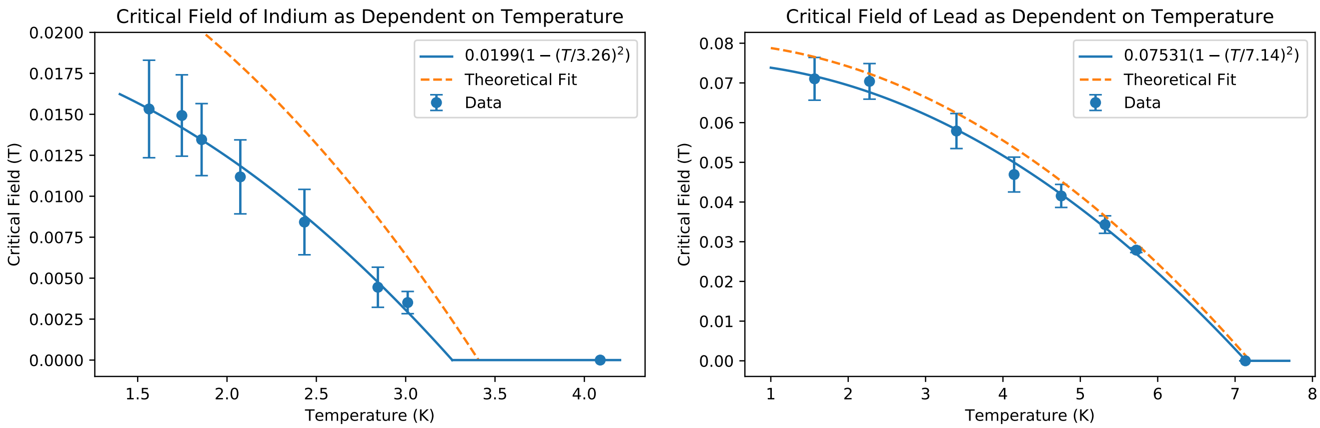

FIG. 7. Determination of critical field for indium and lead as a function of temperature. The fit is to the form of Eqn. 1, which gives the critical temperature and the critical field at zero temperature. We determine that the critical temperatures and fields of Indium and Lead as 3.261 ± 0.043 K, 0.01989 ± 0.00051 T and 7.14 ± 0.12 K, 0.07531 ± 0.0014 T respectively. The theoretical fit from the values in the literature is indicated in orange; differences are discussed in the error analysis section.

RESULTS

We observed that both indium and lead samples when placed as the core of an inductor experienced jumps in frequency, dependent on the magnetic field and temperature, consistent with our expectations for superconductors. From these jumps in frequency we constructed plots of $H_c(T)$ in Fig. 7 and extracted critical temperatures and fields of Indium and Lead as 3.261 ± 0.043 K, 0.01989 ± 0.00051 T and 7.14 ± 0.12 K, 0.07531 ± 0.0014 T respectively from fits to Eqn. 1.

Additionally, our calculations using known values from the literature indicate that Indium is a weak-coupling superconductor while lead is a strong-coupling superconductor. This means that Eqn. 1 is expected to be a good fit for indium and a less good fit for lead. Due to substantial systematic errors, we withhold comment on the appropriateness of Eqn. 1 to fit our data.

Error Analysis

While there are relatively small differences between the critical temperatures we determined and the values literature, there are much larger differences in the critical field. We believe that this difference in critical field could originate from sample dependent features/impurities, however, we suspect that since the difference between the measured and the literature increases linearly with increasing field that the conversion factor from the voltage to the applied field. In particular, the solenoid might produce less than the given 0.1699 T/A. We believe this systematic error might be as high as 20%.

The voltage produced by the lock-in amplifier and the frequency measurements were determined to four significant figures, with random uncertainties. Additionally, since the thermometer and sample were not exactly the same temperature, we estimate that the temperature was known to two significant figures. We conclude that the largest source of error was systematic, and with the determination of the applied magnetic field.

CONCLUSION

Superconductors are perfect diamagnets up to a critical field where they become normal metals. This is a result of a competition between Cooper pairs and magnetic potential energy. Here we used this fact to measure the critical field of indium and lead as a function of temperature by using the samples as the core of an inductor. We used this data within the BCS model to determine the critical temperature and field of our samples. Additionally using known properties of indium and lead, we showed that within the BCS model, indium is a weak-coupling superconductor while lead is a strong-coupling superconductor. If we were to repeat the experiment we would have liked to determine the current to field ratio of field-generating solenoid.

The method of detecting a resonant frequency is general to any detection application where resistance, capacitance, or inductance change and are used to set the resonant frequency of a circuit. In lab 1 we saw the application of an RC circuit to determine the phase transition in the dielectric constant of $BaTiO_3$. Another application using another LC circuit would be to measure the thickness of a deposited film on the surface of a capacitor to the nearest nanometer with a piezoelectric dielectric through shifts in the capacitance.

* stalkington@ucla.edu

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