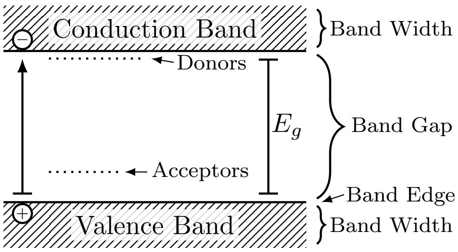
Semiconductors



The fraction of excited electrons is $n\exp(-E_g/k_BT)$, and at 300 K, $k_BT = 0.0259$ eV, and from these carriers, the resistivity is $\rho = m^*/ne^2\tau$.

Effective mass; $E = p^2/2m = \hbar^2k^2/2m_0$:

$$\hbar\frac{dk}{dt} = -eE,\ F = ma \implies \frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}$$

Effective masses, are usually related as:

$$m_e^* < m_h^* < m_e$$

Conductivity is the sum of e^- and h^+:

$$\sigma = \frac{ne^2\tau_e}{m_e} + \frac{pe^2\tau_h}{m_h}$$

Density of states is $D = dN/dk \cdot dk/dE$:

$$N_1 = 2k\left(\frac{L}{2\pi}\right)^1;\ N_2 = \pi k^2\left(\frac{L}{2\pi}\right)^2;\ N_3 = \frac{4\pi k^3}{3}\left(\frac{L}{2\pi}\right)^3$$

The density of conduction electrons:

$$n = \int_{E_g}^{\infty} dE\ f_n(E)D_n(E)$$

$$= 2\left(\frac{m_e k_B T}{2\pi\hbar^2}\right)^{3/2} e^{(\mu - E_g)/k_BT}$$

The chemical potential for undoped:

$$\mu = \frac{E_g}{2} + \frac{3k_BT}{4}\ln\left(\frac{m_h}{m_e}\right)$$

Use hydrogenic model for doped:

$$E = -\frac{e^2}{a_0} \implies E = -\frac{e^2}{\varepsilon r_0} \text{ for } r_0 = \frac{m_e\varepsilon a_0}{m^*}$$

At low temperatures, $k_BT \ll E_g$, for n_0 the coefficient before above in n:

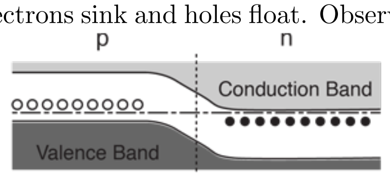
$$n = (n_0N_d)^{1/2}\exp(-E_d/2k_BT)$$

$$p = (n_0N_a)^{1/2}\exp(-E_a/2k_BT)$$

Law of mass action forces equilibrium:

$$np = 4\left(\frac{k_BT}{2\pi\hbar^2}\right)^3(m_em_h)^{3/2}e^{-E_g/k_BT}$$

At pn-junctions, chemical potentials must match, and charge polarizes: electrons sink and holes float. Observe:



Fermi Surfaces

To find the BZ of a given lattice:

- Use $\boldsymbol{a_i}\cdot\boldsymbol{b_j} = 2\pi\delta_{ij}$ to find $\boldsymbol{b_j}$
- Use the Wigner-Seitz Method

Then draw the empty lattice FS:

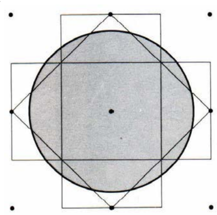
1 e$^-$	$k_F = \sqrt{2/\pi}\cdot\pi/a \approx 0.798\cdot\pi/a$
2 e$^-$	$k_F = \sqrt{4/\pi}\cdot\pi/a \approx 1.128\cdot\pi/a$
3 e$^-$	$k_F = \sqrt{6/\pi}\cdot\pi/a \approx 1.382\cdot\pi/a$

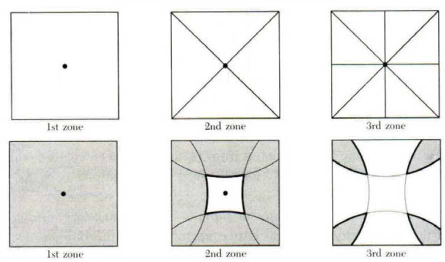
Fold up BZs and select a zone scheme. At each BZ edge, the FS deflects so that $FS \perp BZ$, and a slight gap forms.

Number of electrons in a square lattice:

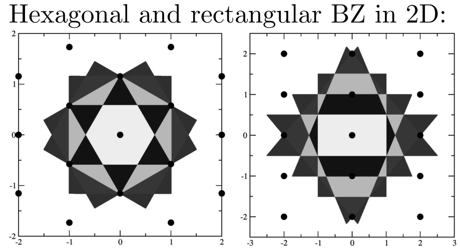
$$N = 1, 2, 3, \dots = \frac{V\cdot\text{spins}}{V_{\text{site}}} = \frac{\pi k_F^2\cdot 2}{\left(\frac{2\pi}{L}\right)^2}$$

An example of a Fermi Surface:





Hexagonal and rectangular BZ in 2D:



The De Haas–van Alphen Effect for the experimental determination of FS: magnetization, $M = \partial_B F$, is $\Delta(1/B)$ periodic where $\Delta = 2\pi e/\hbar A$ for cross sectional area A. Possibly Δ_1, Δ_2, …

In a magnetic field, motion is circular:

$$\hbar\frac{dk}{dt} = e\,\boldsymbol{v}\times\boldsymbol{B} \implies \omega = \omega_c = \frac{eB}{m^*}$$

Free electrons in B-fields form Landau levels, of $E_n = (n + \frac{1}{2})\hbar\omega_c$, levels expand with increases in B, with degeneracy $D = eL^2B/2\pi\hbar$, and total energy $E = D\hbar\omega_c s^2/2 + (s - \frac{1}{2})(N - sD)\hbar\omega_c$.

Quantum Hall Effect

Multiple ways to find the carrier concentration, including with oscillations:

$$\Delta\left(\frac{1}{B}\right) = \frac{2\pi e}{\hbar A};\ n = \frac{2A}{(2\pi)^2} \Rightarrow n = \frac{2e}{h\,\Delta(1/B)}$$

With an expression for Hall Resistance:

$$R_{xy} = \frac{B}{ne} \implies n = \frac{B}{R_{xy}e}$$

Equating Fermi to Landau, E_F to E_n:

$$E_F = \frac{n\hbar^2\pi}{m^*} \implies n = \frac{eB}{2\pi\hbar}\left(L - \frac{1}{2}\right)$$

The Hall Resistance is quantized:

$$R_{xy} = \frac{h}{e^2 i} \approx \frac{25.8}{i}\ \text{k}\Omega$$

Superconductivity I

SC is an electric effect where $\rho = 0$ for a phase bounded by T_c and field H_c.

Meissner effect of perfect diamagnetism:

$$\boldsymbol{B} = \boldsymbol{H}_{\text{app}} + \mu_0\boldsymbol{M} = 0 \implies \boldsymbol{M} = -\boldsymbol{H}/\mu_0$$

London equation for penetration depth:

$$\nabla^2 B = \frac{B}{\lambda_c^2};\ \lambda_c = \sqrt{\frac{m}{ne^2\mu_0}} \sim 10^2\text{–}10^3\ \text{Å}$$

Gibbs energy, $F = U - TS = -k_BT\ln(Z)$:

$$F_N(0,T) - F_S(0,T) = \frac{H_c^2(T)}{2\mu_0}$$

Heat capacity is discontinuous at T_c:

$$(C_S - C_N)\big|_{T=T_c} = \frac{T_c}{\mu_0}\left(\frac{dH_c(T)}{dT}\right)^2_{T=T_c}$$

The intrinsic coherence length of ψ:

$$\xi_0 = 2\hbar v_F/\pi E_g$$

In impure SC, for mean free path ℓ:

$$\xi \approx \sqrt{\xi_0\ell};\qquad \lambda \approx \lambda_c\sqrt{\xi_0/\ell}$$

Critical fields may be estimated as:

$$H_{c1} \approx \Phi_0/\pi\lambda^2;\qquad H_{c2} \approx \Phi_0/\pi\xi^2$$

Natural Constants

$$\alpha = 137.035999^{-1}$$

$$a_0 = 5.29177211\times 10^{-11}\ \text{m}$$

$$c = 2.99792458\times 10^{8}\ \text{m/s}$$

$$4\pi\varepsilon_0 = 1.11265006\times 10^{-10}\ \text{C/(V}\cdot\text{m)}$$

$$e = 1.60217662\times 10^{-19}\ \text{C}$$

$$h = 6.62607004\times 10^{-34}\ \text{J}\cdot\text{s}$$

$$\hbar = 1.05457180\times 10^{-34}\ \text{J}\cdot\text{s}$$

$$k_B = 1.38064852\times 10^{-23}\ \text{J/K}$$

$$m_e = 9.10938356\times 10^{-31}\ \text{kg}$$

$$N_A = 6.02214086\times 10^{23}$$

Superconductivity II

BCS theory: electron phonon pairing creates bosons that all condense to the same energy state. This state is protected by an energy gap and particles cannot individually gain or lose energy.

e^- bound by energy Δ for attraction U, which is an energy gap of $\epsilon_{\mathrm{F}} \pm \Delta$:
$\Delta \approx -2\hbar\omega \exp(-2/UD(\epsilon_{\mathrm{F}})) \sim 1$ meV

BCS theory predicts an energy gap of:
$$E_g = 2\Delta = 3.5 k_{\mathrm{B}} T_c$$

Macroscopic quantum wave function:
$$\psi = \sqrt{n}\, e^{i\theta}$$

Electron tunneling between SC gives DC Josephson effect current, $\delta = \theta_2 - \theta_1$:
$$J = J_0 \sin(\delta)$$

AC effect $\omega = 2eV/\hbar \sim 500$ MHz/μV:
$$J = J_0 \sin(\delta - \omega t)$$

In SC rings there is flux quantization:
$$\Phi_0 = 2\pi\hbar/q = h/2e$$

SQUID behaves like Aharonov-Bohm:
$J = 2J_0 \sin(\delta)\cos(e\Phi/\hbar)$; $\Phi = \text{area} \cdot \|\boldsymbol{B}\|$

Dia- and Paramagnetism

Magnetic susceptibility is $\chi \equiv \mu_0 M/H$. $\chi < 0 \rightarrow$ dia, and $\chi > 0 \rightarrow$ paramagnet. SC are perfect diamagnets with $\chi = -1$. The typical values are $\chi \sim 10^{-3}$–10^{-4}.

Diamagnetism arises in inert gasses, and is captured by Larmor precession:
$$\chi = \frac{\mu_0 (N\mu)}{B} = -\frac{Ze^2\mu_0}{6m} N\langle r^2\rangle$$

Atoms with unpaired electrons, atomic Oxygen, and metals are paramagnetic. The energy is for $\mu_B = e\hbar/2m$:
$$E = -\boldsymbol{\mu} \cdot \boldsymbol{B} = g\mu_B\, \boldsymbol{J} \cdot \boldsymbol{B}$$

In two state systems with $x \equiv \mu B/k_{\mathrm{B}}T$:
$$M = \sum N_i \mu_i = N\mu \frac{e^x - e^{-x}}{e^x + e^{-x}} = N\mu \tanh(x)$$

Curie-Brillouin law for $2J+1$ levels, $x \equiv gJ\mu B/k_{\mathrm{B}}T$, for high temperatures:
$$M = NgJ\mu_B B_J(x) \propto T^{-1}$$

In terms of effective Bohr magnetons:
$$M = \frac{N}{V}\frac{p^2\mu_B^2}{3k_{\mathrm{B}}}\left(\frac{H}{T}\right);\ p = g(J(J+1))^{1/2}$$

Where the Landau g-factor is given as:
$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

Constant Pauli magnetism of metals is:
$M = D(\epsilon_{\mathrm{F}})\mu_B^2 B = 3N\mu^2/2\epsilon_{\mathrm{F}}$ for low T

Electronic State Symbols

Hund's Rules empirically usually work:

- Total spin S_z is maximized
- Orbital momentum, L_z is too
- Total orbital angular momentum:
  - ⋆ $J = |L - S|$ less than half full
  - ⋆ $J = |L + S|$ more than half full

Symbols are written as ${}^{2S+1}L_J$, where:

$L =$	S	P	D	F	G	H	I	J
	↕	↕	↕	↕	↕	↕	↕	↕
$L =$	0	1	2	3	4	5	6	7

(Anti)-Ferromagnetism

Curie-Weiss Law around paramagnetic-ferromagnetic transition temperature:
$$\chi = C/(T - T_c)$$

Exchange interactions (d-orbital) make spins align, as in the Heisenberg model:
$E_{\mathrm{ex}} = -2J\boldsymbol{S}_i \cdot \boldsymbol{S}_j$; with $J \sim 10$ meV
$J < 0 \rightarrow$ anti, and $J > 0 \rightarrow$ ferromagnet.

The Hamiltonian for this system is:
$$H = -\tfrac{1}{2}\sum_{ij}(J\sigma_i\sigma_j) + g\mu_B B \sum_j \sigma_j$$

Approximated using mean field theory:

- Solve exactly for a small region
- Approximate the outside in mean
- Make the solution self-consistent

With $m = M/N\mu$, and $t = k_{\mathrm{B}}T/N\mu^2\lambda$, next solve the transcendental equation:
$m = \tanh(m/t) \Rightarrow$ solns if $t < 1$ i.e. $T < T_c$

Taylor Expansion to find susceptibility:
$$\chi \sim \begin{cases} (T_c - T)^{1/2} & T < T_c \\ 1/(T - T_c) & T_c < T \end{cases}$$

Spins in ferromagnets may behave like phonons, where $i = p$, and $j = p+1$. For small oscillations, $S_z \approx S$, so with the equation of motion $dJ/dt = \boldsymbol{\mu}_p \times \boldsymbol{B}_p$:
$$S_p^{x/y} = a\exp(i(pka - \omega t))$$
Where $\omega = 4Js/\hbar \cdot (1 - \cos(ka))$, which is for long wavelengths, $\hbar\omega = (2JSa^2)k^2$.

For anti-ferromagnets, $T_c \rightarrow T_{\mathrm{N}} = \mu C$, where it has been found that in $T_{\mathrm{N}} < T$:
$$\chi = 2C/(T + T_{\mathrm{N}})$$
Break AFM into two sublattices, and magnons solved with determinantal eqn.
$$M = (C/\mu_B)(H/(T + T_{\mathrm{N}}))$$

Magnetic domain walls have exchange energy density $\sigma_{\mathrm{ex}} = JS^2(\pi/Na)^2 \cdot N$, anisotropy energy $\sigma_{\mathrm{an}} = KNa$, so by infimizing energy, $N = (\pi^2 JS^2/Ka^3)^{1/2}$, so, per wall area $\sigma_w = 2\pi(KJS^2/a)^{1/2}$.

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